Subject – Math AA(Standard Level)
Topic - Functions
Year - May 2021 – Nov 2024
Paper -2
Questions

Question 1

[Maximum mark: 7]

All living plants contain an isotope of carbon called carbon-14. When a plant dies, the isotope decays so that the amount of carbon-14 present in the remains of the plant decreases. The time since the death of a plant can be determined by measuring the amount of carbon-14 still present in the remains.

The amount, A, of carbon-14 present in a plant t years after its death can be modelled by $A = A_0 e^{-kt}$ where $t \geq 0$ and A_0, k are positive constants.

At the time of death, a plant is defined to have 100 units of carbon-14.

(a) Show that $A_0 = 100$. [1]

The time taken for half the original amount of carbon-14 to decay is known to be 5730 years.

(b) Show that $k = \frac{\ln 2}{5730}$. [3]

(c) Find, correct to the nearest 10 years, the time taken after the plant's death for 25% of the carbon-14 to decay. [3]

Question 2

[Maximum mark: 6]

The functions f and g are defined for $x \in \mathbb{R}$ by $f(x) = 6x^2 - 12x + 1$ and $g(x) = -x + c$, where $c \in \mathbb{R}$.

(a) Find the range of f. [2]

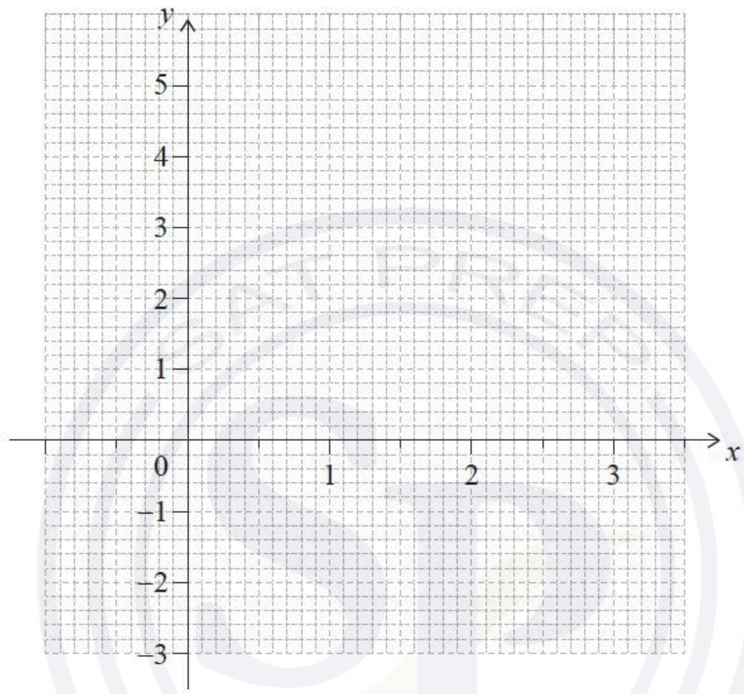
(b) Given that $(g \circ f)(x) \leq 0$ for all $x \in \mathbb{R}$, determine the set of possible values for c. [4]

Question 3

[Maximum mark: 5]

Let $f(x) = 3x - 4^{0.15x^2}$ for $0 \le x \le 3$.

(a) Sketch the graph of f on the grid below. [3]



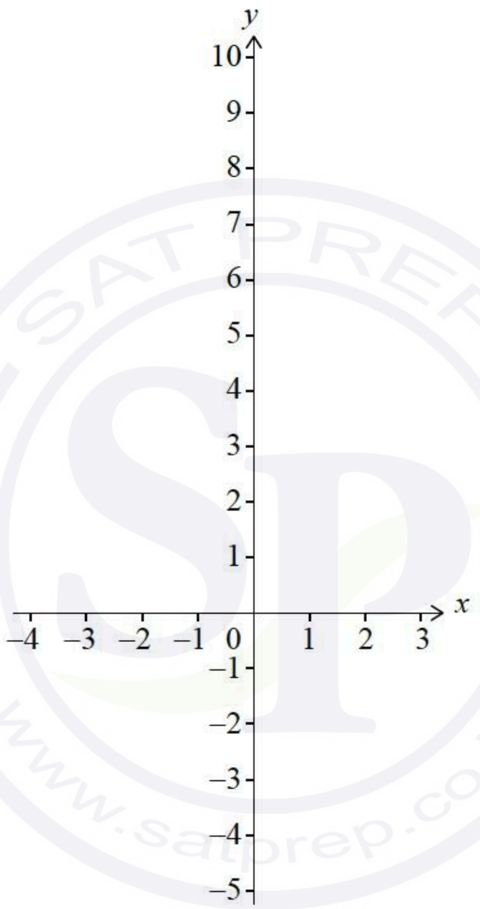
(b) Find the value of x for which $f'(x) = 0$. [2]

Question 4

[Maximum mark: 5]

Consider the function $f(x) = e^x - 2x - 5$.

