Subject – Math AA(Standard Level)
Topic - Geometry and Trigonometry
Year - May 2021 – Nov 2022
Paper -2
Answers

Question 1

(a) correct approach **A1**

eg $\frac{\pi}{6}=\frac{2\pi}{period}$ (or equivalent)

$\text{period}=12$ **A1**

[2 marks]

(b) (i) valid approach **(M1)**

eg $\frac{\max+\min}{2}$ $b=\max-\text{amplitude}$

$\frac{21.8+10.2}{2}$, or equivalent

$b=16$ **A1**

(ii) attempt to substitute into **their** function **(M1)**

$5.8\sin\left(\frac{\pi}{6}(6+1)\right)+16$

$f(6)=13.1$ **A1**

[4 marks]

(c) valid attempt to set up a system of equations **(M1)**

two correct equations **A1**

$p\sin\left(\frac{2\pi}{9}(3-3.75)\right)+q=2.5,\ p\sin\left(\frac{2\pi}{9}(6-3.75)\right)+q=15.1$

valid attempt to solve system **(M1)**

$p=8.4;\ q=6.7$ **A1A1**

[5 marks]

(d) attempt to use $|f(x)-g(x)|$ to find maximum difference **(M1)**

$x=1.64$ **A1**

[2 marks]

Total [13 marks]

Question 2

(a) $\frac{4.2}{60} \times 45$ **A1**

$AB = 3.15$ (km) **A1**

[2 marks]

(b) (i) 66° or $(180 - 114)$ **A1**

$35 + 66$ **A1**

$A\hat{B}C = 101^\circ$ **AG**

(ii) attempt to use cosine rule **(M1)**

$AC^2 = 3.15^2 + 4.6^2 - 2 \times 3.15 \times 4.6 \cos 101^\circ$ (or equivalent) **A1**

$AC = 6.05$ (km) **A1**

[5 marks]

(c) valid approach to find angle BCA **(M1)**

eg sine rule

correct substitution into sine rule **A1**

eg $\frac{\sin(B\hat{C}A)}{3.15} = \frac{\sin 101}{6.0507...}$

$B\hat{C}A = 30.7^\circ$ **A1**

[3 marks]

(d) $B\hat{A}C = 48.267$ (seen anywhere) **A1**

valid approach to find correct bearing **(M1)**

eg $48.267 + 35$

bearing $= 83.3^\circ$ (accept 083°) **A1**

[3 marks]

(e) attempt to use time $= \frac{\text{distance}}{\text{speed}}$ **M1**

$\frac{6.0507}{3.9}$ or 0.065768 km/min **(A1)**

$t = 93$ (minutes) **A1**

[3 marks]

Total [16 marks]

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Question 3

(a) ***METHOD 1***

attempt to use the cosine rule **(M1)**

$\cos\theta = \frac{4^2 + 4^2 - 5^2}{2\times 4\times 4}$ (or equivalent) **A1**

$\theta = 1.35$ **A1**

[3 marks]

METHOD 2

attempt to split triangle AOB into two congruent right triangles **(M1)**

$\sin\left(\frac{\theta}{2}\right) = \frac{2.5}{4}$ **A1**

$\theta = 1.35$ **A1**

[3 marks]

(b) attempt to find the area of the shaded region **(M1)**

$\frac{1}{2}\times 4\times 4\times(2\pi - 1.35...)$ **A1**

$= 39.5\ (\text{cm}^2)$ **A1**

[3 marks]

Total [6 marks]

Question 4

(a) $r = \frac{28}{\theta}$ **A1**

[1 mark]

(b) recognising sum of area of sector and area of triangle required **(M1)**

$\frac{1}{2}r^2\theta + \frac{1}{2}r\times r\times\sin(\pi-\theta)\ \left(= \frac{r^2}{2}(\theta + \sin(\pi-\theta))\right)$ **A1**

