Subject – Math AA(Standard Level)
Topic - Number and Algebra
Year - May 2021 – Nov 2022
Paper -1
Answers

Question 1

(a) attempting to use the change of base rule **M1**

$$\log_9(\cos 2x+2)=\frac{\log_3(\cos 2x+2)}{\log_3 9}$$ **A1**

$$=\frac{1}{2}\log_3(\cos 2x+2)$$ **A1**

$$=\log_3\sqrt{\cos 2x+2}$$ **AG**

[3 marks]

(b) $\log_3(2\sin x)=\log_3\sqrt{\cos 2x+2}$

$2\sin x=\sqrt{\cos 2x+2}$ **M1**

$4\sin^2 x=\cos 2x+2$ (or equivalent) **A1**

use of $\cos 2x=1-2\sin^2 x$ ***(M1)***

$6\sin^2 x=3$

$\sin x=(\pm)\frac{1}{\sqrt{2}}$ **A1**

$x=\frac{\pi}{4}$ **A1**

Note: Award ***A0*** if solutions other than $x=\frac{\pi}{4}$ are included.

[5 marks]

Total [8 marks]

Question 2

(a) attempting to expand the LHS *(M1)*

$\text{LHS} = (4n^2 - 4n + 1) + (4n^2 + 4n + 1)$ ***A1***

$= 8n^2 + 2 (= \text{RHS})$ ***AG***

[2 marks]

(b) ***METHOD 1***

recognition that $2n-1$ and $2n+1$ represent two consecutive odd integers (for $n \in \mathbb{Z}$) ***R1***

$8n^2 + 2 = 2(4n^2 + 1)$ ***A1***

valid reason *eg* divisible by 2 (2 is a factor) ***R1***

so the sum of the squares of any two consecutive odd integers is even ***AG***

[3 marks]

METHOD 2

recognition, *eg* that n and $n+2$ represent two consecutive odd integers (for $n \in \mathbb{Z}$) ***R1***

$n^2 + (n+2)^2 = 2(n^2 + 2n + 2)$ ***A1***

valid reason *eg* divisible by 2 (2 is a factor) ***R1***

so the sum of the squares of any two consecutive odd integers is even ***AG***

[3 marks]

Total [5 marks]

Question 3

METHOD 1 (finding u_1 first, from S_8)

$4(u_1 + 8) = 8$ ***(A1)***

$u_1 = -6$ ***A1***

$u_1 + 7d = 8$ OR $4(2u_1 + 7d) = 8$ (may be seen with their value of u_1) ***(A1)***

attempt to substitute their u_1 ***(M1)***

$d = 2$ ***A1***

METHOD 2 (solving simultaneously)

$u_1 + 7d = 8$ ***(A1)***

$4(u_1 + 8) = 8$ OR $4(2u_1 + 7d) = 8$ OR $u_1 = -3d$ ***(A1)***

attempt to solve linear or simultaneous equations ***(M1)***

$u_1 = -6, d = 2$ ***A1A1***

[5 marks]

Question 4

(a) 3×10^4 OR 30000 (km) (accept $3\bullet10^4$) ***A1***

[1 mark]

(b) $\frac{4}{3}\pi\left(3\times10^4\right)^3$ OR $\frac{4}{3}\pi\left(30000\right)^3$ ***(A1)***

$=\frac{4}{3}\pi\times27\times10^{12}\left(=\pi\left(36\times10^{12}\right)\right)$ OR $=\frac{4}{3}\pi\times27000000000000$ ***(A1)***

$=\pi\left(3.6\times10^{13}\right)$ (km^3) OR $a=3.6, k=13$ ***A1***

[3 marks]
Total [4 marks]

Question 5

EITHER

attempt to use the binomial expansion of $(x+k)^7$ ***(M1)***

${}^7C_0x^7k^0+{}^7C_1x^6k^1+{}^7C_2x^5k^2+\ldots$ (or ${}^7C_0k^7x^0+{}^7C_1k^5x^1+{}^7C_2k^5x^2+\ldots$)

identifying the correct term ${}^7C_2x^5k^2$ (or ${}^7C_5k^2x^5$) ***(A1)***

OR

attempt to use the general term ${}^7C_rx^rk^{7-r}$ (or ${}^7C_rk^rx^{7-r}$) ***(M1)***

$r=2$ (or $r=5$) ***(A1)***

THEN

${}^7C_2=21$ (or ${}^7C_5=21$) (seen anywhere) ***(A1)***

$21x^5k^2=63x^5$ ($21k^2=63$, $k^2=3$) ***A1***

$k=\pm\sqrt{3}$ ***A1***

Note: If working shown, award ***M1A1A1A1A0*** for $k=\sqrt{3}$.