(a) On the following axes, sketch the graph of f for $-4 \leq x \leq 3$. [3]



The function g is defined by $g(x) = e^{3x} - 6x - 7$.

(b) The graph of g is obtained from the graph of f by a horizontal stretch with scale factor k, followed by a vertical translation of c units.

Find the value of k and the value of c. [2]

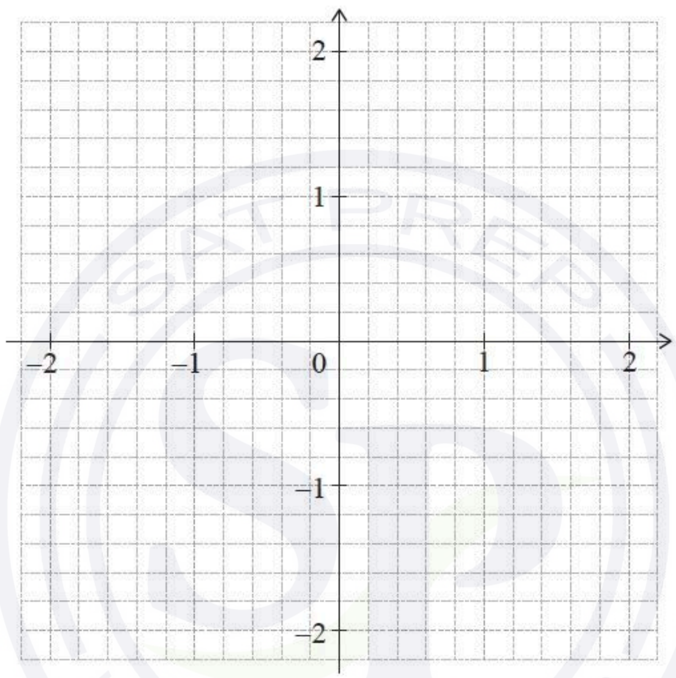
Question 5

[Maximum mark: 5]

Consider the function $f(x) = e^{-x^2} - 0.5$, for $-2 \leq x \leq 2$.

(a) Find the values of x for which $f(x) = 0$. [2]

(b) Sketch the graph of f on the following grid. [3]



Question 5

[Maximum mark: 16]

The function f is defined by $f(x) = \dfrac{4x+1}{x+4}$, where $x \in \mathbb{R}$, $x \neq -4$.

(a) For the graph of f

(i) write down the equation of the vertical asymptote;

(ii) find the equation of the horizontal asymptote. [3]

(b) (i) Find $f^{-1}(x)$.

(ii) Using an algebraic approach, show that the graph of f^{-1} is obtained by a reflection of the graph of f in the y-axis followed by a reflection in the x-axis. [8]

The graphs of f and f^{-1} intersect at $x = p$ and $x = q$, where $p < q$.

(c) (i) Find the value of p and the value of q.

(ii) Hence, find the area enclosed by the graph of f and the graph of f^{-1}. [5]

Question 6

[Maximum mark: 7]

The population of a town t years after 1 January 2014 can be modelled by the function

$$P(t) = 15\,000\mathrm{e}^{kt}, \text{ where } k < 0 \text{ and } t \geq 0.$$

It is known that between 1 January 2014 and 1 January 2022 the population decreased by 11%.

Use this model to estimate the population of this town on 1 January 2041.

