Subject – Math AA(Standard Level)
Topic - Calculus
Year - May 2021 – Nov 2022
Paper -2
Questions

Question 1

[Maximum mark: 7]

The displacement, in centimetres, of a particle from an origin, O, at time t seconds, is given by $s(t) = t^2 \cos t + 2t \sin t$, $0 \le t \le 5$.

(a) Find the maximum distance of the particle from O. [3]

(b) Find the acceleration of the particle at the instant it first changes direction. [4]

Question 2

[Maximum mark: 15]

Consider the function f defined by $f(x) = 90e^{-0.5x}$ for $x \in \mathbb{R}^+$.

The graph of f and the line $y = x$ intersect at point P.

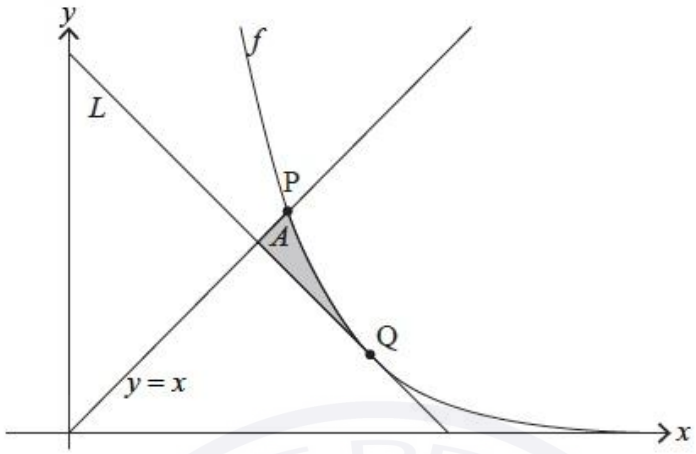
(a) Find the x-coordinate of P. [2]

The line L has a gradient of -1 and is a tangent to the graph of f at the point Q.

(b) Find the exact coordinates of Q. [4]

(c) Show that the equation of L is $y = -x + 2\ln 45 + 2$. [2]

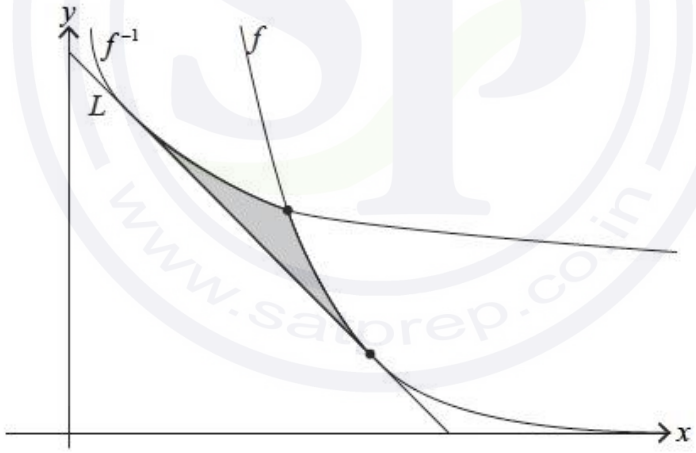
The shaded region A is enclosed by the graph of f and the lines $y = x$ and L.



(d) (i) Find the x-coordinate of the point where L intersects the line $y = x$.

(ii) Hence, find the area of A. [5]

The line L is tangent to the graphs of both f and the inverse function f^{-1}.



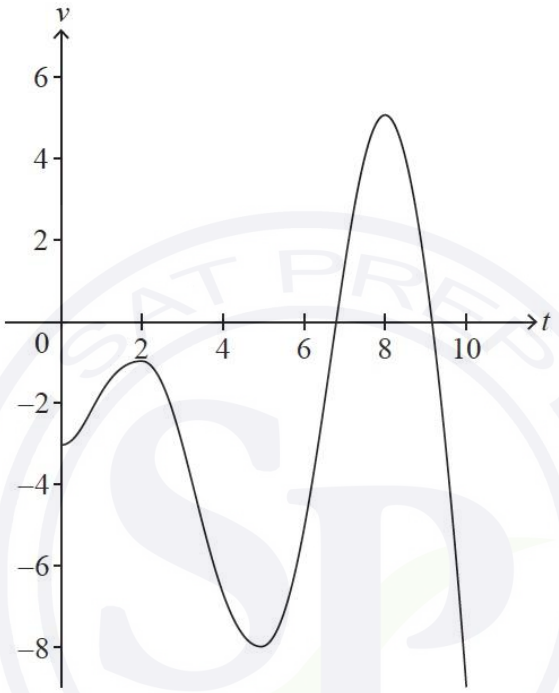
(e) Find the shaded area enclosed by the graphs of f and f^{-1} and the line L. [2]

Question 3

[Maximum mark: 6]

A particle moves in a straight line. The velocity, $v \text{ ms}^{-1}$, of the particle at time t seconds is given by $v(t) = t\sin t - 3$, for $0 \le t \le 10$.

