

Subject – Math AA(Standard Level)
Topic - Functions
Year - May 2021 – Nov 2022
Paper -2
Answers

Question 1

(a) $100 = A_0 e^0$

A1

$A_0 = 100$

AG

[1 mark]

(b) correct substitution of values into exponential equation

(M1)

$50 = 100e^{-5730k}$ OR $e^{-5730k} = \frac{1}{2}$

EITHER

$-5730k = \ln \frac{1}{2}$

A1

$\ln \frac{1}{2} = -\ln 2$ OR $-\ln \frac{1}{2} = \ln 2$

A1

OR

$e^{5730k} = 2$

A1

$5730k = \ln 2$

A1

THEN

$k = \frac{\ln 2}{5730}$

AG

Note: There are many different ways of showing that $k = \frac{\ln 2}{5730}$ which involve showing different steps. Award full marks for at least two correct algebraic steps seen.

[3 marks]

- (c) if 25 % of the carbon-14 has decayed, 75 % remains ie, 75 units remain (A1)

$$75 = 100e^{-\frac{\ln 2}{5730}t}$$

EITHER

using an appropriate graph to attempt to solve for t (M1)

OR

manipulating logs to attempt to solve for t (M1)

$$\ln 0.75 = -\frac{\ln 2}{5730}t$$

$$t = 2378.164\dots$$

THEN

$$t = 2380 \text{ (years) (correct to the nearest 10 years)}$$

A1

[3 marks]

Total [7 marks]

Question 2

- (a) attempting to find the vertex (M1)

$$x = 1 \text{ OR } y = -5 \text{ OR } f(x) = 6(x-1)^2 - 5$$

range is $y \geq -5$

A1

[2 marks]

(b) **METHOD 1**

$$(g \circ f)(x) = -(6x^2 - 12x + 1) + c \quad (= -(6(x-1)^2 - 5) + c) \quad (\text{A1})$$

EITHER

relating to the range of f OR attempting to find $g(-5)$ (M1)

$$5 + c \leq 0 \quad (\text{A1})$$

OR

attempting to find the discriminant of $(g \circ f)(x)$ (M1)

$$144 + 24(c-1) \leq 0 \quad (120 + 24c \leq 0) \quad (\text{A1})$$

THEN

$$c \leq -5 \quad (\text{A1})$$

[4 marks]

METHOD 2

vertical reflection followed by vertical shift (M1)

new vertex is $(1, 5 + c)$ (A1)

$$5 + c \leq 0 \quad (\text{A1})$$

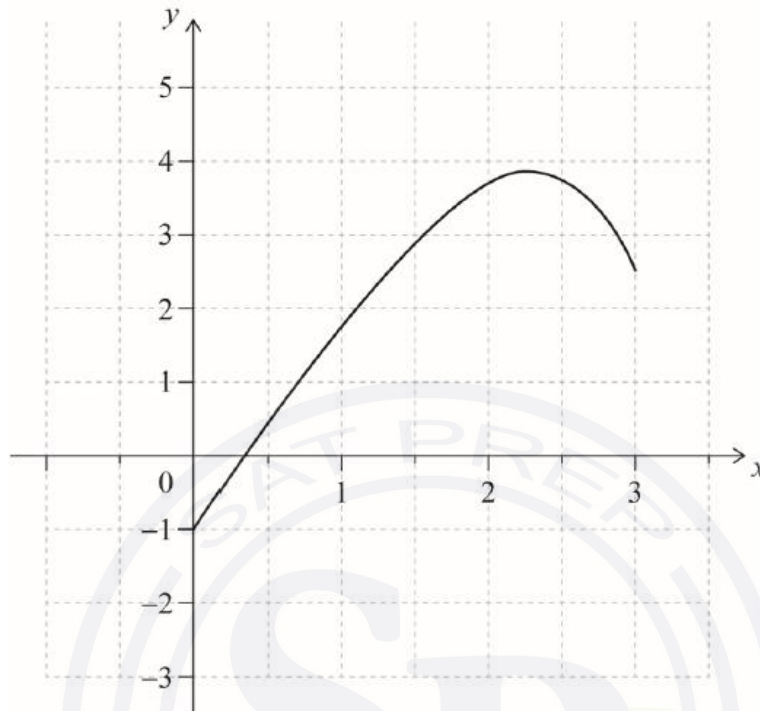
$$c \leq -5 \quad (\text{A1})$$

[4 marks]

Total [6 marks]

Question 3

(a)



A1A1A1

Note: Award **A1** for a smooth concave down curve with generally correct shape. If first mark is awarded, award **A1** for local maximum and x -intercept in approximately correct position, award **A1** for endpoints at $x = 0$ and $x = 3$ with approximately correct y -coordinates.

