Subject – Math AA(Standard Level)
Topic - Calculus
Year - May 2021 – Nov 2022
Paper -2
Answers

Question 1

(a) use of a graph to find the coordinates of the local minimum *(M1)*

$s = -16.513...$ *(A1)*

maximum distance is 16.5 cm (to the left of O) ***A1***

[3 marks]

(b) attempt to find time when particle changes direction *eg* considering the first maximum on the graph of s or the first t – intercept on the graph of s'. *(M1)*

$t = 1.51986...$ *(A1)*

attempt to find the gradient of s' for **their** value of *t*, $s''(1.51986...)$ *(M1)*

$= -8.92 \ (\text{cm/s}^2)$ ***A1***

[4 marks]

Total [7 marks]

Question 2

(a) Attempt to find the point of intersection of the graph of f and the line $y = x$ *(M1)*

$x = 5.56619...$

$= 5.57$ ***A1***

[2 marks]

(b) $f'(x) = -45e^{-0.5x}$ ***A1***

attempt to set the gradient of f equal to -1 *(M1)*

$-45e^{-0.5x} = -1$

Q has coordinates $(2\ln 45, 2)$ (accept $(-2\ln\frac{1}{45}, 2)$) ***A1A1***

> **Note**: Award ***A1*** for each value, even if the answer is not given as a coordinate pair.
>
> Do not accept $\dfrac{\ln\frac{1}{45}}{-0.5}$ or $\dfrac{\ln 45}{0.5}$ as a final value for x. Do not accept 2.0 or 2.00 as a final value for y.

[4 marks]

(c) attempt to substitute coordinates of Q (in any order) into an appropriate equation **(M1)**

$y-2=-(x-2\ln 45)$ OR $2=-2\ln 45+c$ **A1**

equation of L is $y=-x+2\ln 45+2$ **AG**

[2 marks]

(d) (i) $x=\ln 45+1(=4.81)$ **A1**

(ii) appropriate method to find the sum of two areas using integrals of the difference of two functions **(M1)**

Note: Allow absence of incorrect limits.

$$\int_{4.806...}^{5.566...}\left(x-\left(-x+2\ln 45+2\right)\right)\mathrm{d}x+\int_{5.566...}^{7.613...}\left(90\mathrm{e}^{-0.5x}-\left(-x+2\ln 45+2\right)\right)\mathrm{d}x$$ **(A1)(A1)**

Note: Award ***A1*** for one correct integral expression including correct limits and integrand. Award ***A1*** for a second correct integral expression including correct limits and integrand.

$=1.52196...$

$=1.52$ **A1**

[5 marks]

(e) by symmetry $2\times 1.52...$ **(M1)**

$=3.04$ **A1**

Note: Accept any answer that rounds to 3.0 (but do not accept 3).

[2 marks]

Total [15 marks]

Question 3

(a) recognising $v = 0$ *(M1)*

$t = 6.74416...$

$= 6.74$ (sec) ***A1***

> **Note:** Do not award ***A1*** if additional values are given.

[2 marks]

(b) $\int_0^{10} |v(t)| dt$ OR $-\int_0^{6.74416...} v(t)\,dt + \int_{6.74416...}^{9.08837...} v(t)\,dt - \int_{9.08837...}^{10} v(t)\,dt$ *(A1)*

$= 37.0968...$

$= 37.1$ (m) ***A1***

[2 marks]

(c) recognizing acceleration at $t = 7$ is given by $v'(7)$ *(M1)*

acceleration $= 5.93430...$

$= 5.93$ (ms^{-2}) ***A1***

[2 marks]

Total [6 marks]

Question 4

(a) correct integration $3x^2 + 7x + c$ ***A1A1A1***

> **Note:** Award ***A1*** for $3x^2$, ***A1*** for $7x$ and ***A1*** for $+c$

[3 marks]

(b) recognition that $f(x) = \int f'(x)dx$ *(M1)*

$3(1.2)^2 + 7(1.2) + c = 7.32$ *(A1)*

$c = -5.4$

$f(x) = 3x^2 + 7x - 5.4$ ***A1***

[3 marks]

Total [6 marks]

Question 5

(a) recognizing at rest $v = 0$ ***(M1)***

$t = 3.34692...$

$t = 3.35$ (seconds) ***A1***

Note: Award ***(M1)A0*** for additional solutions to $v = 0$ eg $t = -0.205$ or $t = 6.08$.

