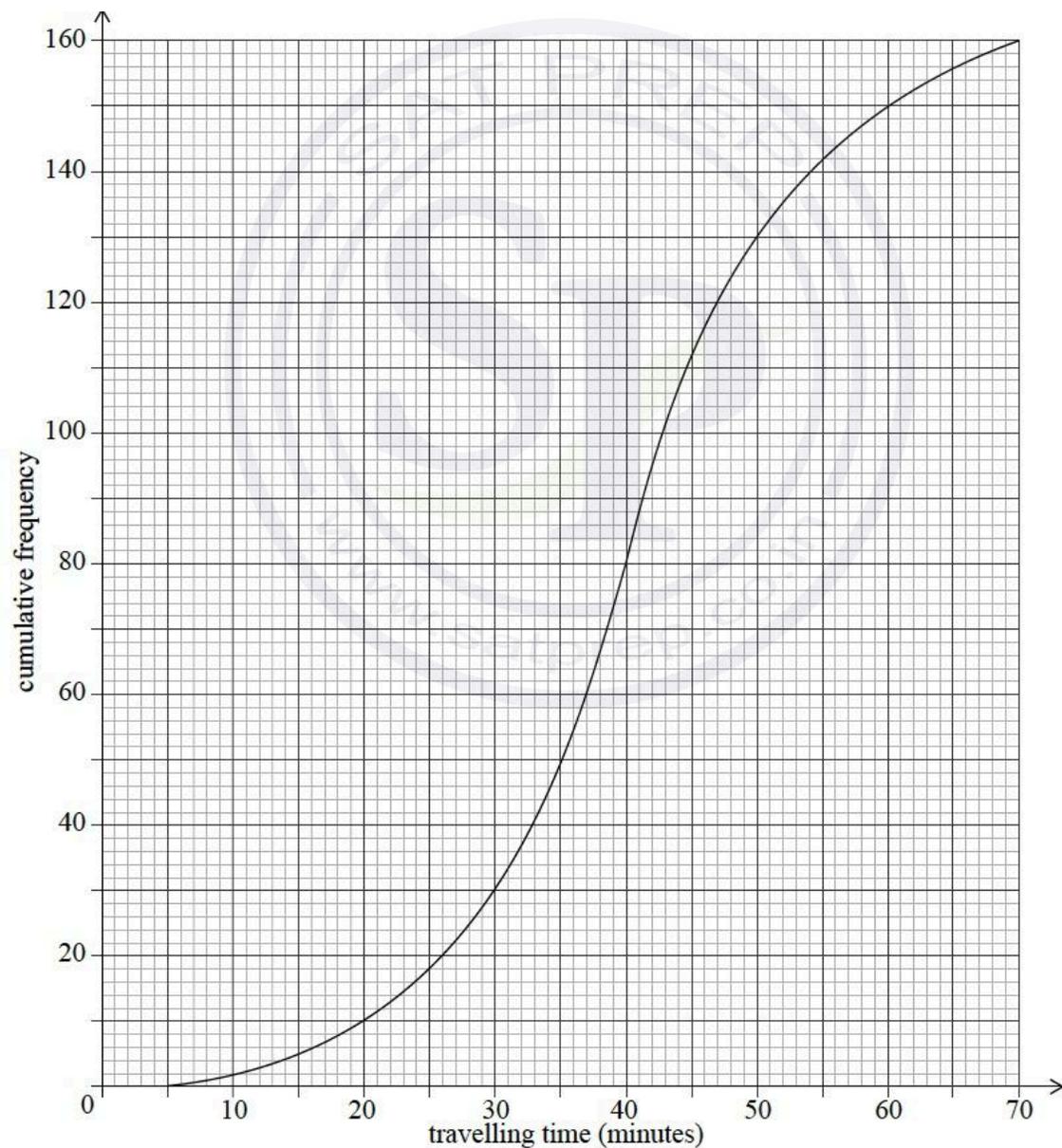


Subject – Math AA(Standard Level)
Topic - Statistics and Probability
Year - May 2021 – Nov 2024
Paper -1
Questions

Question 1

[Maximum mark: 15]

A large company surveyed 160 of its employees to find out how much time they spend traveling to work on a given day. The results of the survey are shown in the following cumulative frequency diagram.