[3 marks]

(b) recognizing that $f'(x) = 0$ at local maximum

(M1)

$$x = 2.33084\dots$$

$$x = 2.33$$

A1

[2 marks]

Total [5 marks]

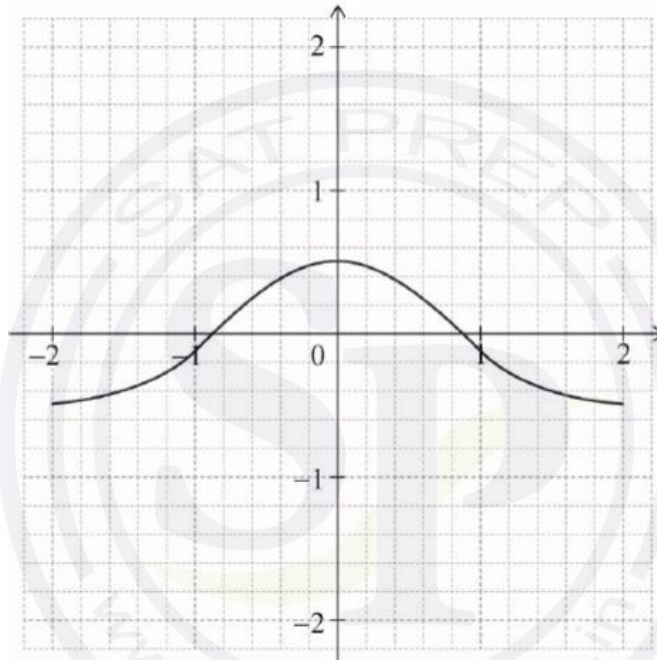
Question 4

- (a) $x = -0.832554\dots, x = 0.832554\dots$
 $x = -0.833, x = 0.833$

A1A1

[2 marks]

- (b)



A1A1A1

Note: Award **A1** for approximately correct shape. Only if this mark is awarded, award **A1** for approximately correct roots and maximum point and **A1** for approximately correct endpoints.
Allow $-1 < x \leq -0.8$, $0.8 \leq x < 1$ for roots, $x = 0$, $0.4 \leq y \leq 0.6$ for maximum and $x = \pm 2$, $-0.6 \leq y \leq -0.4$ for endpoints.

[3 marks]

Total [5 marks]

Question 5

(a) (i) $x = -4$ **A1**

(ii) attempt to substitute into $y = \frac{a}{c}$ OR table with large values of x OR sketch of f showing asymptotic behaviour **(M1)**

$y = 4$ **A1**

[3 marks]

(b) (i) $y = \frac{4x+1}{x+4}$

attempt to interchange x and y (seen anywhere) **M1**

$xy + 4y = 4x + 1$ OR $xy + 4x = 4y + 1$ **(A1)**

$xy - 4x = 1 - 4y$ OR $xy - 4y = 1 - 4x$ **(A1)**

$f^{-1}(x) = \frac{1-4x}{x-4}$ (accept $y = \frac{1-4x}{x-4}$) **A1**

(ii) reflection in y -axis given by $f(-x)$ **(M1)**

$f(-x) = \frac{-4x+1}{-x+4}$ **(A1)**

reflection of their $f(-x)$ in x -axis given by $-f(-x)$ accept "now $-f(x)$ " **M1**

$(-f(-x)) = -\frac{-4x+1}{-x+4}$

$= \frac{-4x+1}{x-4}$ OR $\frac{4x-1}{-x+4}$ **A1**

$= \frac{1-4x}{x-4}$ ($= f^{-1}(x)$) **AG**

Note: If the candidate attempts to show the result using a particular coordinate on the graph of f rather than a general coordinate on the graph of f , where appropriate, award marks as follows:

MOA0 for eg $(2,3) \rightarrow (-2,3)$

MOA0 for $(-2,3) \rightarrow (-2,-3)$

[8 marks]

- (c) (i) attempt to solve $f(x) = f^{-1}(x)$ using graph or algebraically (M1)
 $p = -1$ AND $q = 1$ (A1)

Note: Award (M1)A0 if only one correct value seen.

- (ii) attempt to set up an integral to find area between f and f^{-1} (M1)

$$\int_{-1}^1 \left(\frac{4x+1}{x+4} - \frac{1-4x}{x-4} \right) dx \quad (A1)$$

$$= 0.675231\dots$$

$$= 0.675$$

(A1)

[5 marks]

Total [16 marks]

Question 6

recognition that initial population is 15000 (seen anywhere) (A1)

$$P(0) = 15000 \text{ OR } 0.11 \times 15000 \text{ OR } 0.89 \times 15000$$

population after 11% decrease is $15000 \times 0.89 (=13350)$ (A1)

recognizing that $t = 8$ on 1 January 2022 (seen anywhere) (A1)

substitution of their value of t for 1 January 2022 and their value of $P(8)$ into the model (M1)

$$15000 \times 0.89 = 15000e^{8k} \text{ OR } 13350 = 15000e^{8k}$$

$$k = \frac{\ln 0.89}{8} (-0.014566) \quad (A1)$$

substitution of $t = 2041 - 2014 (= 27)$ and their value for k into the model (M1)

$$P(27) = 15000e^{-0.0145\dots \times 27}$$

$$10122.3\dots$$

$$P(27) = 10100 \text{ (10122)} \quad (A1)$$

Total [7 marks]