Subject – Math AA(Standard Level)
Topic - Geometry and Trigonometry
Year - May 2021 – Nov 2024
Paper -2
Answers

Question 1

(a) correct approach **A1**

eg $\frac{\pi}{6} = \frac{2\pi}{period}$ (or equivalent)

$\text{period} = 12$ **A1**

[2 marks]

(b) (i) valid approach ***(M1)***

eg $\frac{\max + \min}{2}$ $b = \max - \text{amplitude}$

$\frac{21.8 + 10.2}{2}$, or equivalent

$b = 16$ **A1**

(ii) attempt to substitute into **their** function ***(M1)***

$5.8\sin\left(\frac{\pi}{6}(6+1)\right) + 16$

$f(6) = 13.1$ **A1**

[4 marks]

(c) valid attempt to set up a system of equations ***(M1)***

two correct equations **A1**

$p\sin\left(\frac{2\pi}{9}(3-3.75)\right) + q = 2.5,\ p\sin\left(\frac{2\pi}{9}(6-3.75)\right) + q = 15.1$

valid attempt to solve system ***(M1)***

$p = 8.4;\ q = 6.7$ **A1A1**

[5 marks]

(d) attempt to use $|f(x) - g(x)|$ to find maximum difference ***(M1)***

$x = 1.64$ **A1**

[2 marks]

Total [13 marks]

Question 2

(a) $\frac{4.2}{60}\times 45$	***A1***	
$AB = 3.15$ (km)	***A1***	
		[2 marks]
(b) (i) 66° or $(180-114)$	***A1***	
$35+66$	***A1***	
$A\hat{B}C = 101^\circ$	***AG***	
(ii) attempt to use cosine rule	***(M1)***	
$AC^2 = 3.15^2 + 4.6^2 - 2\times 3.15\times 4.6\cos 101^\circ$ (or equivalent)	***A1***	
$AC = 6.05$ (km)	***A1***	
		[5 marks]
(c) valid approach to find angle BCA	***(M1)***	
eg sine rule		
correct substitution into sine rule	***A1***	
eg $\frac{\sin\left(B\hat{C}A\right)}{3.15} = \frac{\sin 101}{6.0507...}$		
$B\hat{C}A = 30.7^\circ$	***A1***	
		[3 marks]
(d) $B\hat{A}C = 48.267$ (seen anywhere)	***A1***	
valid approach to find correct bearing	***(M1)***	
eg $48.267+35$		
bearing $= 83.3^\circ$ (accept 083°)	***A1***	
		[3 marks]
(e) attempt to use time $= \frac{\text{distance}}{\text{speed}}$	***M1***	
$\frac{6.0507}{3.9}$ or 0.065768 km/min	***(A1)***	
$t = 93$ (minutes)	***A1***	
		[3 marks]
		Total [16 marks]

Question 3

(a) ***METHOD 1***

attempt to use the cosine rule **(M1)**

$\cos\theta = \dfrac{4^2 + 4^2 - 5^2}{2\times 4\times 4}$ (or equivalent) **A1**

$\theta = 1.35$ **A1**

[3 marks]

METHOD 2

attempt to split triangle AOB into two congruent right triangles **(M1)**

$\sin\left(\dfrac{\theta}{2}\right) = \dfrac{2.5}{4}$ **A1**

$\theta = 1.35$ **A1**

[3 marks]

(b) attempt to find the area of the shaded region **(M1)**

$\dfrac{1}{2}\times 4\times 4\times(2\pi - 1.35\ldots)$ **A1**

$= 39.5\ (\text{cm}^2)$ **A1**

[3 marks]

Total [6 marks]

Question 4

(a) $r = \dfrac{28}{\theta}$ **A1**

[1 mark]

(b) recognising sum of area of sector and area of triangle required **(M1)**

$\dfrac{1}{2}r^2\theta + \dfrac{1}{2}r\times r\times\sin(\pi-\theta)\ \left(= \dfrac{r^2}{2}(\theta + \sin(\pi-\theta)\right)$ **A1**

$\sin(\pi-\theta) = \sin\theta$ (substitution seen anywhere) **A1**

$\dfrac{1}{2}\left(\dfrac{28}{\theta}\right)^2\theta + \dfrac{1}{2}\left(\dfrac{28}{\theta}\right)^2\sin\theta$ OR $\dfrac{1}{2}\left(\dfrac{28}{\theta}\right)^2(\theta+\sin\theta)$ **A1** area $= \frac{392}{\theta^2}(\theta + \sin\theta)$ **AG**

[4 marks]

(c) $\frac{392}{\theta^2}(\theta + \sin\theta) = 460$ ***(M1)***

$\theta = 1.43917...$

$\theta = 1.44$ ***A1***

[2 marks]

(d) $\frac{\pi - (\pi - \theta)}{2}$ OR $\frac{\theta}{2}$ ***(M1)***

$\text{D}\hat{\text{A}}\text{E} = 0.719588...$
$\text{D}\hat{\text{A}}\text{E} = 0.720$ ***A1***

[2 marks]

(e) (i) $\text{A}\hat{\text{B}}\text{C} = 195 - 180 + 90$ ***(A1)***
$= 105°$ ***A1***

(ii) choosing sine rule ***(M1)***

$\frac{\text{BC}}{\sin \text{D}\hat{\text{A}}\text{E}} = \frac{800}{\sin 105}$ OR $\frac{\text{BC}}{\sin \text{D}\hat{\text{A}}\text{E}} = \frac{800}{\sin 1.83}$ ***A1***

$\text{BC} = 546 \text{ (m)}$ ***A1***

[5 marks]
Total [14 marks]

Question 5

amplitude is $\frac{110}{2} = 55$ ***(A1)***

$a = -55$ ***A1***

$c = 65$ ***A1***

$\frac{2\pi}{b} = 20$ OR $-55\cos(20b) + 65 = 10$ ***(M1)***

$b = \frac{\pi}{10} (= 0.314)$ ***A1***

Total [5 marks]

Question 6

(a) $\tan 0.6 = \dfrac{h}{12}$ **(M1)**

$8.20964...$

8.21 (m) **A1**

[2 marks]

(b) $\tan B = \dfrac{8.2096...}{5}$ OR $\tan^{-1}1.6419...$ **(A1)**

$1.02375...$

1.02 (radians) (accept $58.7°$) **A1**

[2 marks]

(c) $x + 1.8 + 2.5 = 8.20964...$ (or equivalent) **(A1)**

$3.90964...$

3.91 (m) **A1**

[2 marks]

(d) **METHOD 1**

recognition that blade length = amplitude, $p = \dfrac{\max - \min}{2}$ **(M1)**

$p = 3.91$ **A1**

centre of windmill = vertical shift, $q = \dfrac{\max + \min}{2}$ **(M1)**

$q = 8.21$ **A1**

METHOD 2

attempting to form two equations in terms of p and q **(M1) (M1)**

$12.1192... = p\cos\left(\dfrac{3\pi}{10} \cdot 0\right) + q$, $4.3000... = p\cos\left(\dfrac{3\pi}{10} \cdot \dfrac{10}{3}\right) + q$