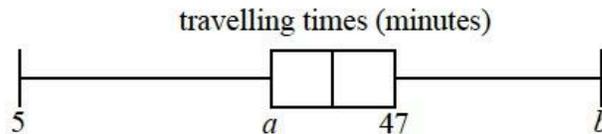
(a) Find the median number of minutes spent traveling to work. [2]

(b) Find the number of employees whose travelling time is within 15 minutes of the median. [3]

Only 10% of the employees spent more than k minutes traveling to work.

(c) Find the value of k . [3]

The results of the survey can also be displayed on the following box-and-whisker diagram.



(d) Write down the value of b . [1]

(e) (i) Find the value of a .

(ii) Hence, find the interquartile range. [4]

Travelling times of less than p minutes are considered outliers.

(f) Find the value of p . [2]

Question 2

[Maximum mark: 5]

Let A and B be events such that $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cup B) = 0.6$.
Find $P(A | B)$.

Question 3

[Maximum mark: 16]

A biased four-sided die, A, is rolled. Let X be the score obtained when die A is rolled. The probability distribution for X is given in the following table.

x	1	2	3	4
$P(X=x)$	p	p	p	$\frac{1}{2}p$

(a) Find the value of p . [2]

(b) Hence, find the value of $E(X)$. [2]

A second biased four-sided die, B, is rolled. Let Y be the score obtained when die B is rolled. The probability distribution for Y is given in the following table.

y	1	2	3	4
$P(Y=y)$	q	q	q	r

(c) (i) State the range of possible values of r .
(ii) Hence, find the range of possible values of q . [3]

(d) Hence, find the range of possible values for $E(Y)$. [3]

Agnes and Barbara play a game using these dice. Agnes rolls die A once and Barbara rolls die B once. The probability that Agnes' score is less than Barbara's score is $\frac{1}{2}$.

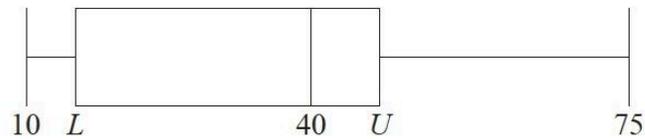
(e) Find the value of $E(Y)$. [6]

Question 4

[Maximum mark: 5]

A research student weighed lizard eggs in grams and recorded the results. The following box and whisker diagram shows a summary of the results where L and U are the lower and upper quartiles respectively.

diagram not to scale



The interquartile range is 20 grams and there are no outliers in the results.