The following diagram shows the graph of v.



(a) Find the smallest value of t for which the particle is at rest. [2]

(b) Find the total distance travelled by the particle. [2]

(c) Find the acceleration of the particle when $t = 7$. [2]

Question 4

[Maximum mark: 6]

(a) Find $\int (6x+7)\,dx$. [3]

(b) Given $f'(x) = 6x + 7$ and $f(1.2) = 7.32$, find $f(x)$. [3]

Question 5

[Maximum mark: 7]

A particle moves along a straight line so that its velocity, $v\,\text{m s}^{-1}$, after t seconds is given by $v(t) = e^{\sin t} + 4\sin t$ for $0 \le t \le 6$.

(a) Find the value of t when the particle is at rest. [2]

(b) Find the acceleration of the particle when it changes direction. [3]

(c) Find the total distance travelled by the particle. [2]

Question 6

[Maximum mark: 7]

A particle moves in a straight line such that its velocity, $v\,\text{m s}^{-1}$, at time t seconds is given by $v = \frac{(t^2+1)\cos t}{4}$, $0 \le t \le 3$.

(a) Determine when the particle changes its direction of motion. [2]

(b) Find the times when the particle's acceleration is $-1.9\,\text{m s}^{-2}$. [3]

(c) Find the particle's acceleration when its speed is at its greatest. [2]

Question 7

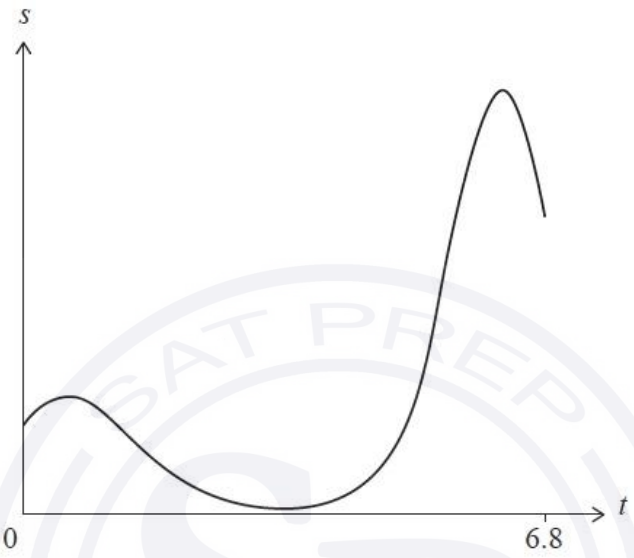
[Maximum mark: 5]

The derivative of a function g is given by $g'(x) = 3x^2 + 5e^x$, where $x \in \mathbb{R}$. The graph of g passes through the point $(0, 4)$. Find $g(x)$.

Question 8

[Maximum mark: 16]

A particle moves in a straight line. Its displacement, s metres, from a fixed point P at time t seconds is given by $s(t) = 3(t+2)^{\cos t}$, for $0 \le t \le 6.8$, as shown in the following graph.



(a) Find the particle's initial displacement from the point P. [2]

(b) Find the particle's velocity when $t = 2$. [2]

(c) Determine the intervals of time when the particle is moving away from the point P. [5]

The acceleration of the particle is zero when $t = b$ and $t = c$, where $b < c$.

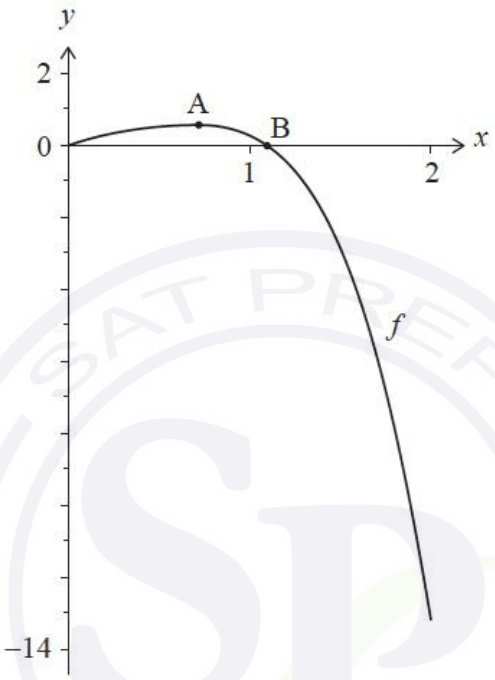
(d) Find the value of b and the value of c. [4]

(e) Find the total distance travelled by the particle for $b \le t \le c$. [3]

Question 9

[Maximum mark: 6]

The function f is defined as $f(x) = \ln(xe^x + 1) - x^4$, for $0 \le x \le 2$. The graph of f is shown in the following diagram.



The graph of f has a local maximum at point A. The graph intersects the x-axis at the origin and at point B.

(a) Find the coordinates of A. [2]

(b) Find the x-coordinate of B. [1]

(c) Find the total area enclosed by the graph of f, the x-axis and the line $x = 2$. [3]