$p = 3.91$ **A1**

$q = 8.21$ **A1**

[4 marks]

(e) appropriate working towards finding the period **(M1)**

$$\text{period} = \frac{2\pi}{\frac{3\pi}{10}} \left(=6.6666\ldots\right)$$

rotations per minute $= \dfrac{60}{\text{their period}}$ **(M1)**

$n = 9$ (must be an integer) (accept $n = 10$, $n = 18$, $n = 19$) **A1**

[3 marks]

Total [13 marks]

Question 7

(a) $12 = \dfrac{2\pi}{b}$ OR $b = \dfrac{2\pi}{12}$ **A1**

$b = \dfrac{\pi}{6}$ **AG**

[1 mark]

(b) $a = \dfrac{6.8 - 2.2}{2}$ OR $a = \dfrac{\max - \min}{2}$ **(M1)**

$= 2.3$ (m) **A1**

[2 marks]

(c) $d = \dfrac{6.8 + 2.2}{2}$ OR $d = \dfrac{\max + \min}{2}$ **(M1)**

$= 4.5$ (m) **A1**

[2 marks]

(d) **METHOD 1**

substituting $t = 4.5$ and $H = 6.8$ for example into their equation for H ***(A1)***

$$6.8 = 2.3\sin\left(\frac{\pi}{6}(4.5 - c)\right) + 4.5$$

attempt to solve their equation ***(M1)***

$c = 1.5$ ***A1***

METHOD 2

using horizontal translation of $\frac{12}{4}$ ***(M1)***

$4.5 - c = 3$ ***(A1)***

$c = 1.5$ ***A1***

METHOD 3

$H'(t) = (2.3)\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}(t - c)\right)$ ***(A1)***

attempts to solve their $H'(4.5) = 0$ for c ***(M1)***

$$(2.3)\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}(4.5 - c)\right) = 0$$

$c = 1.5$ ***A1***

[3 marks]

(e) attempt to find H when $t = 12$ or $t = 0$, graphically or algebraically ***(M1)***

$H = 2.87365...$

$H = 2.87\,(\text{m})$ ***A1***

[2 marks]

(f) attempt to solve $5=2.3\sin\left(\frac{\pi}{6}(t-1.5)\right)+4.5$ ***(M1)***

times are $t=1.91852...$ and $t=7.08147...$, $(t=13.9185...,t=19.0814...)$ ***(A1)***

total time is $2\times(7.081...-1.919...)$

$10.3258...$

$=10.3$ (hours) ***A1***

Note: Accept 10.

[3 marks]

Total [13 marks]

Question 8

(a) (i) $\frac{\text{AP}}{42}$ OR $\frac{215}{84}$ OR $\frac{65}{42}+\frac{215}{84}$ ***(M1)***

time = $4.10714\ldots$ (hours)

time = 4.11 (hours) ***A1***

(ii) $\text{AB}=\sqrt{215^2+65^2}\ (=224.610...)$ ***(A1)***

time = $5.34787\ldots$ (hours)

time = 5.35 (hours) ***A1***

[4 marks]

(b) (i) $\text{AD}=\sqrt{(215-x)^2+65^2}$ ***(A1)***

$t=\frac{\sqrt{(215-x)^2+65^2}}{42}$ ***(A1)***

$T=\frac{\sqrt{(215-x)^2+65^2}}{42}+\frac{x}{84}\left(=\frac{\sqrt{x^2-430x+50450}}{42}+\frac{x}{84}\right)$ ***A1***

(ii) valid approach to find the minimum for T (may be seen in (iii)) ***(M1)***

graph of T OR $T'=0$ OR graph of T'

$x=177.472...$ km

$x=177$ km ***A1***

(iii) $T = 3.89980...$

$T = 3.90$ (hours) ***A1***

(c) (i) $C = 200 \cdot \frac{\sqrt{(215-x)^2 + 65^2}}{42} + 150 \cdot \frac{x}{84}$ ***(A1)***

valid approach to find the minimum for $C(x)$ (may be seen in (ii)) ***(M1)***

graph of C OR $C' = 0$ OR graph of C'

$x = 188.706...$ km

$x = 189$ km ***A1***

Note: Only allow ***FT*** from (b) if the function T has a minimum in $0 < x < 215$.

(ii) $C = 670.864$

$C = \$671$ ***A1***

Note: Only allow ***FT*** from (c)(i) if the function C has a minimum in $0 < x < 215$.

[4 marks]

Total [14 marks]

Question 9

(a) attempt to find the area of either shaded region in terms of r and θ ***(M1)***

Note: Do not award ***M1*** if they have only copied from the booklet and not applied to the shaded area.

Area of segment $=\frac{1}{2}r^2\theta-\frac{1}{2}r^2\sin\theta$ ***A1***

Area of triangle $=\frac{1}{2}r^2\sin(\pi-\theta)$ ***A1***

correct equation in terms of θ only ***(A1)***

$\theta-\sin\theta=\sin(\pi-\theta)$

$\theta-\sin\theta=\sin\theta$ ***A1***

$\theta=2\sin\theta$ ***AG***

Note: Award a maximum of ***M1A1A0A0A0*** if a candidate uses degrees (i.e., $\frac{1}{2}r^2\sin(180°-\theta)$), even if later work is correct.

Note: If a candidate directly states that the area of the triangle is $\frac{1}{2}r^2\sin\theta$, award a maximum of ***M1A1A0A1A1***.

[5 marks]

(b) $\theta=1.89549...$

$\theta=1.90$ ***A1***

Note: Award ***A0*** if there is more than one solution. Award ***A0*** for an answer in degrees.

[1 mark]

Total [6 marks]

Question 10

EITHER

attempt to use cosine rule *(M1)*

$12^2 + AB^2 - 2\times12\times\cos25°\times AB = 7^2$ OR $AB^2 - 21.7513...AB + 95 = 0$ *(A1)*

at least one correct value for AB *(A1)*

$AB = 6.05068...$ OR $AB = 15.7007...$

using their smaller value for AB to find minimum perimeter *(M1)*

$12 + 7 + 6.05068...$

OR

attempt to use sine rule *(M1)*

$\frac{\sin B}{12} = \frac{\sin 25°}{7}$ OR $\sin B = 0.724488...$ OR $\hat{B} = 133.573...°$ OR $\hat{B} = 46.4263...°$ *(A1)*

at least one correct value for C *(A1)*

$\hat{C} = 21.4263...°$ OR $\hat{C} = 108.573...°$

using their acute value for $\hat{C}$ to find minimum perimeter *(M1)*

$12 + 7 + \sqrt{12^2 + 7^2 - 2\times12\times7\cos21.4263...°}$ OR $12 + 7 + \frac{7\sin21.4263...°}{\sin25°}$

THEN

$25.0506\ldots$

minimum perimeter $= 25.1$. ***A1***

Total [5 marks]