$\sin(\pi-\theta) = \sin\theta$ (substitution seen anywhere) **A1**

$\frac{1}{2}\left(\frac{28}{\theta}\right)^2\theta + \frac{1}{2}\left(\frac{28}{\theta}\right)^2\sin\theta$ OR $\frac{1}{2}\left(\frac{28}{\theta}\right)^2(\theta + \sin\theta)$ **A1** area $= \dfrac{392}{\theta^2}(\theta + \sin\theta)$ **AG**

[4 marks]

(c) $\dfrac{392}{\theta^2}(\theta + \sin\theta) = 460$ ***(M1)***

$\theta = 1.43917...$

$\theta = 1.44$ ***A1***

[2 marks]

(d) $\dfrac{\pi - (\pi - \theta)}{2}$ OR $\dfrac{\theta}{2}$ ***(M1)***

$\text{D}\hat{\text{A}}\text{E} = 0.719588...$

$\text{D}\hat{\text{A}}\text{E} = 0.720$ ***A1***

[2 marks]

(e) (i) $\text{A}\hat{\text{B}}\text{C} = 195 - 180 + 90$ ***(A1)***

$= 105°$ ***A1***

(ii) choosing sine rule ***(M1)***

$\dfrac{\text{BC}}{\sin \text{D}\hat{\text{A}}\text{E}} = \dfrac{800}{\sin 105}$ OR $\dfrac{\text{BC}}{\sin \text{D}\hat{\text{A}}\text{E}} = \dfrac{800}{\sin 1.83}$ ***A1***

$\text{BC} = 546 \text{ (m)}$ ***A1***

[5 marks]

Total [14 marks]

Question 5

amplitude is $\dfrac{110}{2} = 55$ ***(A1)***

$a = -55$ ***A1***

$c = 65$ ***A1***

$\dfrac{2\pi}{b} = 20$ OR $-55\cos(20b) + 65 = 10$ ***(M1)***

$b = \dfrac{\pi}{10} (= 0.314)$ ***A1***

Total [5 marks]

Question 6

(a) $\tan 0.6 = \frac{h}{12}$ ***(M1)***

$8.20964...$

8.21 (m) ***A1***

[2 marks]

(b) $\tan B = \frac{8.2096...}{5}$ OR $\tan^{-1}1.6419...$ ***(A1)***

$1.02375...$

1.02 (radians) (accept $58.7°$) ***A1***

[2 marks]

(c) $x + 1.8 + 2.5 = 8.20964...$ (or equivalent) ***(A1)***

$3.90964...$

3.91 (m) ***A1***

[2 marks]

(d) **METHOD 1**

recognition that blade length = amplitude, $p = \frac{\max - \min}{2}$ ***(M1)***

$p = 3.91$ ***A1***

centre of windmill = vertical shift, $q = \frac{\max + \min}{2}$ ***(M1)***

$q = 8.21$ ***A1***

METHOD 2

attempting to form two equations in terms of p and q ***(M1) (M1)***

$12.1192... = p\cos\left(\frac{3\pi}{10} \cdot 0\right) + q$, $4.3000... = p\cos\left(\frac{3\pi}{10} \cdot \frac{10}{3}\right) + q$

$p = 3.91$ ***A1***

$q = 8.21$ ***A1***

[4 marks]

(e) appropriate working towards finding the period **(M1)**

$$\text{period} = \frac{2\pi}{\frac{3\pi}{10}} (= 6.6666...)$$

rotations per minute $= \dfrac{60}{\text{their period}}$ **(M1)**

$n = 9$ (must be an integer) (accept $n = 10$, $n = 18$, $n = 19$) **A1**

[3 marks]

Total [13 marks]

Question 7

(a) $12 = \dfrac{2\pi}{b}$ OR $b = \dfrac{2\pi}{12}$ **A1**

$b = \dfrac{\pi}{6}$ **AG**

[1 mark]