[2 marks]

(b) recognizing particle changes direction when $v = 0$ OR when $t = 3.34692...$ ***(M1)***

$a = -4.71439...$

$a = -4.71\,(\text{ms}^{-2})$ ***A2***

[3 marks]

(c) distance travelled = $\int_0^6 |v|\,\mathrm{d}t$ OR

$$\int_0^{3.34...}\left(\mathrm{e}^{\sin(t)} + 4\sin(t)\right)\mathrm{d}t - \int_{3.34...}^{6}\left(\mathrm{e}^{\sin(t)} + 4\sin(t)\right)\mathrm{d}t \quad (= 14.3104... + 6.44300...)$$ ***(A1)***

$= 20.7534...$

$= 20.8$ (metres) ***A1***

[2 marks]

Total [7 marks]

Question 6

(a) recognises the need to find the value of t when $v=0$ **(M1)**

$t=1.57079...\left(=\frac{\pi}{2}\right)$

$t=1.57\left(=\frac{\pi}{2}\right)$ (s) **A1**

[2 marks]

(b) recognises that $a(t)=v'(t)$ **(M1)**

$t_1=2.26277...,\ t_2=2.95736...$

$t_1=2.26,\ t_2=2.96$ (s) **A1A1**

(c) speed is greatest at $t=3$ **(A1)**

$a=-1.83778...$

$a=-1.84$ (m s^{-2}) **A1**

[2 marks]

Total [7 marks]

Question 7

METHOD 1

recognises that $g(x)=\int\left(3x^2+5e^x\right)\,dx$ **(M1)**

$g(x)=x^3+5e^x(+C)$ **(A1)(A1)**

METHOD 2

attempts to write both sides in the form of a definite integral **(M1)**

$\int_0^x g'(t)\,dt=\int_0^x\left(3t^2+5e^t\right)\,dt$ **(A1)**

$g(x)-4=x^3+5e^x-5e^0$ **(A1)(A1)**

$g(x)=x^3+5e^x-1$ **A1**

[5 marks]

Question 8

(a) initial displacement is $s(0)$ *(M1)*

6 (m) ***A1***

[2 marks]

(b) velocity is s' *(M1)*

-2.29920

-2.30 (m/s) ***A1***

[2 marks]

(c) attempting to find t when the particle changes direction *(M1)*

$t = 0.433007\ldots \text{AND } 3.25575\ldots \text{AND } 6.33965\ldots$ (may be seen on a graph) *(A1)*

particle travels away from P when $v > 0$ OR when $s' > 0$ *(M1)*

$0 \le t < 0.433007\ldots,\ 3.25575\ldots < t < 6.33965\ldots$

$0 \le t < 0.433,\ 3.26 < t < 6.34$ ***A1A1***

[5 marks]

(d) recognizing that acceleration is $a(t) = v'(t)$ OR $a(t) = s''(t)$ *(M1)*

attempting to find max/min on graph of velocity OR finding zeros on graph of acceleration *(M1)*

$b = 1.23140\ldots,\ c = 5.68959\ldots$

$b = 1.23,\ c = 5.69$ ***A1A1***

[4 marks]

(e) **METHOD 1** (using integral of velocity)

correct integral (accept absence of $\mathrm{d}t$) ***(A1)***

$\int_{1.23140...}^{5.68959...} |v(t)|\, \mathrm{d}t$ OR $\int_b^c |s'(t)|\, \mathrm{d}t$ OR $-\int_{1.23140...}^{3.25575...} v(t)\, \mathrm{d}t + \int_{3.25575...}^{5.68959...} v(t)\, \mathrm{d}t$ OR

$3.8560 + 15.696$

$19.5525\ldots$

total distance $= 19.6$ (m) ***A2***

METHOD 2 (using differences in displacement)

finding displacement at b, c **and** local min on displacement graph ***(A1)***

$(b,\ 4.43306),\ (c,\ 16.2734), (3.25575,\ 0.577001)$ OR $4.43306, 0.577001,\ 16.2734$

correct approach ***(A1)***

$(4.43306 - 0.577001) + (16.2734 - 0.577001)$ OR towards P 3.85606 + away from P 15.696

$19.5525\ldots$

total distance $= 19.6$ (m) ***A1***

[3 marks]

Total [16 marks]

Question 9

(a) $(0.708519..., 0.639580...)$

$(0.709, 0.640)$ $(x = 0.709,\ y = 0.640)$ ***A1A1***

[2 marks]

(b) $1.09885...$

$x = 1.10$ accept $(1.10, 0)$ ***A1***

[1 mark]

(c) **METHOD 1**

$\int_0^2 |f(x)|dx$ *(A1)*

4.61117...

area = 4.61 ***A2***

METHOD 2

$-\int_{1.09885...}^{2} f(x)dx$ OR $\int_{1.09885...}^{2} |f(x)|dx$ OR 4.17527… *(A1)*

$\int_0^{1.09885...} f(x)dx - \int_{1.09885...}^{2} f(x)dx$ OR 0.435901… + 4.17527… *(A1)*

4.61117...

area = 4.61 ***A1***

[3 marks]

Total [6 marks]