[5 marks]

Question 6

attempt to subtract squares of integers *(M1)*

$(n+1)^2 - n^2$

EITHER

correct order of subtraction and correct expansion of $(n+1)^2$, seen anywhere ***A1A1***

$= n^2 + 2n + 1 - n^2 \, (= 2n+1)$

OR

correct order of subtraction and correct factorization of difference of squares ***A1A1***

$= (n+1-n)(n+1+n)(= 2n+1)$

THEN

$= n + n + 1$ =RHS ***A1***

Note: Do not award final ***A1*** unless all previous working is correct.

which is the sum of n and $n+1$ ***AG***

Note: If expansion and order of subtraction are correct, award full marks for candidates who find the sum of the integers as $2n+1$ and then show that the difference of the squares (subtracted in the correct order) is $2n+1$.

[4 marks]

Question 6

(a) $f\left(\frac{2}{3}\right)=4$ OR $a^{\frac{2}{3}}=4$ **(M1)**

$a=4^{\frac{3}{2}}$ OR $a=\left(2^2\right)^{\frac{3}{2}}$ OR $a^2=64$ OR $\sqrt[3]{a}=2$ **A1**

$a=8$ **AG**

[2 marks]

(b) $f^{-1}(x)=\log_8 x$ **A1**

> **Note**: Accept $f^{-1}(x)=\log_a x$.
>
> Accept any equivalent expression for f^{-1} e.g. $f^{-1}(x)=\frac{\ln x}{\ln 8}$.

[1 mark]

(c) correct substitution **(A1)**

$\log_8 \sqrt{32}$ OR $8^x=32^{\frac{1}{2}}$

correct working involving log/index law **(A1)**

$\frac{1}{2}\log_8 32$ OR $\frac{5}{2}\log_8 2$ OR $\log_8 2=\frac{1}{3}$ OR $\log_2 2^{\frac{5}{2}}$ OR $\log_2 8=3$ OR $\frac{\ln 2^{\frac{5}{2}}}{\ln 2^3}$ OR $2^{3x}=2^{\frac{5}{2}}$

$f^{-1}\left(\sqrt{32}\right)=\frac{5}{6}$ **A1**

[3 marks]

(d) (i) **METHOD 1**

equating a pair of differences *(M1)*

$u_2 - u_1 = u_4 - u_3 (= u_3 - u_2)$

$\log_8 p - \log_8 27 = \log_8 125 - \log_8 q$

$\log_8 125 - \log_8 q = \log_8 q - \log_8 p$

$\log_8\left(\frac{p}{27}\right) = \log_8\left(\frac{125}{q}\right),\ \log_8\left(\frac{125}{q}\right) = \log_8\left(\frac{q}{p}\right)$ ***A1A1***

$\frac{p}{27} = \frac{125}{q}$ and $\frac{125}{q} = \frac{q}{p}$ ***A1***

$27, p, q$ and 125 are in geometric sequence ***AG***

Note: If candidate assumes the sequence is geometric, award no marks for part (i). If $r = \frac{5}{3}$ has been found, this will be awarded marks in part (ii).

METHOD 2

expressing a pair of consecutive terms, in terms of d *(M1)*

$p = 8^d \times 27$ and $q = 8^{2d} \times 27$ OR $q = 8^{2d} \times 27$ and $125 = 8^{3d} \times 27$

two correct pairs of consecutive terms, in terms of d ***A1***

$\frac{8^d \times 27}{27} = \frac{8^{2d} \times 27}{8^d \times 27} = \frac{8^{3d} \times 27}{8^{2d} \times 27}$ (must include 3 ratios) ***A1***

all simplify to 8^d ***A1***

$27, p, q$ and 125 are in geometric sequence ***AG***

(ii) **METHOD 1 (geometric, finding *r*)**

$u_4 = u_1 r^3$ OR $125 = 27(r)^3$ ***(M1)***

$r = \frac{5}{3}$ (seen anywhere) ***A1***

$p = 27r$ OR $\frac{125}{q} = \frac{5}{3}$ ***(M1)***

$p = 45,\ q = 75$ ***A1A1***

METHOD 2 (arithmetic)