Question 11

(a) attempt to use the distance formula to find AV *(M1)*

$\sqrt{(1-(-1))^2+(5-1)^2+(0-6)^2}$

$=7.48331...$

$=7.48\ \text{(cm)}\ \left(=\sqrt{56}\text{ or }2\sqrt{14}\right)$ ***A1***

[2 marks]

(b) **METHOD 1**

attempt to apply cosine rule OR sine rule to find AB *(M1)*

$(\text{AB}=)\ \sqrt{7.48...^2+7.48...^2-2\times7.48...\times7.48...\cos\left(40^\circ\right)}$ OR $\frac{\text{AB}}{\sin 40^\circ}=\frac{\sqrt{56}}{\sin 70^\circ}$ *(A1)*

$=5.11888...$

$=5.12\ \text{(cm)}$ ***A1***

METHOD 2

Let M be the midpoint of [AB]

attempt to apply right-angled trigonometry on triangle AVM *(M1)*

$=2\times7.48...\times\sin(20^\circ)$ *(A1)*

$=5.11888...$

$=5.12\ \text{(cm)}$ ***A1***

[3 marks]

(c) **METHOD 1**

equating volume of pyramid formula to 57.2 *(M1)*

$\frac{1}{3}\times5.11...^2\times h=57.2$ *(A1)*

$h=6.54886...$

$h=6.55\ \text{(cm)}$ ***A1***

METHOD 2

Let M be the midpoint of [AB]

$AV^2 = AM^2 + MX^2 + XV^2$ *(M1)*

$\Rightarrow XV = \sqrt{7.48...^2 - \left(\frac{5.11...}{2}\right)^2 - \left(\frac{5.11...}{2}\right)^2}$ *(A1)*

$h = 6.54886...$

$h = 6.55$ (cm) ***A1***

[3 marks]

(d) $V = x \times 2x \times y = 57.2$ *(A1)*

$S = 2\left(2x^2 + xy + 2xy\right)$ ***A1***

Note: Condone use of A.

attempt to substitute $y = \frac{57.2}{2x^2}$ into their expression for surface area *(M1)*

$\left(S(x) =\right)\ 4x^2 + 6x\left(\frac{57.2}{2x^2}\right)$

EITHER

attempt to find minimum turning point on graph of area function *(M1)*

OR

$\frac{dS}{dx} = 8x - 171.6x^{-2} = 0$ OR $x = 2.77849...$ *(M1)*

THEN

$92.6401...$

minimum surface area $= 92.6$ (cm^2) ***A1***

[5 marks]

Total [13 marks]

Question 12

(a) valid approach to find area of segment by finding area of sector – area of triangle *(M1)*

$\frac{1}{2}r^2\theta - \frac{1}{2}r^2 \sin\theta$

$\frac{1}{2}(2)^2\theta - \frac{1}{2}(2)^2 \sin\theta$ *(A1)*

area = $2\theta - 2\sin\theta$ ***A1***

[3 marks]

(b) **EITHER**

area of logo = area of rectangle – area of segments *(M1)*

$5\times4-2\times(2\theta-2\sin\theta)=13.4$ *(A1)*

OR

area of one segment = $\frac{20-13.4}{2}$ $(=3.3)$ *(M1)*

$2\theta-2\sin\theta=3.3$ *(A1)*

THEN

$\theta=2.35672...$

$\theta=2.36$ (do not accept an answer in degrees) ***A1***

Note: Award ***(M1)(A1)A0*** if there is more than one solution.
Award ***(M1)(A1FT)A0*** if the candidate works in degrees and obtains a final answer of $135.030\ldots$

[3 marks]

Total [6 marks]

Question 13

(a) (i) 32 (cm) **A1**

(ii) $h_A(0) = \sin(6) + 27$ ***(M1)***

$= 26.7205...$

$= 26.7$ (cm) **A1**

[3 marks]

(b) attempts to solve $h_A(t) = h_B(t)$ for t ***(M1)***

$t = 4.00746..., 4.70343..., 5.88332...$

$t = 4.01, 4.70, 5.88$ (weeks) **A2**

[3 marks]

(c) recognises that $h_A'(t)$ and $h_B'(t)$ are required ***(M1)***

attempts to solve $h_A'(t) = h_B'(t)$ for t ***(M1)***

$t = 1.18879...$ and $2.23598...$ OR $4.33038...$ and $5.37758...$ OR $7.47197...$ and $8.51917...$ ***(A1)***

> **Note:** Award full marks for $t = \frac{4\pi}{3} - 3, \frac{5\pi}{3} - 3, \left(\frac{7\pi}{3} - 3, \frac{8\pi}{3} - 3, \frac{10\pi}{3} - 3, \frac{11\pi}{3} - 3\right)$.
> *Award* subsequent marks for correct use of these exact values.

$1.18879... < t < 2.23598...$ OR $4.33038... < t < 5.37758...$ OR
$7.47197... < t < 8.51917...$ ***(A1)***

attempts to calculate the total amount of time ***(M1)***

$3(2.2359... - 1.1887...)$ $\left(= 3\left(\left(\frac{5\pi}{3} - 3\right) - \left(\frac{4\pi}{3} - 3\right)\right)\right)$

$= 3.14$ $(= \pi)$ (weeks) **A1**

[6 marks]

Total [12 marks]

Question 14

(a) $A\hat{B}C = 27°$ *(A1)*

attempt to substitute into cosine rule *(M1)*

$175^2 + 230^2 - 2(175)(230)\cos 27°$ *(A1)*

108.62308…

$AC = 109$ (m) ***A1***

[4 marks]

(b) correct substitution into area formula *(A1)*

$\frac{1}{2} \times 175 \times 230 \times \sin 27°$

9136.55...

$\text{area} = 9140\left(\text{m}^2\right)$ ***A1***

[2 marks]

(c) attempt to substitute into sine rule or cosine rule *(M1)*

$\frac{\sin 27°}{108.623...} = \frac{\sin \hat{A}}{175}$ OR $\cos A = \frac{(108.623...)^2 + 230^2 - 175^2}{2 \times 108.623... \times 230}$ *(A1)*

47.0049…

$C\hat{A}B = 47.0°$ ***A1***

[3 marks]

(d) **METHOD 1**

recognizing that for areas to be equal, AD=DC *(M1)*

$AD = \frac{1}{2}AC = 54.3115...$ ***A1***

attempt to substitute into cosine rule to find BD *(M1)*

correct substitution into cosine rule *(A1)*

$BD^2 = 230^2 + 54.3115^2 - 2(230)(54.3115)\cos 47.0049°$

$BD = 197.009...$

$BD = 197(\text{m})$ ***A1***

[5 marks]

METHOD 2

correct expressions for areas of triangle BDA and triangle BCD using BD **_A1_**

$\frac{1}{2}\times \mathrm{BD}\times 230\times \sin x^\circ$ and $\frac{1}{2}\times \mathrm{BD}\times 175\times \sin(27-x)^\circ$ OR