(b) $a = \dfrac{6.8 - 2.2}{2}$ OR $a = \dfrac{\max - \min}{2}$ **(M1)**

$= 2.3$ (m) **A1**

[2 marks]

(c) $d = \dfrac{6.8 + 2.2}{2}$ OR $d = \dfrac{\max + \min}{2}$ **(M1)**

$= 4.5$ (m) **A1**

[2 marks]

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(d) **METHOD 1**

substituting $t = 4.5$ and $H = 6.8$ for example into their equation for H ***(A1)***

$6.8 = 2.3\sin\left(\frac{\pi}{6}(4.5 - c)\right) + 4.5$

attempt to solve their equation ***(M1)***

$c = 1.5$ ***A1***

METHOD 2

using horizontal translation of $\frac{12}{4}$ ***(M1)***

$4.5 - c = 3$ ***(A1)***

$c = 1.5$ ***A1***

METHOD 3

$H'(t) = (2.3)\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}(t - c)\right)$ ***(A1)***

attempts to solve their $H'(4.5) = 0$ for c ***(M1)***

$(2.3)\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}(4.5 - c)\right) = 0$

$c = 1.5$ ***A1***

[3 marks]

(e) attempt to find H when $t = 12$ or $t = 0$, graphically or algebraically ***(M1)***

$H = 2.87365...$

$H = 2.87\,(\text{m})$ ***A1***

[2 marks]

(f) attempt to solve $5=2.3\sin\left(\frac{\pi}{6}(t-1.5)\right)+4.5$ ***(M1)***

times are $t=1.91852...$ and $t=7.08147...$, $(t=13.9185...,t=19.0814...)$ ***(A1)***

total time is $2\times(7.081...-1.919...)$

$10.3258...$

$=10.3$ (hours) ***A1***

Note: Accept 10.

[3 marks]

Total [13 marks]

Question 7

(a) (i) $\frac{\text{AP}}{42}$ OR $\frac{215}{84}$ OR $\frac{65}{42}+\frac{215}{84}$ ***(M1)***

time = $4.10714\ldots$ (hours)

time = 4.11 (hours) ***A1***

(ii) $\text{AB}=\sqrt{215^2+65^2}\ (=224.610...)$ ***(A1)***

time = $5.34787\ldots$ (hours)

time = 5.35 (hours) ***A1***

[4 marks]

(b) (i) $\text{AD}=\sqrt{(215-x)^2+65^2}$ ***(A1)***

$t=\frac{\sqrt{(215-x)^2+65^2}}{42}$ ***(A1)***

$T=\frac{\sqrt{(215-x)^2+65^2}}{42}+\frac{x}{84}\left(=\frac{\sqrt{x^2-430x+50450}}{42}+\frac{x}{84}\right)$ ***A1***

(ii) valid approach to find the minimum for T (may be seen in (iii)) ***(M1)***

graph of T OR $T'=0$ OR graph of T'

$x=177.472...$ km

$x=177$ km ***A1***

(iii) $T = 3.89980...$

$T = 3.90$ (hours) ***A1***

(c) (i) $C = 200 \cdot \frac{\sqrt{(215-x)^2 + 65^2}}{42} + 150 \cdot \frac{x}{84}$ ***(A1)***

valid approach to find the minimum for $C(x)$ (may be seen in (ii)) ***(M1)***

graph of C OR $C' = 0$ OR graph of C'

$x = 188.706...$ km

$x = 189$ km ***A1***

Note: Only allow ***FT*** from (b) if the function T has a minimum in $0 < x < 215$.

(ii) $C = 670.864$

$C = \$671$ ***A1***

Note: Only allow ***FT*** from (c)(i) if the function C has a minimum in $0 < x < 215$.

[4 marks]

Total [14 marks]

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Question 9

(a) attempt to find the area of either shaded region in terms of r and θ ***(M1)***

Note*:* Do not award ***M1*** if they have only copied from the booklet and not applied to the shaded area.