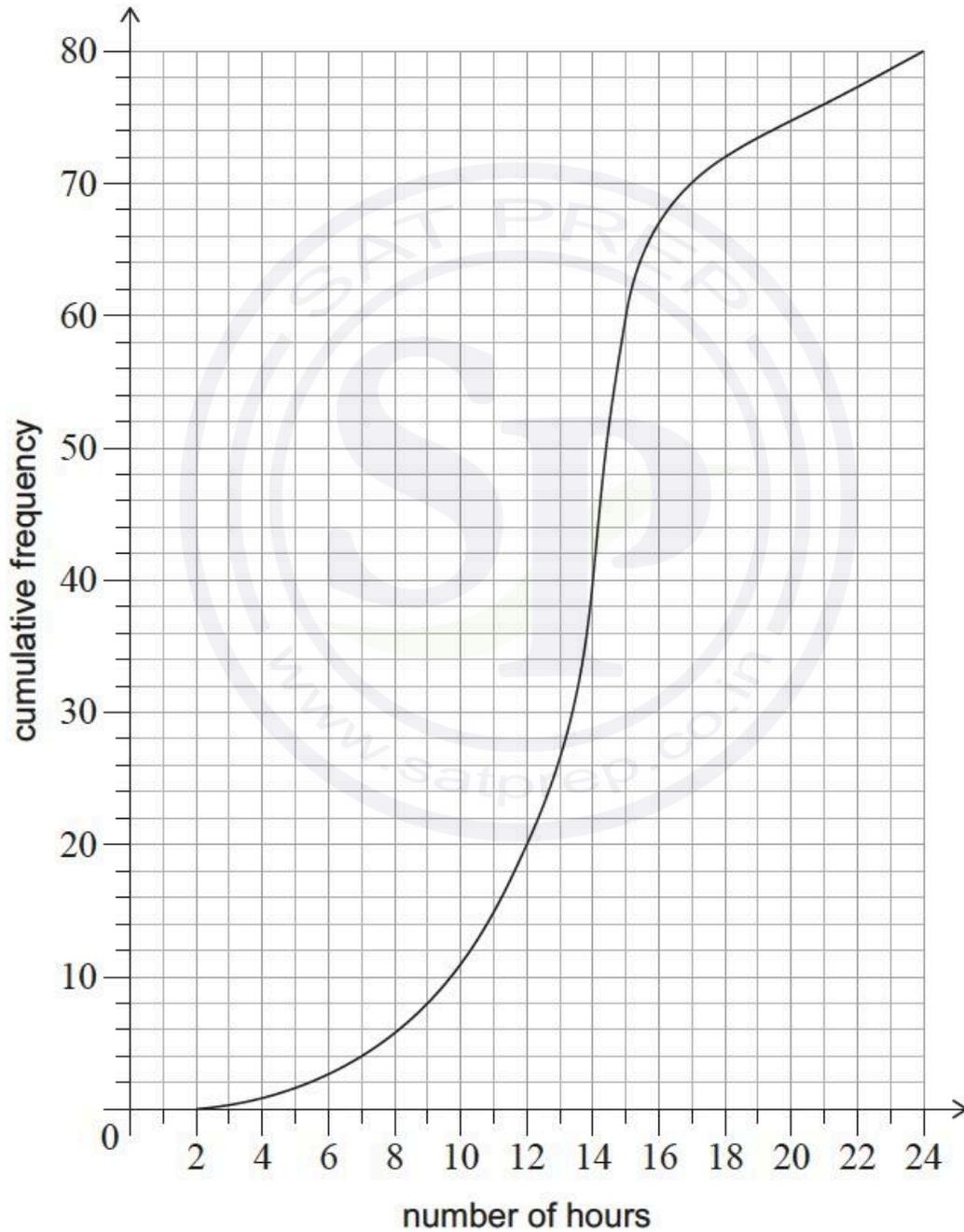
- (a) Find the minimum possible value of U . [3]
- (b) Hence, find the minimum possible value of L . [2]

Question 5

[Maximum mark: 14]

A large school has students from Year 6 to Year 12.

A group of 80 students in Year 12 were randomly selected and surveyed to find out how many hours per week they each spend doing homework. Their results are represented by the following cumulative frequency graph.



(a) Find the median number of hours per week these Year 12 students spend doing homework. [2]

(b) Given that 10% of these Year 12 students spend more than k hours per week doing homework, find the value of k . [3]

This same information is represented by the following table.

Hours (h) spent doing homework	$2 < h \leq 7$	$7 < h \leq 15$	$15 < h \leq 21$	$21 < h \leq 24$
Frequency	4	p	16	q

(c) Find the value of p and the value of q . [4]

There are 320 students in Year 12 at this school.

(d) Estimate the number of Year 12 students that spend more than 15 hours each week doing homework. [3]

(e) (i) Explain why this sampling method might not provide an accurate representation of the amount of time **all** of the students in the school spend doing homework.

(ii) Suggest a more appropriate sampling method. [2]

Question 6

[Maximum mark: 5]

Box 1 contains 5 red balls and 2 white balls.

Box 2 contains 4 red balls and 3 white balls.

(a) A box is chosen at random and a ball is drawn. Find the probability that the ball is red. [3]

Let A be the event that "box 1 is chosen" and let R be the event that "a red ball is drawn".