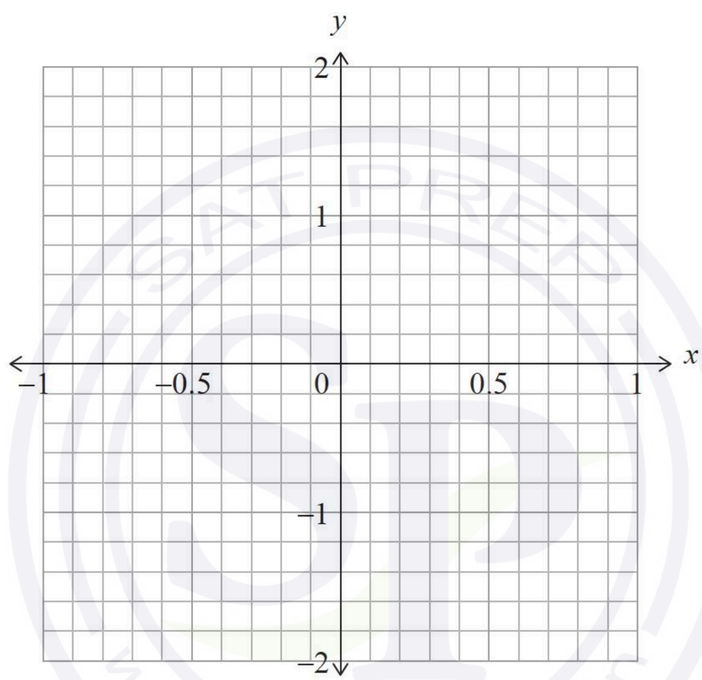
Question 7

[Maximum mark: 5]

The functions f and g are defined by $f(x) = 2x - x^3$ and $g(x) = \tan x$.

(a) Find $(f \circ g)(x)$. [2]

(b) On the following grid, sketch the graph of $y = (f \circ g)(x)$ for $-1 \le x \le 1$. Write down and clearly label the coordinates of any local maximum or minimum points. [3]



Question 8

[Maximum mark: 5]

The amount of a drug, in milligrams (mg), in a patient's body can be modelled by the function $A(t) = 500e^{-kt}$, where k is a positive constant and t is the time in hours after the initial dose is given.

(a) Write down the amount of the drug in the patient's body when $t = 0$. [1]

After three hours, the amount of the drug in the patient's body has decreased to $280\,\text{mg}$.

(b) Find the value of k. [2]

The second dose is given T hours after the initial dose, when the amount of the drug in the patient's body is $140\,\text{mg}$.

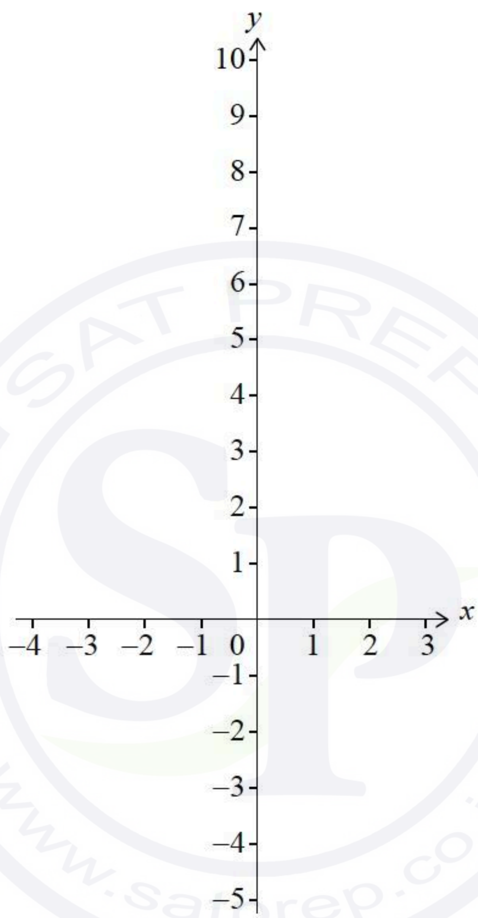
(c) Find the value of T. [2]

Question 9

[Maximum mark: 5]

Consider the function $f(x) = e^{x} - 3x - 4$.

(a) On the following axes, sketch the graph of f for $-4 \le x \le 3$. [3]



The function g is defined by $g(x) = e^{2x} - 6x - 7$.

(b) The graph of g is obtained from the graph of f by a horizontal stretch with scale factor k, followed by a vertical translation of c units.

Find the value of k and the value of c. [2]

Question 10

[Maximum mark: 6]

The loudness of a sound, L, measured in decibels, is related to its intensity, I units, by $L = 10\log_{10}(I \times 10^{12})$.

Consider two sounds, S_1 and S_2.

S_1 has an intensity of 10^{-6} units and a loudness of 60 decibels.

S_2 has an intensity that is twice that of S_1.

(a) State the intensity of S_2. [1]

(b) Determine the loudness of S_2. [2]

The maximum loudness of thunder in a thunderstorm was measured to be 115 decibels.