$u_4 = u_1 + 3d$ OR $\log_8 125 = \log_8 27 + 3d$ ***(M1)***

$d = \log_8\left(\frac{5}{3}\right)$ (seen anywhere) ***A1***

$\log_8 p = \log_8 27 + \log_8\left(\frac{5}{3}\right)$ OR $\log_8 q = \log_8 27 + 2\log_8\left(\frac{5}{3}\right)$ ***(M1)***

$p = 45,\ q = 75$ ***A1A1***

METHOD 3 (geometric using proportion)

recognizing proportion ***(M1)***

$pq = 125 \times 27$ OR $q^2 = 125p$ OR $p^2 = 27q$

two correct proportion equations ***A1***

attempt to eliminate either p or q ***(M1)***

$q^2 = 125 \times \frac{125 \times 27}{q}$ OR $p^2 = 27 \times \frac{125 \times 27}{p}$

$p = 45,\ q = 75$ ***A1A1***

[9 marks]
Total [15 marks]

Question 7

(a) (i) **EITHER**

attempt to use a ratio from consecutive terms **_M1_**

$$\frac{p\ln x}{\ln x}=\frac{\frac{1}{3}\ln x}{p\ln x} \quad \text{OR} \quad \frac{1}{3}\ln x=(\ln x)r^2 \quad \text{OR} \quad p\ln x=\ln x\left(\frac{1}{3p}\right)$$

> **Note:** Candidates may use $\ln x^1+\ln x^p+\ln x^{\frac{1}{3}}+\ldots$ and consider the powers of x in geometric sequence.
>
> Award ***M1*** for $\frac{p}{1}=\frac{\frac{1}{3}}{p}$.

OR

$r=p$ and $r^2=\frac{1}{3}$ **_M1_**

THEN

$p^2=\frac{1}{3}$ OR $r=\pm\frac{1}{\sqrt{3}}$ **_A1_**

$p=\pm\frac{1}{\sqrt{3}}$ **_AG_**

> **Note:** Award ***M0A0*** for $r^2=\frac{1}{3}$ or $p^2=\frac{1}{3}$ with no other working seen.

(ii) $\dfrac{\ln x}{1-\frac{1}{\sqrt{3}}}$ $\left(=3+\sqrt{3}\right)$ ***(A1)***

$\ln x=3-\frac{3}{\sqrt{3}}+\sqrt{3}-\frac{\sqrt{3}}{\sqrt{3}}$ OR $\ln x=3-\sqrt{3}+\sqrt{3}-1$ $(\Rightarrow \ln x=2)$ **_A1_**

$x=\mathrm{e}^2$ **_A1_**

[5 marks]

(b) (i) **METHOD 1**

attempt to find a difference from consecutive terms or from u_2 ***M1***

correct equation ***A1***

$p\ln x-\ln x=\frac{1}{3}\ln x-p\ln x$ OR $\frac{1}{3}\ln x=\ln x+2\left(p\ln x-\ln x\right)$

Note: Candidates may use $\ln x^1+\ln x^p+\ln x^{\frac{1}{3}}+\ldots$ and consider the powers of x in arithmetic sequence. Award ***M1A1*** for $p-1=\frac{1}{3}-p$.

$2p\ln x=\frac{4}{3}\ln x \quad \left(\Rightarrow 2p=\frac{4}{3}\right)$ ***A1***

$p=\frac{2}{3}$ ***AG***

METHOD 2

attempt to use arithmetic mean $u_2=\frac{u_1+u_3}{2}$ ***M1***

$p\ln x=\frac{\ln x+\frac{1}{3}\ln x}{2}$ ***A1***

$2p\ln x=\frac{4}{3}\ln x \quad \left(\Rightarrow 2p=\frac{4}{3}\right)$ ***A1***

$p=\frac{2}{3}$ ***AG***

METHOD 3

attempt to find difference using u_3 **M1**

$$\frac{1}{3}\ln x = \ln x + 2d \quad \left(\Rightarrow d = -\frac{1}{3}\ln x\right)$$

$u_2 = \ln x + \frac{1}{2}\left(\frac{1}{3}\ln x - \ln x\right)$ OR $p\ln x - \ln x = -\frac{1}{3}\ln x$ **A1**

