Subject – Math AA(Standard Level)
Topic - Number and Algebra
Year - May 2021 – Nov 2022
Paper -1
Questions

Question 1

[Maximum mark: 8]

(a) Show that $\log_9(\cos 2x + 2) = \log_3 \sqrt{\cos 2x + 2}$. [3]

(b) Hence or otherwise solve $\log_3(2\sin x) = \log_9(\cos 2x + 2)$ for $0 < x < \frac{\pi}{2}$. [5]

Question 2

[Maximum mark: 5]

(a) Show that $(2n-1)^2 + (2n+1)^2 = 8n^2 + 2$, where $n \in \mathbb{Z}$. [2]

(b) Hence, or otherwise, prove that the sum of the squares of any two consecutive odd integers is even. [3]

Question 3

[Maximum mark: 5]

Consider an arithmetic sequence where $u_8 = S_8 = 8$. Find the value of the first term, u_1, and the value of the common difference, d.

Question 4

[Maximum mark: 4]

The diameter of a spherical planet is $6 \times 10^4\,\text{km}$.

(a) Write down the radius of the planet. [1]

The volume of the planet can be expressed in the form $\pi(a \times 10^k)\,\text{km}^3$ where $1 \le a < 10$ and $k \in \mathbb{Z}$.

(b) Find the value of a and the value of k. [3]

Question 5

[Maximum mark: 4]

Consider two consecutive positive integers, n and $n+1$.

Show that the difference of their squares is equal to the sum of the two integers.

Question 6

[Maximum mark: 15]

Consider the series $\ln x + p\ln x + \frac{1}{3}\ln x + \ldots$, where $x \in \mathbb{R}$, $x > 1$ and $p \in \mathbb{R}$, $p \neq 0$.

(a) Consider the case where the series is geometric.

(i) Show that $p = \pm\frac{1}{\sqrt{3}}$.

(ii) Given that $p > 0$ and $S_\infty = 3 + \sqrt{3}$, find the value of x. [5]

(b) Now consider the case where the series is arithmetic with common difference d.

(i) Show that $p = \frac{2}{3}$.

(ii) Write down d in the form $k\ln x$, where $k \in \mathbb{Q}$.

(iii) The sum of the first n terms of the series is $-3\ln x$.

Find the value of n. [10]

Question 7

[Maximum mark: 15]

Consider the function $f(x) = a^x$ where $x, a \in \mathbb{R}$ and $x > 0$, $a > 1$.

The graph of f contains the point $\left(\frac{2}{3}, 4\right)$.

(a) Show that $a = 8$. [2]

(b) Write down an expression for $f^{-1}(x)$. [1]

(c) Find the value of $f^{-1}\left(\sqrt{32}\right)$. [3]

(d) Consider the arithmetic sequence $\log_8 27$, $\log_8 p$, $\log_8 q$, $\log_8 125$, where $p > 1$ and $q > 1$.

(i) Show that 27, p, q and 125 are four consecutive terms in a geometric sequence.

(ii) Find the value of p and the value of q. [9]

Question 8

[Maximum mark: 7]

Consider the binomial expansion $(x+1)^7 = x^7 + ax^6 + bx^5 + 35x^4 + \ldots + 1$ where $x \neq 0$ and $a, b \in \mathbb{Z}^+$.

(a) Show that $b = 21$. [2]

The third term in the expansion is the mean of the second term and the fourth term in the expansion.

(b) Find the possible values of x. [5]

Question 9

[Maximum mark: 6]

Consider any three consecutive integers, $n-1$, n and $n+1$.

(a) Prove that the sum of these three integers is always divisible by 3. [2]

(b) Prove that the sum of the squares of these three integers is never divisible by 3. [4]

Question 10

[Maximum mark: 5]

The n^{th} term of an arithmetic sequence is given by $u_n = 15 - 3n$.

(a) State the value of the first term, u_1. [1]

(b) Given that the n^{th} term of this sequence is -33, find the value of n. [2]

(c) Find the common difference, d. [2]

Question 11

[Maximum mark: 15]

(a) Calculate the value of each of the following logarithms:

(i) $\log_2 \frac{1}{16}$;

(ii) $\log_9 3$;

(iii) $\log_{\sqrt{3}} 81$. [7]

(b) It is given that $\log_{ab} a = 3$, where $a, b \in \mathbb{R}^+, ab \neq 1$.

(i) Show that $\log_{ab} b = -2$.

(ii) Hence find the value of $\log_{ab} \frac{\sqrt[3]{a}}{\sqrt{b}}$. [8]