$\frac{1}{2}\times \mathrm{BD}\times 230\times \sin(27-x)^\circ$ and $\frac{1}{2}\times \mathrm{BD}\times 175\times \sin x^\circ$

correct equation in terms of x **_(A1)_**

$175\sin(27-x)=230\sin x$ or $175\sin\mathrm{x}=230\sin(27-x)$

$x=11.6326...$ or $x=15.3673...$ **_(A1)_**

substituting their value of x into equation to solve for BD **_(M1)_**

$\frac{1}{2}\times \mathrm{BD}\times 230\times \sin 11.6326...=\frac{1}{2}\times \mathrm{BD}\times 175\times \sin 15.3673...$ or

$\frac{1}{2}\times \mathrm{BD}\times 230\times \sin 11.6326...=\frac{1}{2}\times 9136.55...$

$\mathrm{BD}=197(\mathrm{m})$ **_A1_**

[5 marks]

Total [14 marks]

Question 15

(a) **EITHER**

uses the cosine rule *(M1)*

$AB^2 = 5^2 + 5^2 - 2 \times 5 \times 5 \times \cos 1.9$ *(A1)*

OR

uses right-angled trigonometry *(M1)*

$\dfrac{\frac{AB}{2}}{5} = \sin 0.95$ *(A1)*

OR

uses the sine rule *(M1)*

$\alpha = \frac{1}{2}(\pi - 1.9)(= 0.6207...)$

$\dfrac{AB}{\sin 1.9} = \dfrac{5}{\sin 0.6207...}$ *(A1)*

THEN

$AB = 8.13415...$

$AB = 8.13$ (m) ***A1***

[3 marks]

(b) let the shaded area be A

METHOD 1

attempt at finding reflex angle *(M1)*

$A\hat{O}B = 2\pi - 1.9 \ (= 4.3831...)$

substitution into area formula *(A1)*

$A = \frac{1}{2} \times 5^2 \times 4.3831...$ OR $\left(\dfrac{2\pi - 1.9}{2\pi}\right) \times \pi\left(5^2\right)$

$= 54.7898...$

$= 54.8$ (m^2) ***A1***

METHOD 2

let the area of the circle be A_C and the area of the unshaded sector be A_U

$A = A_C - A_U$ ***(M1)***

$A = \pi \times 5^2 - \frac{1}{2} \times 5^2 \times 1.9 \ (= 78.5398... - 23.75)$ ***(A1)***

$= 54.7898...$

$= 54.8$ (m^2) ***A1***

[3 marks]

Total [6 marks]

Question 16

(a) attempt to use right angled trigonometry or sine rule to find AE in terms of r and α ***(M1)***

$\tan\alpha = \frac{r}{\text{AE}}$ OR $\frac{\text{AE}}{\sin\left(\frac{\pi}{2} - \alpha\right)} = \frac{r}{\sin\alpha}$

$\text{AE} = \frac{r}{\tan\alpha}$ OR $\text{AE} = \frac{r\sin\left(\frac{\pi}{2} - \alpha\right)}{\sin\alpha}$ OR $\text{AE} = \frac{r\cos\alpha}{\sin\alpha}$ ***A1***

valid approach to find the area of ADOE ***(M1)***

2x area of triangle AOE OR area of triangle AED + area of triangle OED OR $\text{OE} \times \text{AE}$

Area ADOE = $2\left(\frac{1}{2} \cdot \frac{r}{\tan\alpha} \cdot r\right)$ OR $r \times \text{AE}$ ***A1***

Area ADOE = $\frac{r^2}{\tan\alpha}$ ***AG***

[4 marks]

(b) (i) recognizing that the sum of the angles of a kite is 2π ***(M1)***

$$\mathrm{D\hat{O}E} + \mathrm{O\hat{E}A} + \mathrm{E\hat{A}D} + \mathrm{A\hat{D}O} = 2\pi \text{ OR } 2\alpha + 2\cdot\frac{\pi}{2} + \mathrm{D\hat{O}E} = 2\pi$$

$\mathrm{D\hat{O}E} = \pi - 2\alpha$ ***A1***

Note: Award ***M1A0*** if candidate uses degrees (i.e.

$\mathrm{D\hat{O}E} + \mathrm{O\hat{E}A} + \mathrm{E\hat{A}D} + \mathrm{A\hat{D}O} = 360°$ or $2\alpha + 2\cdot\frac{\pi}{2} + \mathrm{D\hat{O}E} = 360°$) and obtains

$\mathrm{D\hat{O}E} = 180° - 2\alpha$.

(ii) valid approach to find the area of R ***(M1)***

area of kite – area of sector OR 2(area of triangle AOE – 0.5 area of sector OED)

Area of sector = $\frac{1}{2}r^2.\mathrm{D\hat{O}E}\left(=\frac{1}{2}r^2(\pi - 2\alpha)\right)$ seen anywhere ***(A1)***

Area of R = $\frac{r^2}{\tan\alpha} - \frac{1}{2}r^2(\pi - 2\alpha)$ ***A1***

Note: Accept $\frac{r^2}{\tan\alpha} - \frac{1}{2}r^2.\mathrm{D\hat{O}E}$.

[5 marks]

(c) equating their area formula to πr^2 **(M1)**

$$\frac{r^2}{\tan\alpha}-\frac{1}{2}r^2\left(\pi-2\alpha\right)=\pi r^2$$

correct equation in terms of α **A1**

$$\frac{1}{\tan\alpha}-\frac{1}{2}\left(\pi-2\alpha\right)=\pi$$

valid approach to solve the equation **(M1)**

$\alpha = 0.218979...$

$\alpha = 0.219$ **A1**

[4 marks]

Total [13 marks]

Question 17

(a) attempt to use sine rule **(M1)**

$$\frac{24}{\sin 113^\circ}=\frac{17}{\sin \mathrm{B\hat{A}C}}$$ OR $\left(\sin \mathrm{B\hat{A}C}=\right)0.652024...$ **(A1)**

$40.6943\ldots$

$\mathrm{B\hat{A}C} = 40.7^\circ$ **A1**

[3 marks]

(b) **METHOD 1** (cosine rule with $\hat{ABC}$ or $\hat{BAC}$)

attempt to use the cosine rule ***(M1)***

$24^2 = AB^2 + 17^2 - 2 \cdot 17 \cdot AB \cdot \cos 113°$ $\left(AB^2 + 13.2848...AB - 287 = 0\right)$ OR

$17^2 = AB^2 + 24^2 - 2 \cdot 24 \cdot AB \cdot \cos 40.6943...°$ $\left(AB^2 - 36.3935...AB + 287 = 0\right)$ ***(A1)***

$11.5543\ldots$

$AB = 11.6$ ***A1***

METHOD 2 (cosine rule with $B\hat{C}A$)

attempt to use cosine rule ***(M1)***

correct substitution ***(A1)***

$AB^2 = 17^2 + 24^2 - 2 \cdot 17 \cdot 24 \cdot \cos 26.3056...°$ OR $AB^2 = 133.502...$

$11.5543\ldots$

$AB = 11.6$ ***A1***

METHOD 3 (sine rule)

attempt to use sine rule ***(M1)***

correct substitution ***(A1)***

$\frac{AB}{\sin 26.3056...°} = \frac{24}{\sin 113°} = \frac{17}{\sin 40.6943...°}$ OR $AB = \frac{24 \cdot \sin\left(180° - 113° - 40.6943...°\right)}{\sin 113°}$

$11.5543\ldots$

$AB = 11.6$ ***A1***

[3 marks]

Total [6 marks]

Question 18

(a) Let N be North

$N\hat{J}D = 34°$ OR $D\hat{J}L = 56°$ (must be labelled or indicated in diagram): ***(A1)***

$J\hat{D}L = 99(°)$ ***A1***

Note: Accept $\frac{11\pi}{20}, 1.73$ (radians).