Area of segment $=\frac{1}{2}r^2\theta-\frac{1}{2}r^2\sin\theta$ ***A1***

Area of triangle $=\frac{1}{2}r^2\sin(\pi-\theta)$ ***A1***

correct equation in terms of θ only ***(A1)***

$\theta-\sin\theta=\sin(\pi-\theta)$

$\theta-\sin\theta=\sin\theta$ ***A1***

$\theta=2\sin\theta$ ***AG***

Note*:* Award a maximum of ***M1A1A0A0A0*** if a candidate uses degrees (i.e., $\frac{1}{2}r^2\sin(180°-\theta)$), even if later work is correct.

Note*:* If a candidate directly states that the area of the triangle is $\frac{1}{2}r^2\sin\theta$, award a maximum of ***M1A1A0A1A1***.

[5 marks]

(b) $\theta=1.89549...$

$\theta=1.90$ ***A1***

Note: Award ***A0*** if there is more than one solution. Award ***A0*** for an answer in degrees.

[1 mark]

Total [6 marks]

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Question 10

EITHER

attempt to use cosine rule **(M1)**

$12^2 + AB^2 - 2\times12\times\cos 25°\times AB = 7^2$ OR $AB^2 - 21.7513...AB + 95 = 0$ **(A1)**

at least one correct value for AB **(A1)**

$AB = 6.05068...$ OR $AB = 15.7007...$

using their smaller value for AB to find minimum perimeter **(M1)**

$12 + 7 + 6.05068...$

OR

attempt to use sine rule **(M1)**

$\frac{\sin B}{12} = \frac{\sin 25°}{7}$ OR $\sin B = 0.724488...$ OR $\hat{B} = 133.573...°$ OR $\hat{B} = 46.4263...°$ **(A1)**

at least one correct value for C **(A1)**

$\hat{C} = 21.4263...°$ OR $\hat{C} = 108.573...°$

using their acute value for $\hat{C}$ to find minimum perimeter **(M1)**

$12 + 7 + \sqrt{12^2 + 7^2 - 2\times12\times7\cos 21.4263...°}$ OR $12 + 7 + \frac{7\sin 21.4263...°}{\sin 25°}$

THEN

$25.0506\ldots$

minimum perimeter = 25.1. **A1**

Total [5 marks]

Question 11

(a) attempt to use the distance formula to find AV **(M1)**

$\sqrt{(1-(-1))^2+(5-1)^2+(0-6)^2}$

$=7.48331\ldots$

$=7.48\ \text{(cm)}\ \left(=\sqrt{56}\text{ or }2\sqrt{14}\right)$ **A1**

[2 marks]

(b) **METHOD 1**

attempt to apply cosine rule OR sine rule to find AB **(M1)**

$(\text{AB}=)\ \sqrt{7.48\ldots^2+7.48\ldots^2-2\times7.48\ldots\times7.48\ldots\cos\left(40^\circ\right)}$ OR $\dfrac{\text{AB}}{\sin 40^\circ}=\dfrac{\sqrt{56}}{\sin 70^\circ}$ **(A1)**

$=5.11888\ldots$

$=5.12\ \text{(cm)}$ **A1**

METHOD 2

Let M be the midpoint of [AB]

attempt to apply right-angled trigonometry on triangle AVM **(M1)**

$=2\times7.48\ldots\times\sin(20^\circ)$ **(A1)**

$=5.11888\ldots$

$=5.12\ \text{(cm)}$ **A1**

[3 marks]

