

Subject – Math AA(Standard Level)
Topic - Functions
Year - May 2021 – Nov 2022
Paper -2
Questions

Question 1

[Maximum mark: 7]

All living plants contain an isotope of carbon called carbon-14. When a plant dies, the isotope decays so that the amount of carbon-14 present in the remains of the plant decreases. The time since the death of a plant can be determined by measuring the amount of carbon-14 still present in the remains.

The amount, A , of carbon-14 present in a plant t years after its death can be modelled by $A = A_0 e^{-kt}$ where $t \geq 0$ and A_0, k are positive constants.

At the time of death, a plant is defined to have 100 units of carbon-14.

(a) Show that $A_0 = 100$. [1]

The time taken for half the original amount of carbon-14 to decay is known to be 5730 years.

(b) Show that $k = \frac{\ln 2}{5730}$. [3]

(c) Find, correct to the nearest 10 years, the time taken after the plant's death for 25% of the carbon-14 to decay. [3]

Question 2

[Maximum mark: 6]

The functions f and g are defined for $x \in \mathbb{R}$ by $f(x) = 6x^2 - 12x + 1$ and $g(x) = -x + c$, where $c \in \mathbb{R}$.

(a) Find the range of f . [2]

(b) Given that $(g \circ f)(x) \leq 0$ for all $x \in \mathbb{R}$, determine the set of possible values for c . [4]

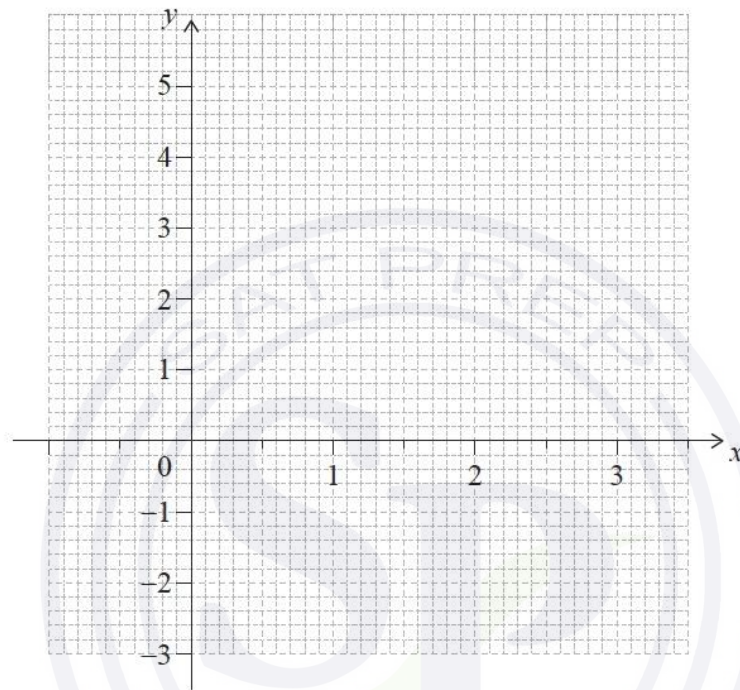
Question 3

[Maximum mark: 5]

Let $f(x) = 3x - 4^{0.15x^2}$ for $0 \leq x \leq 3$.

(a) Sketch the graph of f on the grid below.

[3]



(b) Find the value of x for which $f'(x) = 0$.

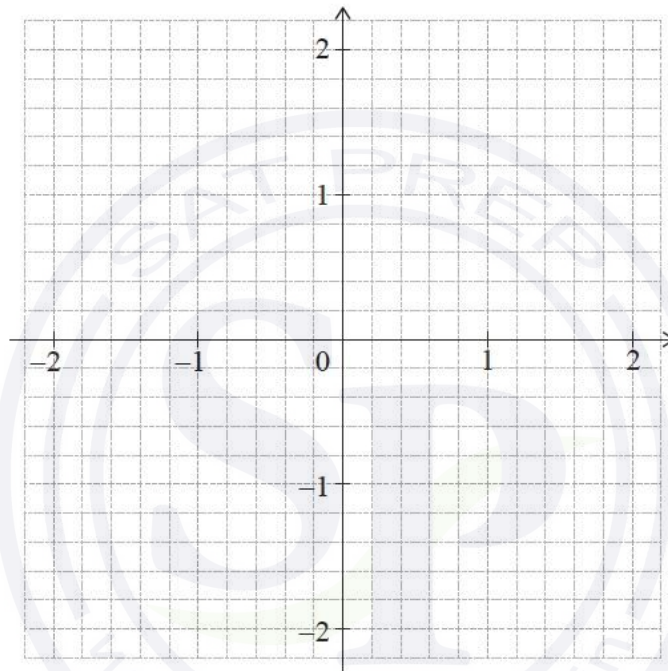
[2]

Question 4

[Maximum mark: 5]

Consider the function $f(x) = e^{-x^2} - 0.5$, for $-2 \leq x \leq 2$.

- (a) Find the values of x for which $f(x) = 0$. [2]
- (b) Sketch the graph of f on the following grid. [3]



Question 5

[Maximum mark: 16]

The function f is defined by $f(x) = \frac{4x+1}{x+4}$, where $x \in \mathbb{R}$, $x \neq -4$.

- (a) For the graph of f
- (i) write down the equation of the vertical asymptote;
 - (ii) find the equation of the horizontal asymptote. [3]
- (b) (i) Find $f^{-1}(x)$.
- (ii) Using an algebraic approach, show that the graph of f^{-1} is obtained by a reflection of the graph of f in the y -axis followed by a reflection in the x -axis. [8]

The graphs of f and f^{-1} intersect at $x = p$ and $x = q$, where $p < q$.

- (c) (i) Find the value of p and the value of q .
- (ii) Hence, find the area enclosed by the graph of f and the graph of f^{-1} . [5]

Question 6

[Maximum mark: 7]

The population of a town t years after 1 January 2014 can be modelled by the function

$$P(t) = 15\,000e^{kt}, \text{ where } k < 0 \text{ and } t \geq 0.$$

It is known that between 1 January 2014 and 1 January 2022 the population decreased by 11%.

Use this model to estimate the population of this town on 1 January 2041.