[2 marks]

(b) attempt to apply the sine rule ***(M1)***

$\frac{DL}{\sin 56°} = \frac{500}{\sin 99°}$ OR $\frac{DL}{\sin 0.977384...} = \frac{500}{\sin 1.72787...}$ ***(A1)***

$419.685...$

$DL = 420$ (km) ***A1***

Note: Award ***M1A1A0*** for 261 (km) from use of degrees with GDC set in radians (with or without working).

[3 marks]

Total [5 marks]

Question 19

(a) use of sector area formula to find area of at least one sector **(M1)**

$\frac{1}{2}\times 4.8\times 100-\frac{1}{2}\times 4.8\times r^2$ OR $10^2\pi-\pi r^2-\left(\frac{1}{2}10^2\times(2\pi-4.8)-\frac{1}{2}\times(2\pi-4.8)r^2\right)$ **A1**

(area) $=240-2.4r^2$ **AG**

[2 marks]

(b) (i) $240-2.4r^2=176$ **(A1)**

$r=5.16397\ldots$

$=5.16$ (cm) $\left(\frac{4\sqrt{15}}{3}\text{ exact}\right)$ **A1**

(ii) 10×4.8 OR $5.16\ldots\times 4.8$ **(A1)**

substituting their value of r into $10\times 4.8+r\times 4.8+2(10-r)$ (or equivalent) **(M1)**

Perimeter $=10\times 4.8+5.16\ldots\times 4.8+2(10-5.16\ldots)$ $(=48+24.7870\ldots+9.67204\ldots)$

$= 82.4591\ldots$

$= 82.5$ (cm) (82.4 from 3 sf) **A1**

[5 marks]

Total [7 marks]

Question 20

(a) $BV = \sqrt{(8-4)^2 + (6-3)^2 + (0-10)^2}$ ***(A1)***

$= 11.1803...$

$= 11.2\ (= \sqrt{125} = 5\sqrt{5})$ ***A1***

[2 marks]

(b) **METHOD 1**

$BV = VC$ AND $BC = 6$ (seen anywhere) ***(A1)***

attempt to use the cosine rule on triangle BVC for any angle ***(M1)***

Note: Recognition must be shown in context either in terms of labelled sides or side lengths.

$\cos B\hat{V}C = \dfrac{11.1...^2 + 11.1...^2 - 6^2}{2 \times 11.1... \times 11.1...}$ OR

$6^2 = 11.1....^2 + 11.1...^2 - 2 \times 11.1... \times 11.1... \cos B\hat{V}C$ ***(A1)***

$B\hat{V}C = 0.543314...$

$B\hat{V}C = 0.543$ (0.542 from 3 sf) (accept 31.1°) ***A1***

METHOD 2

let M be the midpoint of BC

$\text{BM} = 3$ (seen anywhere) ***(A1)***

attempt to use sine or cosine in triangle BMV or CMV ***(M1)***

$\arcsin\frac{3}{\sqrt{125}}$ OR $\frac{\pi}{2} - \arccos\frac{3}{\sqrt{125}}$ OR $0.271657\ldots$ ***(A1)***

$\text{B}\hat{\text{V}}\text{C} = 0.543314\ldots$

$\text{B}\hat{\text{V}}\text{C} = 0.543$ (0.542 from 3 sf) (accept 31.1°) ***A1***

[4 marks]

Total [6 marks]

Question 21

(a) $7.8 = \frac{2\pi}{\text{period}}$ ***(M1)***

$\frac{2\pi}{7.8} = 0.805536\ldots$

$\text{period} = 0.806 \left(= \frac{10\pi}{39}\right)$ ***A1***

[2 marks]

(b) **METHOD 1**

(i) $\text{amplitude} = \dfrac{\max - \min}{2}$ ***(M1)***

$\dfrac{1.8-1}{2}$

$a = -0.4$ ***A1***

(ii) $b = 1.4$ ***A1***

METHOD 2

attempt to form two simultaneous equations in a and b ***(M1)***

$H(0) = 1 \Rightarrow a + b = 1,\ H\left(\dfrac{\pi}{7.8}\right) = 1.8 \Rightarrow -a + b = 1.8$

$a = -0.4,\ b = 1.4$ ***A1A1***

[3 marks]

(c) **EITHER**

$\dfrac{5}{\text{period}} = 6.207\ldots < 6\tfrac{1}{2}$ ***(A1)***

OR

consideration of number of maximums on graph in first 5 seconds ***(A1)***

OR

maximums when $t = 0.403, 1.21, 2.01, 2.82, 3.62, 4.43$ ***(A1)***

THEN

6 times ***A1***

[2 marks]

(d) recognizing that $H(t) = 1.5$ ***(M1)***

$-0.4\cos(7.8t) + 1.4 = 1.5$

$0.233779\ldots$

$t = 0.234$ (seconds) ***A1***

[2 marks]

[Total Marks 9]

Question 22

(a) attempt to substitute into cosine rule **(M1)**

$154^2 = 150^2 + 90^2 - 2(150)(90)\cos A\hat{P}B$ OR $\cos A\hat{P}B = \dfrac{150^2 + 90^2 - 154^2}{2(150)(90)}$ **(A1)**

$A\hat{P}B = 75.2286...^\circ$ OR $1.31298...$ radians

$A\hat{P}B = 75.2^\circ$ OR 1.31 radians **A1**

[3 marks]

(b) valid approach to find θ **(M1)**

$\theta = \dfrac{180^\circ - A\hat{P}B}{2}$ OR $\theta = \dfrac{180^\circ - 75.2286...^\circ}{2}$ $(= 52.3856...)$ OR

$\theta = \dfrac{\pi - 1.31298...}{2} (= 0.914302...)$

valid approach to express h in terms of θ **(M1)**

$\sin\theta = \dfrac{h}{150}$ OR $h = 150\sin 52.3856...^\circ$

$h = 118.820...$

$h = 119$ (m) **A1**

[3 marks]

Total [6 marks]

Question 23

(a) use of sector area formula to find area of at least one sector **(M1)**

$\frac{1}{2}\times5.2\times100-\frac{1}{2}\times5.2\times r^2$ OR $10^2\pi-\frac{1}{2}10^2\times(2\pi-5.2)-\left(\pi r^2-\frac{1}{2}\times(2\pi-5.2)\times r^2\right)$ **A1**

$(\text{area})=260-2.6r^2$ **AG**

[2 marks]