(c) **METHOD 1**

equating volume of pyramid formula to 57.2 **(M1)**

$\dfrac{1}{3}\times5.11\ldots^2\times h=57.2$ **(A1)**

$h=6.54886\ldots$

$h=6.55\ \text{(cm)}$ **A1**

METHOD 2

Let M be the midpoint of $[\mathrm{AB}]$

$\mathrm{AV}^2 = \mathrm{AM}^2 + \mathrm{MX}^2 + \mathrm{XV}^2$ ***(M1)***

$\Rightarrow \mathrm{XV} = \sqrt{7.48...^2 - \left(\frac{5.11...}{2}\right)^2 - \left(\frac{5.11...}{2}\right)^2}$ ***(A1)***

$h = 6.54886...$

$h = 6.55\ (\text{cm})$ ***A1***

[3 marks]

(d) $V = x \times 2x \times y = 57.2$ ***(A1)***

$S = 2\left(2x^2 + xy + 2xy\right)$ ***A1***

Note: Condone use of A.

attempt to substitute $y = \frac{57.2}{2x^2}$ into their expression for surface area ***(M1)***

$\left(S(x) =\right)\ 4x^2 + 6x\left(\frac{57.2}{2x^2}\right)$

EITHER

attempt to find minimum turning point on graph of area function ***(M1)***

OR

$\frac{\mathrm{d}S}{\mathrm{d}x} = 8x - 171.6x^{-2} = 0$ OR $x = 2.77849...$ ***(M1)***

THEN

$92.6401...$

minimum surface area $= 92.6\ (\text{cm}^2)$ ***A1***

[5 marks]

Total [13 marks]

Question 12

(a) valid approach to find area of segment by finding area of sector – area of triangle ***(M1)***

$\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$

$\frac{1}{2}(2)^2\theta - \frac{1}{2}(2)^2\sin\theta$ ***(A1)***

area = $2\theta - 2\sin\theta$ ***A1***

[3 marks]

(b) **EITHER**

area of logo = area of rectangle – area of segments ***(M1)***

$5\times 4 - 2\times(2\theta - 2\sin\theta) = 13.4$ ***(A1)***

OR

area of one segment = $\frac{20-13.4}{2}$ $(=3.3)$ ***(M1)***

$2\theta - 2\sin\theta = 3.3$ ***(A1)***

THEN

$\theta = 2.35672...$

$\theta = 2.36$ (do not accept an answer in degrees) ***A1***

Note: Award ***(M1)(A1)A0*** if there is more than one solution.
Award ***(M1)(A1FT)A0*** if the candidate works in degrees and obtains a final answer of $135.030\ldots$

[3 marks]

Total [6 marks]

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Question 13

(a) (i) 32 (cm) **A1**

(ii) $h_A(0) = \sin(6) + 27$ ***(M1)***

$= 26.7205...$

$= 26.7$ (cm) **A1**

[3 marks]

(b) attempts to solve $h_A(t) = h_B(t)$ for t ***(M1)***

$t = 4.00746..., 4.70343..., 5.88332...$

$t = 4.01, 4.70, 5.88$ (weeks) **A2**

[3 marks]

(c) recognises that $h_A'(t)$ and $h_B'(t)$ are required ***(M1)***

attempts to solve $h_A'(t) = h_B'(t)$ for t ***(M1)***

$t = 1.18879...$ and $2.23598...$ OR $4.33038...$ and $5.37758...$ OR $7.47197...$ and $8.51917...$ ***(A1)***

> **Note:** Award full marks for $t = \frac{4\pi}{3} - 3, \frac{5\pi}{3} - 3, \left(\frac{7\pi}{3} - 3, \frac{8\pi}{3} - 3, \frac{10\pi}{3} - 3, \frac{11\pi}{3} - 3\right)$.
> *Award* subsequent marks for correct use of these exact values.