$p\ln x = \frac{2}{3}\ln x$ **A1**

$p = \frac{2}{3}$ **AG**

(ii) $d = -\frac{1}{3}\ln x$ **A1**

(iii) **METHOD 1**

$$S_n = \frac{n}{2}\left[2\ln x + (n-1)\times\left(-\frac{1}{3}\ln x\right)\right]$$

attempt to substitute into S_n and equate to $-3\ln x$ ***(M1)***

$$\frac{n}{2}\left[2\ln x + (n-1)\times\left(-\frac{1}{3}\ln x\right)\right] = -3\ln x$$

correct working with S_n (seen anywhere) ***(A1)***

$\frac{n}{2}\left[2\ln x - \frac{n}{3}\ln x + \frac{1}{3}\ln x\right]$ OR $n\ln x - \frac{n(n-1)}{6}\ln x$ OR
$\frac{n}{2}\left(\ln x + \left(\frac{4-n}{3}\right)\ln x\right)$

correct equation without $\ln x$ ***A1***

$\frac{n}{2}\left(\frac{7}{3} - \frac{n}{3}\right) = -3$ OR $n - \frac{n(n-1)}{6} = -3$ or equivalent

Note: Award as above if the series $1 + p + \frac{1}{3} + \ldots$ is considered leading to
$\frac{n}{2}\left(\frac{7}{3} - \frac{n}{3}\right) = -3$.

attempt to form a quadratic $= 0$ ***(M1)***

$n^2 - 7n - 18 = 0$

attempt to solve their quadratic ***(M1)***

$(n-9)(n+2) = 0$

$n = 9$ ***A1***

METHOD 2

listing the first 7 terms of the sequence **(A1)**

$$\ln x + \frac{2}{3}\ln x + \frac{1}{3}\ln x + 0 - \frac{1}{3}\ln x - \frac{2}{3}\ln x - \ln x + \ldots$$

recognizing first 7 terms sum to 0 **M1**

8th term is $-\frac{4}{3}\ln x$ **(A1)**

9th term is $-\frac{5}{3}\ln x$ **(A1)**

sum of 8th and 9th terms $= -3\ln x$ **(A1)**

$n = 9$ **A1**

[10 marks]

Total [15 marks]

Question 8

(a) **EITHER**

recognises the required term (or coefficient) in the expansion ***(M1)***

$bx^5 = {}^7C_2 x^5 1^2$ OR $b = {}^7C_2$ OR ${}^7C_5$

$b = \frac{7!}{2!5!}\left(= \frac{7!}{2!(7-2)!}\right)$

correct working ***A1***

$\frac{7\times6\times5\times4\times3\times2\times1}{2\times1\times5\times4\times3\times2\times1}$ OR $\frac{7\times6}{2!}$ OR $\frac{42}{2}$

OR

lists terms from row 7 of Pascal's triangle ***(M1)***

$1, 7, 21, \ldots$ ***A1***

THEN

$b = 21$ ***AG***

[2 marks]

(b) $a = 7$ ***(A1)***

correct equation ***A1***

$21x^5 = \frac{ax^6 + 35x^4}{2}$ OR $21x^5 = \frac{7x^6 + 35x^4}{2}$

correct quadratic equation ***A1***

$7x^2 - 42x + 35 = 0$ OR $x^2 - 6x + 5 = 0$ (or equivalent)

valid attempt to solve **their** quadratic ***(M1)***

$(x-1)(x-5) = 0$ OR $x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(5)}}{2(1)}$

$x = 1, x = 5$ ***A1***

Note: Award final ***A0*** for obtaining $x = 0, x = 1, x = 5$.

[5 marks]

Total [7 marks]

Question 9

(a) $(n-1)+n+(n+1)$ **(A1)**

$=3n$ **A1**

which is always divisible by 3 **AG**

[2 marks]

(b) $(n-1)^2+n^2+(n+1)^2$ (= $n^2-2n+1+n^2+n^2+2n+1$) **A1**

attempts to expand either $(n-1)^2$ or $(n+1)^2$ (do not accept n^2-1 or n^2+1) **(M1)**

$=3n^2+2$ **A1**

demonstrating recognition that 2 is not divisible by 3 or $\frac{2}{3}$ seen after correct

expression divided by 3 **R1**

$3n^2$ is divisible by 3 and so $3n^2+2$ is never divisible by 3

OR the first term is divisible by 3, the second is not

OR $3\left(n^2+\frac{2}{3}\right)$ OR $\frac{3n^2+2}{3}=n^2+\frac{2}{3}$

hence the sum of the squares is never divisible by 3 **AG**

[4 marks]