(b) (i) $260-2.6r^2=64$ **(A1)**

$r=8.68243\ldots$

$=8.68\,(\text{cm})\left(\frac{14\sqrt{65}}{13}\text{ exact}\right)$ **A1**

(ii) 10×5.2 OR $8.68\ldots\times5.2$ **(A1)**

substituting their value of r into $10\times5.2+r\times5.2+2(10-r)$ (or equivalent) **(M1)**

Perimeter = $10\times5.2+8.68\ldots\times5.2+2(10-8.68\ldots)$ $(=52+45.1486\ldots+2.63513\ldots)$

= 99.7837…

= 99.8 (cm) **A1**

[5 marks]

Total [7 marks]

Question 24

(a) $BV=\sqrt{(6-3)^2+(8-4)^2+(0-9)^2}$ ***(A1)***

$=10.2956...$

$=10.3\ (=\sqrt{106})$ ***A1***

[2 marks]

(b) **METHOD 1**

$BV=VC$ AND $BC=8$ (seen anywhere) ***(A1)***

attempt to use the cosine rule on triangle BVC for any angle ***(M1)***

$\cos B\hat{V}C=\dfrac{10.2...^2+10.2...^2-8^2}{2\times 10.2...\times 10.2...}$ OR

$8^2=10.2....^2+10.2...^2-2\times 10.2...\times 10.2...\cos B\hat{V}C$ ***(A1)***

$B\hat{V}C=0.798037...$

$B\hat{V}C=0.798$ (accept 45.7°) ***A1***

METHOD 2

let M be the midpoint of BC

$BM=4$ (seen anywhere) ***(A1)***

attempt to use sine or cosine in triangle BMV or CMV ***(M1)***

$\arcsin\dfrac{4}{\sqrt{106}}$ OR $\dfrac{\pi}{2}-\arccos\dfrac{4}{\sqrt{106}}$ OR 0.399018 ***(A1)***

$B\hat{V}C=0.798037...$

$B\hat{V}C=0.798$ (accept 45.7°) ***A1***

[4 marks]

Total [6 marks]

Question 25

(a) **METHOD 1**

let M be the midpoint of $[\mathrm{AB}]$ and so $\mathrm{AB} = 2\mathrm{AM}$

attempts to use Pythagoras' theorem to find AM^2 OR AM ***(M1)***

$\mathrm{AM}^2 = 20^2 - 14^2 \ (= 204)$ OR $\mathrm{AM} = \sqrt{20^2 - 14^2} \ \left(= 14.2828... = \sqrt{204} = 2\sqrt{51}\right)$

recognizes that $\mathrm{AB} = 2\mathrm{AM}$ ***(A1)***

$\mathrm{AB} = 2 \times 14.2828... \ (= 28.5657...) \ \left(= 2\sqrt{204} = 4\sqrt{51}\right)$ ***A1***

$\mathrm{AB} = 28.5657...$

$\mathrm{AB} = 28.57$ (m) ***AG***

METHOD 2

let M be the midpoint of $[\mathrm{AB}]$ and so $\mathrm{AB} = 2\mathrm{AM}$

let $\theta = \mathrm{A\hat{S}M}$

$\theta = 0.795398... \left(= \cos^{-1}\frac{14}{20}\right)$ ***(A1)***

attempts to use a valid trigonometric ratio ***(M1)***

EITHER

$\mathrm{AM} = 14\tan(0.795398...) \left(= 14.2828... = 14\tan\left(\cos^{-1}\frac{14}{20}\right)\right)$ ***A1***

OR

$\mathrm{AM} = 20\sin(0.795398...) \left(= 14.2828... = 20\sin\left(\cos^{-1}\frac{14}{20}\right)\right)$ ***A1***

THEN

$\mathrm{AB} = 28.5657...$

$\mathrm{AB} = 28.57$ (m) ***AG***

[3 marks]

(b) **EITHER**

the sprinkler rotates through (an angle of) 2π (radians) every 16 seconds and

hence rotates through $\frac{2\pi}{16}$ (radians) in 1 second ***A1***

OR

$\left(\frac{2\pi}{n}=16 \Rightarrow n=\right)\frac{2\pi}{16}\ \left(=\frac{\pi}{8}\right)$ ***A1***

THEN

sprinkler rotates through an angle of $\frac{\pi}{8}$ radians in one second ***AG***

[1 mark]

(c)

attempts to find 2θ where $\theta = \mathrm{A\hat{S}M}$ ***(M1)***

$$=2(0.795398...) \quad \left(1.59079... = 2\cos^{-1}\frac{14}{20}\right)$$

uses $\frac{\theta}{t}$ (rad/s) or similar to form an equation involving T ***(M1)***

$$\frac{2\pi}{16} = \frac{1.59079...}{T} \quad \left(\frac{2\pi}{16} = \frac{2\cos^{-1}\frac{14}{20}}{T}\right)$$ ***(A1)***

$$T = 4.05093... \quad \left(= \frac{1.59079...}{\frac{2\pi}{16}}\right)\left(= \frac{2\cos^{-1}\frac{14}{20}}{\frac{2\pi}{16}}\right)$$

$T = 4.05$ (s) ***A1***

[4 marks]

(d) $\alpha = \frac{\pi t}{8}$ ***A1***

[1 mark]

(e) applies sine rule in ΔASD ***A1***

$$\frac{d}{\sin\alpha} = \frac{20}{\sin \text{A}\hat{\text{D}}\text{S}}$$

attempts to find $\text{A}\hat{\text{D}}\text{S}$ in terms of α ***M1***

$$\text{A}\hat{\text{D}}\text{S} = \pi - \beta - \alpha \ \ (= \pi - 0.7754 - \alpha)(= 2.366... - \alpha) \ \ (= 2.37 - \alpha)$$

$$d = \frac{20\sin\alpha}{\sin(2.366... - \alpha)} \left(= \frac{20\sin\alpha}{\sin(2.37 - \alpha)} \right) \text{ (accept } d = \frac{20\sin\alpha}{\sin(\pi - \beta - \alpha)} \text{)}$$ ***A1***

$$d = \frac{20\sin\left(\frac{\pi t}{8}\right)}{\sin\left(2.37 - \frac{\pi t}{8}\right)}$$ ***AG***

[3 marks]

(f) 18 (m) ***A1***

[1 mark]

(g) (i) $w = \left| 0.05t^2 + 1.1t + 18 - \dfrac{20\sin\left(\dfrac{\pi t}{8}\right)}{\sin\left(2.37 - \dfrac{\pi t}{8}\right)} \right|$ ***A1***

(ii) attempts to solve $w = 0$ for t ***(M1)***

$t = 3.34880...(12.7765...)$

$t = 3.35$ (s) ***A1***

$22.2444...$

22.2 (m) (south of A) ***A1***

[4 marks]

Total [17 marks]

Question 26

(a) (i) **METHOD 1**

attempt to use the sine rule to find $\hat{ABO}$ ***(M1)***

$$\frac{25}{\sin 28°} = \frac{50}{\sin A\hat{B}O}$$

$\left(A\hat{B}O =\right) 69.8748\ldots°$ or $\left(A\hat{B}O =\right) 110.125\ldots°$ ***(A1)(A1)***