$1.18879... < t < 2.23598...$ OR $4.33038... < t < 5.37758...$ OR
$7.47197... < t < 8.51917...$ ***(A1)***

attempts to calculate the total amount of time ***(M1)***

$3(2.2359... - 1.1887...)$ $\left(= 3\left(\left(\frac{5\pi}{3} - 3\right) - \left(\frac{4\pi}{3} - 3\right)\right)\right)$

$= 3.14$ $(= \pi)$ (weeks) **A1**

[6 marks]

Total [12 marks]

Question 14

(a) $\hat{ABC} = 27°$ *(A1)*

attempt to substitute into cosine rule *(M1)*

$175^2 + 230^2 - 2(175)(230)\cos 27°$ *(A1)*

$108.62308\ldots$

$AC = 109$ (m) ***A1***

[4 marks]

(b) correct substitution into area formula *(A1)*

$\frac{1}{2} \times 175 \times 230 \times \sin 27°$

$9136.55\ldots$

$\text{area} = 9140(\text{m}^2)$ ***A1***

[2 marks]

(c) attempt to substitute into sine rule or cosine rule *(M1)*

$\frac{\sin 27°}{108.623\ldots} = \frac{\sin \hat{A}}{175}$ OR $\cos A = \frac{(108.623\ldots)^2 + 230^2 - 175^2}{2 \times 108.623\ldots \times 230}$ *(A1)*

$47.0049\ldots$

$\hat{CAB} = 47.0°$ ***A1***

[3 marks]

(d) **METHOD 1**

recognizing that for areas to be equal, AD=DC *(M1)*

$AD = \frac{1}{2} AC = 54.3115\ldots$ ***A1***

attempt to substitute into cosine rule to find BD *(M1)*

correct substitution into cosine rule *(A1)*

$BD^2 = 230^2 + 54.3115^2 - 2(230)(54.3115)\cos 47.0049°$

$BD = 197.009\ldots$

$BD = 197(\text{m})$ ***A1***

[5 marks]

METHOD 2

correct expressions for areas of triangle BDA and triangle BCD using BD ***A1***

$\frac{1}{2}\times \text{BD}\times 230\times \sin x°$ and $\frac{1}{2}\times \text{BD}\times 175\times \sin(27-x)°$ OR

$\frac{1}{2}\times \text{BD}\times 230\times \sin(27-x)°$ and $\frac{1}{2}\times \text{BD}\times 175\times \sin x°$

correct equation in terms of x ***(A1)***

$175\sin(27-x)=230\sin x$ or $175\sin\text{x}=230\sin(27-x)$

$x=11.6326...$ or $x=15.3673...$ ***(A1)***

substituting their value of x into equation to solve for BD ***(M1)***

$\frac{1}{2}\times \text{BD}\times 230\times \sin 11.6326...=\frac{1}{2}\times \text{BD}\times 175\times \sin 15.3673...$ or

$\frac{1}{2}\times \text{BD}\times 230\times \sin 11.6326...=\frac{1}{2}\times 9136.55...$

$\text{BD}=197(\text{m})$ ***A1***

[5 marks]

Total [14 marks]

Question 15

(a) **EITHER**

uses the cosine rule **(M1)**

$AB^2 = 5^2 + 5^2 - 2\times5\times5\times\cos1.9$ **(A1)**

OR

uses right-angled trigonometry **(M1)**

$\frac{\frac{AB}{2}}{5} = \sin0.95$ **(A1)**

OR

uses the sine rule **(M1)**

$\alpha = \frac{1}{2}(\pi - 1.9)(= 0.6207...)$

$\frac{AB}{\sin1.9} = \frac{5}{\sin0.6207...}$ **(A1)**

THEN

$AB = 8.13415...$

$AB = 8.13$ (m) ***A1***

[3 marks]

(b) let the shaded area be A

METHOD 1

attempt at finding reflex angle **(M1)**

$A\hat{O}B = 2\pi - 1.9$ $(= 4.3831...)$

substitution into area formula **(A1)**

$A = \frac{1}{2}\times5^2\times4.3831...$ OR $\left(\frac{2\pi-1.9}{2\pi}\right)\times\pi\left(5^2\right)$