(b) Determine whether events A and R are independent. [2]

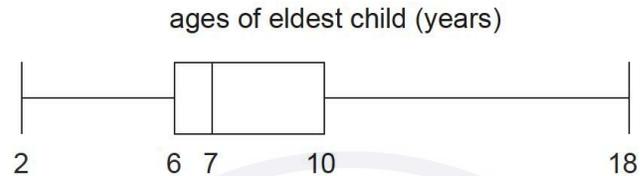
Question 7

[Maximum mark: 7]

A survey at a swimming pool is given to one adult in each family. The age of the adult, a years old, and of their eldest child, c years old, are recorded.

The ages of the eldest child are summarized in the following box and whisker diagram.

diagram not to scale



- (a) Find the largest value of c that would not be considered an outlier. [3]

The regression line of a on c is $a = \frac{7}{4}c + 20$. The regression line of c on a is $c = \frac{1}{2}a - 9$.

- (b) (i) One of the adults surveyed is 42 years old. Estimate the age of their eldest child. [4]
(ii) Find the mean age of all the adults surveyed.

Question 8

[Maximum mark: 16]

A biased four-sided die with faces labelled 1, 2, 3 and 4 is rolled and the result recorded. Let X be the result obtained when the die is rolled. The probability distribution for X is given in the following table where p and q are constants.

x	1	2	3	4
$P(X=x)$	p	0.3	q	0.1

For this probability distribution, it is known that $E(X) = 2$.

(a) Show that $p = 0.4$ and $q = 0.2$. [5]

(b) Find $P(X > 2)$. [2]

Nicky plays a game with this four-sided die. In this game she is allowed a maximum of five rolls. Her score is calculated by adding the results of each roll. Nicky wins the game if her score is at least ten.

After three rolls of the die, Nicky has a score of four.

(c) Assuming that rolls of the die are independent, find the probability that Nicky wins the game. [5]

David has two pairs of unbiased four-sided dice, a yellow pair and a red pair.

Both yellow dice have faces labelled 1, 2, 3 and 4. Let S represent the sum obtained by rolling the two yellow dice. The probability distribution for S is shown below.

s	2	3	4	5	6	7	8
$P(S=s)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

The first red die has faces labelled 1, 2, 2 and 3. The second red die has faces labelled 1, a , a and b , where $a < b$ and $a, b \in \mathbb{Z}^+$. The probability distribution for the sum obtained by rolling the red pair is the same as the distribution for the sum obtained by rolling the yellow pair.

(d) Determine the value of b . [2]

(e) Find the value of a , providing evidence for your answer. [2]

Question 9

[Maximum mark: 6]

Events A and B are such that $P(A) = 0.3$ and $P(B) = 0.8$.

- (a) Determine the value of $P(A \cap B)$ in the case where the events A and B are independent. [1]
- (b) Determine the minimum possible value of $P(A \cap B)$. [3]
- (c) Determine the maximum possible value of $P(A \cap B)$, justifying your answer. [2]

Question 10

[Maximum mark: 5]

Events A and B are such that $P(A) = 0.4$, $P(A|B) = 0.25$ and $P(A \cup B) = 0.55$.

Find $P(B)$.

Question 11

[Maximum mark: 6]

Consider an arithmetic sequence with $u_1 = 0.6$ and $u_4 = 0.15$.

- (a) Find the common difference, d . [2]

The following table shows the probability distribution of a discrete random variable X such that $P(X = n) = \frac{u_n}{k}$, where $n \in \mathbb{Z}^+$, $1 \leq n \leq 4$ and $k \in \mathbb{R}^+$.

n	1	2	3	4
$P(X = n)$	$\frac{0.6}{k}$	$\frac{u_2}{k}$	$\frac{u_3}{k}$	$\frac{0.15}{k}$

- (b) Find the value of k . [4]

Question 12

[Maximum mark: 6]

On a Monday at an amusement park, a sample of 40 visitors was randomly selected as they were leaving the park. They were asked how many times that day they had been on a ride called *The Dragon*. This information is summarized in the following frequency table.

Number of times on <i>The Dragon</i>	Frequency
0	6
1	16
2	13
3	2
4	3

It can be assumed that this sample is representative of all visitors to the park for the following day.