Note: Award ***A1*** for each value.

attempt to find at least one possible angle for $O\hat{A}B$ ***(M1)***

$\left(O\hat{A}B =\right) 180° - 28° - 69.8748\ldots°$ OR $\left(O\hat{A}B =\right) 180° - 28° - 110.125\ldots°$

$O\hat{A}B = 82.1251\ldots°, 41.8748\ldots°$

$O\hat{A}B = 82.1°, 41.9°$ ***A1***

METHOD 2

attempt to use the cosine rule to find OB ***(M1)***

$25^2 = 50^2 + OB^2 - 2(50)(OB)\cos 28°$

$OB = 52.7491\ldots$ or $35.5455\ldots$ ***(A1) (A1)***

attempt to use the sine rule to find $O\hat{A}B$ ***(M1)***

$$\frac{25}{\sin 28°} = \frac{OB}{\sin O\hat{A}B}$$

$O\hat{A}B = 82.1251\ldots°, 41.8748\ldots°$

$O\hat{A}B = 82.1°, 41.9°$ ***A1***

(ii) attempt to substitute two sides of triangle OAB and one of their angles into the area formula $\frac{1}{2}ab\sin C$ ***(M1)***

$\frac{1}{2}(\text{OA})(\text{AB})\sin 82.1251\ldots^\circ$ OR $\frac{1}{2}(50)(25)\sin 41.8748\ldots^\circ$ OR

$\frac{1}{2}(50)(52.7491...)\sin 28^\circ$ OR $\frac{1}{2}(50)(35.5455)\sin 28^\circ$

Area $= 619.106\ldots$ OR $= 417.190\ldots$

$= 619\ (\text{m}^2)$ OR $= 417\ (\text{m}^2)$ ***A1A1***

[8 marks]

(b) attempt to use the cosine rule in triangle OCD ***(M1)***

$10^2 = x^2 + y^2 - 2xy\cos 28^\circ$ ***A1***

attempt to use the area formula in triangle OCD ***(M1)***

$\frac{1}{2}xy\sin 28^\circ = 60$ ***A1***

Note: Award ***(M1)A1*** for use of the area formula independently of the ***(M1)A1*** for use of the cosine rule.

$xy = \frac{120}{\sin 28^\circ}$

$100 = x^2 + y^2 - \frac{240\cos 28^\circ}{\sin 28^\circ}$ ***A1***

$x^2 + y^2 = 100 + \frac{240}{\tan 28^\circ}$ ***AG***

[5 marks]

(c) **EITHER**

attempt to eliminate y or x *(M1)*

$$100 = x^2 + \left(\frac{120}{x\sin 28^\circ}\right)^2 - \frac{240}{\tan 28^\circ}$$

OR

attempt to find the intersection of the graph of their cosine rule equation and the graph of their area formula *(M1)*

Note: Award ***(M1)*** only if their graphs are of functions with the same subject e.g. both "$y = ...$" or both "$y^2 = ...$".

THEN

$x = 13.1300...$ or $x = 19.4673...$

$x = 13.1$ or $x = 19.5$ ***A1A1***

$\text{OC} = 13.1(\text{m})$ or $19.5(\text{m})$

[3 marks]

Total [16 marks]

Question 27

(a) **EITHER**

attempt to find value of t for the first low tide OR the first high tide ***(M1)***

$11.2619\ldots - 5.13801\ldots$

$= 6.12396\ldots$ ***(A1)***

OR

attempt to find half of the period ***(M1)***

$\frac{1}{2} \times \frac{2\pi}{0.513}$

$= 6.12396\ldots$ ***(A1)***

THEN

$m = (6.12396\ldots - 6) \times 60 = 7.43773...$

$m = 7$ ***A1***

[3 marks]

(b) attempt to solve $H(t) = 1$ ***(M1)***

$3.56919\ldots$ OR $6.70684\ldots$ OR $15.8171\ldots$ OR $18.9547\ldots$

$(6.70684\ldots - 3.56919\ldots =)\ 3.13764...$

$= 3.14$ (hours) ***A1***

[2 marks]

(c) recognition that $H'(13)$ is required ***(M1)***

$= -0.650622\ldots$

$= -0.651$ (m/h) ***A1***

[2 marks]

(d)

Note: In part (d), award the marks for a, b, c and d independent of each other.

METHOD 1

$a = 1.17$ ***A1***

$d = 1.57$ ***A1***

attempt to find time between low and high tide in hours ***(M1)***

6 hours and 21 minutes = 6.35 hours

(period =) 12.7 ***(A1)***

$b = \frac{2\pi}{12.7} = 0.494739\ldots$

$b = 0.495 \left(= \frac{60\pi}{381} \right)$ ***A1***

attempt to find mean of low and high tide times OR substitute values of a known point ***(M1)***

$c = \frac{1}{2}\left(2\frac{41}{60} + 9\frac{2}{60}\right)$ OR eg $0.40 = 1.17\sin\left(0.495(2.68333... - c)\right) + 1.57$

$c = 5.85833...$

$c = 5.86$ ***A1***

Note: Award ***(M1)A1*** for $c = 18.6$.
Award ***(M1)A0*** for $c = -6.84$.

[7 marks]

METHOD 2

$a = 1.17$ **_A1_**

$d = 1.57$ **_A1_**

substituting at least one point into $h(t)$ **_(M1)_**

$1.17\sin\left(b\left(2\frac{41}{60} - c\right)\right) + 1.57 = 0.4$ OR $1.17\sin\left(b\left(9\frac{2}{60} - c\right)\right) + 1.57 = 2.74$

$b\left(2\frac{41}{60} - c\right) = -\frac{\pi}{2}(= -1.57)$ **AND** $b\left(9\frac{2}{60} - c\right) = \frac{\pi}{2}(= 1.57)$ **_(A1)_**

Note: accept any angles of the form $-\frac{\pi}{2} + 2\pi k$ and $\frac{\pi}{2} + 2\pi k$.

EITHER

use of graph or table to find their intersection **_(M1)_**

OR

attempt to solve their equations simultaneously **_(M1)_**

$$\frac{2\frac{41}{60} - c}{9\frac{2}{60} - c} = -1$$

THEN

$c = 5.85833...$

$c = 5.86$ **_A1_**

$b = 0.494739...$

$b = 0.495$ **_A1_**

[7 marks]

(e) attempt to find point of intersection of two graphs **_(M1)_**

$T = 4.16292\ldots$ OR $T = 4.16417\ldots$(using 3 sf)

$T = 4.16$ **_A1_**

[2 marks]

Total [16 marks]

Question 28

(a) attempt to use trigonometry to find the radius of the cone OR Oliver's distance from centre $(r+5)$ ***(M1)***

$\tan 58° = \frac{18.2}{r+5}$ OR $\frac{r+5}{\sin 32°} = \frac{18.2}{\sin 58°}$ OR $(r+5=)11.3726...$ ***(A1)***

$r = 6.37262...$ (m)

$(r =) 6.37$ (m) ***A1***

[3 marks]

(b) attempt to substitute $h = 20$ and their radius into the correct volume of cone formula ***(M1)***

$V = \frac{\pi(6.37262...)^2(20)}{3}$

$= 850.540...$

$= 851\ (\text{m}^3)$ ***A1***

Note: Accept $849.840...$ (850) obtained from using $r = 6.37$.