(c) Find the corresponding intensity, I, of the thunder. [3]

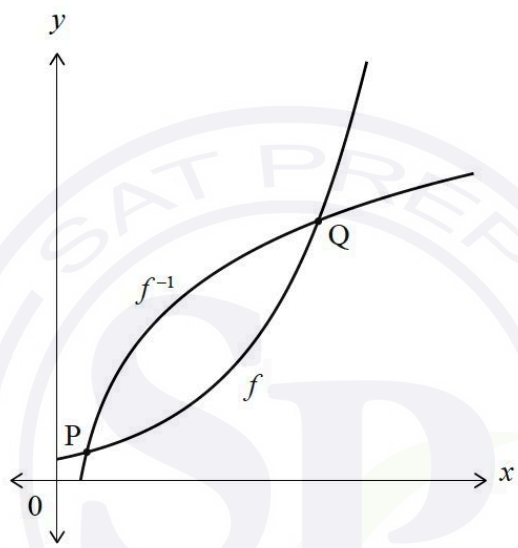
Question 11

[Maximum mark: 12]

Consider the function defined by $f(x)=\frac{3}{2}e^{x-2}$, $0 \le x \le 4$.

(a) Show that the inverse function is given by $f^{-1}(x)=2+\ln\left(\frac{2x}{3}\right)$. [3]

The graphs of f and f^{-1} intersect at two points P and Q, as shown on the following diagram.



(b) Find PQ. [3]

The graph of f is reflected in the x-axis and then translated parallel to the y-axis by 5 units in the positive direction to give the graph of a function g.

(c) Write down

(i) an expression for $g(x)$;

(ii) the domain of g. [3]

(d) Solve the equation $f(x)=g(x)$. Give your answer in the form $x=a+\ln b$, where $a, b \in \mathbb{Q}$. [3]

Question 12

[Maximum mark: 5]

Consider the function $h(x)=\log_{10}(4x^2-rx+r-1)$, where $x \in \mathbb{R}$.

Find the possible values of r. [5]

Question 13

[Maximum mark: 7]

Consider the function $f(x) = 11\sqrt{x} - 2x - 11$, where $0 \le x \le 20$.

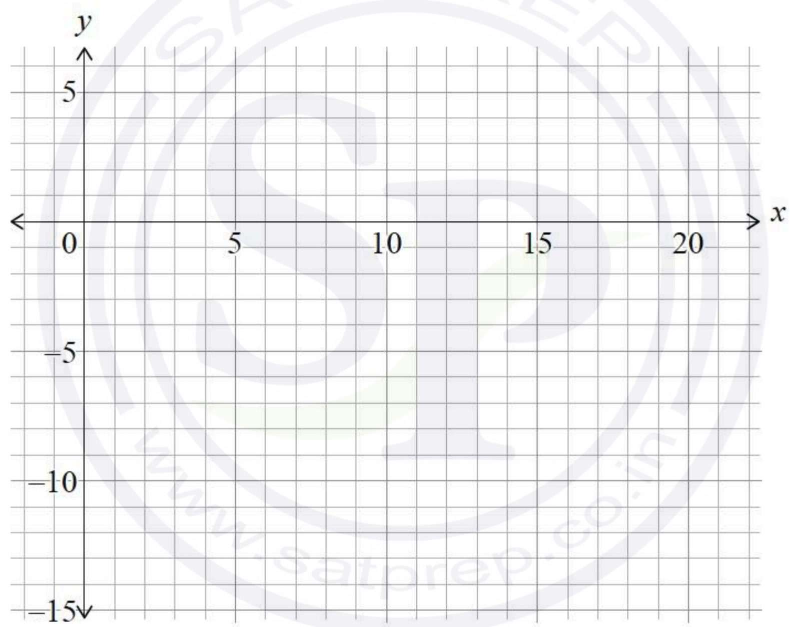
(a) Find the value of

(i) $f(0)$;

(ii) $f(20)$. [2]

(b) Find the two roots of $f(x) = 0$. [2]

(c) Sketch the graph of $y = f(x)$ on the following grid. [3]



Question 14

[Maximum mark: 5]

Consider the function $h(x) = \log_{10}(3x^2 - rx + r - 2)$, where $x \in \mathbb{R}$.

Find the possible values of r. [5]

Question 15

[Maximum mark: 7]

Consider the function $f(x) = 7\sqrt{x} - x - 7$, where $0 \le x \le 60$.

(a) Find the value of

(i) $f(0)$;

(ii) $f(60)$. [2]

(b) Find the two roots of $f(x) = 0$. [2]

(c) Sketch the graph of $y = f(x)$ on the following grid. [3]