(a) For the following day, Tuesday, estimate

(i) the probability that a randomly selected visitor will ride *The Dragon*;

(ii) the expected number of times a visitor will ride *The Dragon*.

[4]

It is known that 1000 visitors will attend the amusement park on Tuesday. *The Dragon* can carry a maximum of 10 people each time it runs.

(b) Estimate the minimum number of times *The Dragon* must run to satisfy demand.

[2]

Question 13

[Maximum mark: 17]

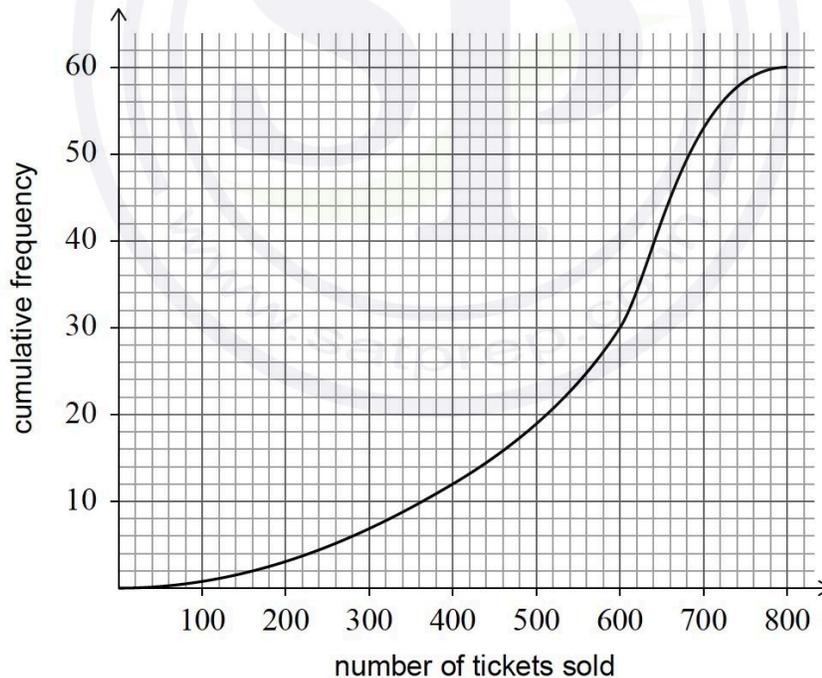
A ballet company performs *The Nutcracker* every year. Last year they gave a total of 60 performances at their theatre which has a maximum capacity of 800. The number of tickets sold, n , at each performance is shown in the following frequency table.

Number of tickets sold, n	Number of performances
$0 < n \leq 200$	3
$200 < n \leq 400$	p
$400 < n \leq 600$	18
$600 < n \leq 800$	30

- (a) (i) Find the value of p .
(ii) Write down the modal class.

[2]

The following cumulative frequency diagram also displays these data.



- (b) Use the cumulative frequency curve to estimate
- (i) the median number of tickets sold;
- (ii) the number of performances where at least 80% of the tickets were sold.

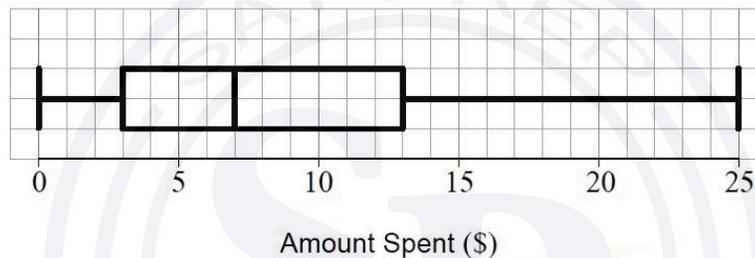
[4]

After a performance, the company decides to conduct a survey to obtain feedback from the audience.

- (c) (i) State one disadvantage of the company surveying only the first 5% of the audience as they leave the theatre.
- (ii) Describe briefly how the company could collect feedback from 5% of the audience using the systematic sampling method.
- (iii) State the sampling method which should be used if the survey is to be representative of the number of children and the number of adults in the audience. [4]

Last year 36 000 tickets were sold to *The Nutcracker*.