[2 marks]

Total [5 marks]

Question 29

(a) (i) 6 *A1*

(ii) attempt to find period *(M1)*

$\frac{2\pi}{b}$ or $\frac{2\pi}{\pi}$

period = 2 *A1*

[3 marks]

(b) (i) evidence of considering the graph of $6\cos(\pi x) - 8\sin(\pi x)$ (seen in i, ii, or iii) *(M1)*

$a = 10$ *A1*

(ii) $b = \pi$ *A1*

(iii) $1.70483\ldots$

$c = 1.70$ *A1*

[4 marks]

Question 30

(a) **METHOD 1**

attempt to use right triangle trigonometry **(M1)**

$\tan \mathrm{B\hat{A}E} = \dfrac{12}{7}$ OR $\tan\left(90° - \mathrm{B\hat{A}E}\right) = \dfrac{7}{12}$ **(A1)**

$59.7435...$

$\mathrm{B\hat{A}E} = 59.7°$ **A1**

Note: Award ***(M1)(A1)A0*** for the equivalent radian value of 1.04.

METHOD 2

attempt to find $\mathrm{B\hat{A}E}$ using sine rule OR cosine rule **(M1)**

$\dfrac{\sin \mathrm{B\hat{A}E}}{12} = \dfrac{\sin 90}{\sqrt{12^2 + 7^2}}$ OR $12^2 = 7^2 + 193 - 2 \times 7 \times \sqrt{12^2 + 7^2} \times \cos \mathrm{B\hat{A}E}$ **(A1)**

$\mathrm{B\hat{A}E} = 59.7435...$

$\mathrm{B\hat{A}E}=59.7°$ **A1**

Note: Award ***(M1)(A1)A0*** for the equivalent radian value of 1.04.

[3 marks]

(b) (i) **METHOD 1**

attempt to find DE using right angle trigonometry **(M1)**

$\sin 59.7435...° = \dfrac{350}{\mathrm{DE}}$ OR equivalent **(A1)**

$\mathrm{DE} = 405.196...$

$\mathrm{CE} = 405.196... + 50$

$= 455.196...$

$= 455$ (cm) **A1**

METHOD 2

Let $\mathrm{DE} = \mathrm{EF} = x$

attempt to find DE using their $\mathrm{D\hat{E}F}$ and the sine rule OR cosine rule **(M1)**

$\dfrac{700}{\sin(119.487...)} = \dfrac{\mathrm{DE}}{\sin(30.2564...)}$ OR $x^2 = 700^2 + x^2 - 2 \times 700 \times x \times \cos 30.2564...$ **(A1)**

$\mathrm{DE} = 405.196...$

$\mathrm{CE} = 405.196... + 50$

$= 455.196...$

$= 455$ (cm) **A1**

METHOD 3

Let G be the midpoint of DF

$\text{EG} = \frac{7}{12} \times 350 \left(= \frac{1225}{6} = 204.166... \right)$ ***(A1)***

use of Pythagoras' with their EG to find DE ***(M1)***

$\text{DE} = \sqrt{204.166...^2 + 350^2} \ (= 405.196...)$

$\text{CE} = 405.196... + 50$

$= 455.196...$

$= 455$ (cm) ***A1***

(ii) $\tan(59.7435...°) = \frac{30}{x}$ OR $\frac{12}{7} = \frac{30}{x}$ ***(A1)***

$x = 17.5$

$\text{BA} = 455.196... + 17.5$

$= 472.696...$

$= 473$ (cm) ***A1***

[5 marks]

Total [8 marks]

Question 31

(a) (i) 3 ***A1***

(ii) attempt to find period ***(M1)***

$\frac{2\pi}{b}$ or $\frac{2\pi}{4\pi}$

period $= \frac{1}{2}$ ***A1***

[3 marks]

(b) (i) evidence of considering the graph of $3\sin(4\pi x) - 4\cos(4\pi x)$ (seen in i, ii,or iii) ***(M1)***

$a = 5$ ***A1***

(ii) $b = 4\pi$ ***A1***

(iii) $0.198792\ldots$

$c = 0.199$ ***A1***

[4 marks]

Question 32

(a) **METHOD 1**

attempt to use right triangle trigonometry **(M1)**

$\tan B\hat{A}E = \dfrac{15}{4}$ OR $\tan\left(90° - B\hat{A}E\right) = \dfrac{4}{15}$ **(A1)**

$75.0685...$

$B\hat{A}E = 75.1°$ **A1**

Note: Award ***(M1)(A1)A0*** for the equivalent radian value of 1.31.

METHOD 2

attempt to find $B\hat{A}E$ using sine rule OR cosine rule **(M1)**

$\dfrac{\sin B\hat{A}E}{15} = \dfrac{\sin 90}{\sqrt{15^2 + 4^2}}$ OR $15^2 = 4^2 + 241 - 2 \times 4 \times \sqrt{15^2 + 4^2} \times \cos B\hat{A}E$ **(A1)**

$75.0685...$

$B\hat{A}E = 75.1°$ **A1**

Note: Award ***(M1)(A1)A0*** for the equivalent radian value of 1.31.

[3 marks]

(b) (i) **METHOD 1**

attempt to find DE using right angle trigonometry **(M1)**

$\sin 75.0685...° = \dfrac{350}{DE}$ OR equivalent **(A1)**

$DE = 362.230...$

$CE = 362.230... + 40$

$= 402.230...$

$= 402$ (cm) **A1**

METHOD 2

Let $DE = EF = x$

attempt to find DE using their $D\hat{E}F$ and the sine rule OR cosine rule **(M1)**

$\dfrac{700}{\sin(150.137...)} = \dfrac{DE}{\sin(14.9314...)}$ OR $x^2 = 700^2 + x^2 - 2 \times 700 \times x \times \cos 14.9314...$ **(A1)**

$DE = 362.230...$

$CE = 362.230... + 40$

$= 402.230...$

$= 402$ (cm) **A1**

METHOD 3

Let G be the midpoint of DF

$\text{EG} = \frac{4}{15} \times 350 \left(= \frac{280}{3} = 93.3333... \right)$ ***(A1)***

use of Pythagoras' with their EG to find DE ***(M1)***

$(\text{DE} =) \sqrt{93.3333...^2 + 350^2} \ (= 362.230...)$

$\text{CE} = 362.230... + 40$

$= 402.230...$

$= 402$ (cm) ***A1***

(ii) $\tan(75.0685...°) = \frac{20}{x}$ OR $\frac{15}{4} = \frac{20}{x}$ ***(A1)***

$x = 5.33$

$\text{BA} = 402.230... + 5.33333...$

$= 407.564...$

$= 408$ (cm) (accept 407 from previous 3sf answer) ***A1***

[5 marks]

Total [8 marks]