Total [6 marks]

Question 10

(a) $u_1 = 12$ ***A1***

[1 mark]

(b) $15 - 3n = -33$ ***(A1)***

$n = 16$ ***A1***

[2 marks]

(c) valid approach to find d ***(M1)***

$u_2 - u_1 = 9 - 12$ OR recognize gradient is -3 OR attempts to solve $-33 = 12 + 15d$

$d = -3$ ***A1***

[2 marks]

Total [5 marks]

Question 11

(a) (i) valid approach to find the required logarithm **(M1)**

$2^x = \frac{1}{16}$ OR $2^x = 2^{-4}$ OR $\frac{1}{16} = 2^{-4}$ OR $\log_2 1 - \log_2 16$

$\log_2 \frac{1}{16} = -4$ **A1**

(ii) valid approach to find the required logarithm **(M1)**

$9^x = 3$ OR $3^{2x} = 3$ OR $3 = 9^{\frac{1}{2}}$ OR $\frac{\log_3 3}{\log_3 9}$

$\log_9 3 = \frac{1}{2}$ **A1**

(iii) $\left(\sqrt{3}\right)^x = 81$ OR $\frac{\log_3 81}{\log_3 \sqrt{3}}$ **(A1)**

$(3)^{\frac{x}{2}} = 3^4$ OR $\frac{x}{2} = 4$ OR $\frac{4}{\frac{1}{2}}$ **(A1)**

$x = 8$ **A1**

[7 marks]

(b) (i)

Note: There are many valid approaches to the question, and the steps may be seen in different ways. Some possible methods are given here, but candidates may use a combination of one or more of these methods.

In all methods, the final ***A*** mark is awarded for working which leads directly to the ***AG***.

METHOD 1

$(ab)^3 = a$ ***(A1)***

attempt to isolate b or a power of b ***(M1)***

correct working ***(A1)***

$b = \dfrac{a}{a^3b^2}$ OR $b^3 = a^{-2}$ OR $b^{-1} = (ab)^2$ OR $b^3 = \dfrac{1}{a^2}$

$b = \dfrac{1}{a^2b^2}$ OR $b = (ab)^{-2}$ OR $3\log_{ab} b = -2\log_{ab} a$ OR $-\log_{ab} b = 2\log_{ab} ab$ ***A1***

$\log_{ab} b = -2$ ***AG***

METHOD 3

$\log_{ab} a = 3$

writing in terms of base a **(M1)**

$\dfrac{\log_a a}{\log_a ab}(=3)$

correct application of log rules **(A1)**

$\dfrac{\log_a a}{\log_a a + \log_a b}(=3)$ OR $\dfrac{1}{1+\log_a b}(=3)$ OR $3\log_a b = -2$ OR

$\log_a b = -\dfrac{2}{3}$

writing $\log_{ab} b$ in terms of base a **(A1)**

$\log_{ab} b = \dfrac{\log_a b}{\log_a a + \log_a b}$

correct working **A1**

$\log_{ab} b = \dfrac{-\frac{2}{3}}{1-\frac{2}{3}}$ OR $\dfrac{\left(-\frac{2}{3}\right)}{\left(\frac{1}{3}\right)}$

$\log_{ab} b = -2$ **AG**

METHOD 4

$\log_{ab} ab = 1$ **A2**

$\log_{ab} a + \log_{ab} b = 1$ ***(A1)***

$3 + \log_{ab} b = 1$ ***A1***

$\log_{ab} b = -2$ ***AG***

(ii) applying the quotient rule or product rule for logs

$\log_{ab} \frac{\sqrt[3]{a}}{\sqrt{b}} = \log_{ab} \sqrt[3]{a} - \log_{ab} \sqrt{b}$ OR $\log_{ab} \frac{\sqrt[3]{a}}{\sqrt{b}} = \log_{ab} \sqrt[3]{a} + \log_{ab} \frac{1}{\sqrt{b}}$ ***(A1)***

correct working ***(A1)***

$= \frac{1}{3}\log_{ab} a - \frac{1}{2}\log_{ab} b$ OR $\log_{ab} ab - \log_{ab} \sqrt{b}$

$= \frac{1}{3} \cdot 3 - \frac{1}{2}(-2)$ ***(A1)***

$= 2$ ***A1***

Note: Award ***A1A0A0A1*** for a correct answer with no working.

[8 marks]

Total [15 marks]