- (d) The following box and whisker diagram displays the amount spent by the audience at the souvenir shop when they attended the performance.



- (i) Estimate the number of people who spent between \$3 and \$25.
- (ii) Half the audience spent less than \$ a . Estimate the value of a . [3]
- (e) This year the company will again give 60 performances and expects to sell 17 additional tickets for each performance.
- (i) Calculate the mean number of tickets the company expects to sell this year for each performance.
- (ii) State what effect, if any, this increase in ticket sales would have on the variance of the number of tickets sold for each performance. [4]

Question 14

[Maximum mark: 4]

Events A and B are such that $P(A) = 0.7$, $P(B) = 0.75$ and $P(A \cap B) = 0.55$.

- (a) Find $P(A \cup B)$. [2]
- (b) Hence, or otherwise, find $P(A' \cap B')$. [2]

Question 15

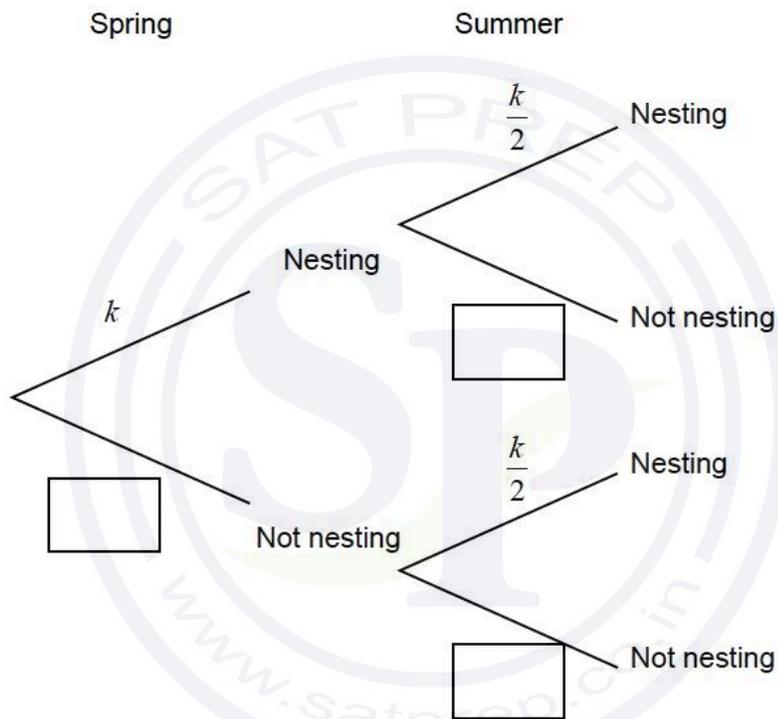
[Maximum mark: 6]

A species of bird can nest in two seasons: Spring and Summer.

The probability of nesting in Spring is k .

The probability of nesting in Summer is $\frac{k}{2}$.

This is shown in the following tree diagram.



- (a) Complete the tree diagram to show the probabilities of not nesting in each season. Write your answers in terms of k . [2]

It is known that the probability of not nesting in Spring and not nesting in Summer is $\frac{5}{9}$.

- (b) (i) Show that $9k^2 - 27k + 8 = 0$.
(ii) Both $k = \frac{1}{3}$ and $k = \frac{8}{3}$ satisfy $9k^2 - 27k + 8 = 0$.

State why $k = \frac{1}{3}$ is the only valid solution. [4]

Question 16

[Maximum mark: 17]

A bag contains buttons which are either red or blue.

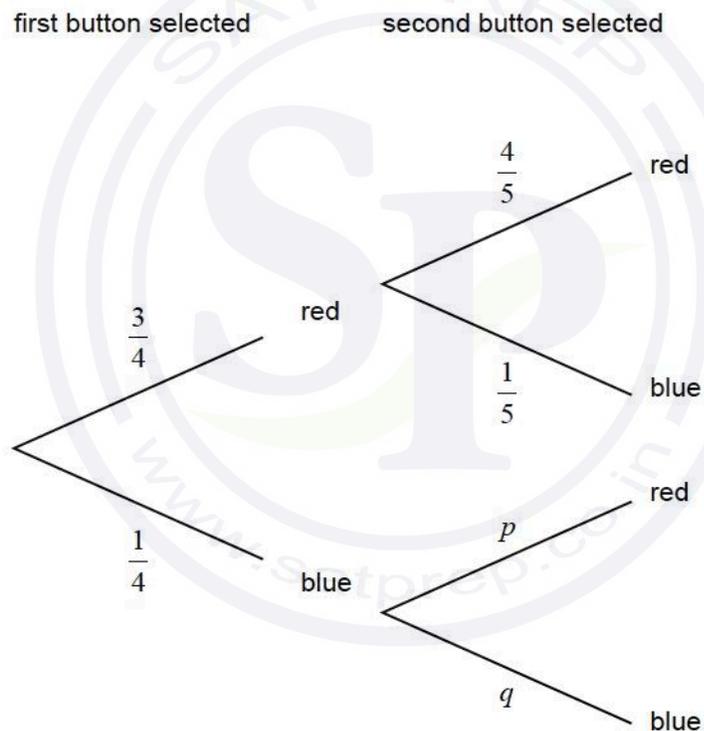
Initially, the bag contains three red buttons and one blue button.

Francine randomly selects one button from the bag. She then replaces the button and adds one extra button of the same colour.

For example, if she selects a red button, she then replaces it and adds one extra red button so that the bag then contains four red buttons and one blue button.

Francine then randomly selects a second button from the bag.

The following tree diagram represents the probabilities of the first two selections.



- (a) Find the value of p and the value of q . [2]
- (b) Show that the probability that Francine selects two buttons of the same colour is $\frac{7}{10}$. [2]
- (c) Given that Francine selects two buttons of the same colour, find the probability that she selects two red buttons. [3]

The random variable X is defined as the number of red buttons selected by Francine.

The following table shows the probability distribution of X .

x	0	1	2
$P(X=x)$	$\frac{1}{10}$	a	b

(d) Find the value of a and the value of b . [2]

(e) Hence, find the expected number of red buttons selected by Francine. [2]

Francine restarts the process with three red buttons and one blue button in the bag. She selects buttons as before, replacing the button and adding one extra button of the same colour each time. She repeats this until she selects a blue button.

(f) Given that the first two buttons she selects are red, write down the probability that the next button she selects is blue. [1]

The probability that she selects the first blue button after n selections in total is $\frac{3}{56}$.

(g) Find the value of n . [5]



Question 17

[Maximum mark: 6]

Claire rolls a six-sided die 16 times.

The scores obtained are shown in the following frequency table.

Score	Frequency
1	p
2	q
3	4
4	2
5	0
6	3

It is given that the mean score is 3.

- (a) Find the value of p and the value of q . [5]

Each of Claire's scores is multiplied by 10 in order to determine the final score for a game she is playing.

- (b) Write down the mean final score. [1]

Question 18

[Maximum mark: 6]

Two events A and B are such that $P(A) = 0.65$, $P(B) = 0.45$ and $P(A \cup B) = 0.85$.

- (a) Find $P(A \cap B)$. [3]

- (b) Find $P(A' | B')$. [3]

Question 19

[Maximum mark: 6]

Consider the following set of ordered data.

22	22	26	27	28	32	36	38	39	42
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(a) Write down

- (i) the mode;
- (ii) the range;
- (iii) the median.

[3]

(b) Find the interquartile range.

[3]

Question 20

[Maximum mark: 6]

Two events A and B are such that $P(A) = 0.45$, $P(B) = 0.65$ and $P(A \cup B) = 0.8$.

(a) Find $P(A \cap B)$.

[3]

(b) Find $P(A' | B')$.

[3]

Question 21

[Maximum mark: 6]

Consider the following set of ordered data.

27	27	31	32	33	37	41	46	54	57
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(a) Write down

- (i) the mode;
- (ii) the range;
- (iii) the median.

[3]

(b) Find the interquartile range.

[3]