$= 54.7898...$

$= 54.8$ (m^2) ***A1***

METHOD 2

let the area of the circle be A_C and the area of the unshaded sector be A_U

$A = A_C - A_U$ ***(M1)***

$A = \pi \times 5^2 - \frac{1}{2} \times 5^2 \times 1.9 \ (= 78.5398... - 23.75)$ ***(A1)***

$= 54.7898...$

$= 54.8$ (m^2) ***A1***

[3 marks]

Total [6 marks]

Question 16

(a) attempt to use right angled trigonometry or sine rule to find AE in terms of r and α ***(M1)***

$\tan\alpha = \frac{r}{\text{AE}}$ OR $\frac{\text{AE}}{\sin\left(\frac{\pi}{2} - \alpha\right)} = \frac{r}{\sin\alpha}$

$\text{AE} = \frac{r}{\tan\alpha}$ OR $\text{AE} = \frac{r\sin\left(\frac{\pi}{2} - \alpha\right)}{\sin\alpha}$ OR $\text{AE} = \frac{r\cos\alpha}{\sin\alpha}$ ***A1***

valid approach to find the area of ADOE ***(M1)***

2x area of triangle AOE OR area of triangle AED + area of triangle OED OR $\text{OE} \times \text{AE}$

Area ADOE = $2\left(\frac{1}{2} \cdot \frac{r}{\tan\alpha} \cdot r\right)$ OR $r \times \text{AE}$ ***A1***

Area ADOE = $\frac{r^2}{\tan\alpha}$ ***AG***

[4 marks]

(b) (i) recognizing that the sum of the angles of a kite is 2π ***(M1)***

$D\hat{O}E + O\hat{E}A + E\hat{A}D + A\hat{D}O = 2\pi$ OR $2\alpha + 2\cdot\frac{\pi}{2} + D\hat{O}E = 2\pi$

$D\hat{O}E = \pi - 2\alpha$ ***A1***

Note: Award ***M1A0*** if candidate uses degrees (i.e.

$D\hat{O}E + O\hat{E}A + E\hat{A}D + A\hat{D}O = 360°$ or $2\alpha + 2\cdot\frac{\pi}{2} + D\hat{O}E = 360°$) and obtains

$D\hat{O}E = 180° - 2\alpha$.

(ii) valid approach to find the area of R ***(M1)***

area of kite – area of sector OR 2(area of triangle AOE – 0.5 area of sector OED)

Area of sector = $\frac{1}{2}r^2.D\hat{O}E\left(=\frac{1}{2}r^2(\pi-2\alpha)\right)$ seen anywhere ***(A1)***

Area of R = $\frac{r^2}{\tan\alpha} - \frac{1}{2}r^2(\pi - 2\alpha)$ ***A1***

Note: Accept $\frac{r^2}{\tan\alpha} - \frac{1}{2}r^2.D\hat{O}E$.

[5 marks]

(c) equating their area formula to πr^2 *(M1)*

$$\frac{r^2}{\tan\alpha} - \frac{1}{2}r^2(\pi - 2\alpha) = \pi r^2$$

correct equation in terms of α ***A1***

$$\frac{1}{\tan\alpha} - \frac{1}{2}(\pi - 2\alpha) = \pi$$

valid approach to solve the equation *(M1)*

$\alpha = 0.218979...$

$\alpha = 0.219$ ***A1***

[4 marks]

Total [13 marks]

Question 17

(a) attempt to use sine rule *(M1)*

$\frac{24}{\sin 113^\circ} = \frac{17}{\sin B\hat{A}C}$ OR $\left(\sin B\hat{A}C = \right) 0.652024...$ *(A1)*

$40.6943...$

$B\hat{A}C = 40.7^\circ$ ***A1***

[3 marks]

(b) **METHOD 1** (cosine rule with $\hat{ABC}$ or $\hat{BAC}$)

attempt to use the cosine rule ***(M1)***

$24^2 = AB^2 + 17^2 - 2 \cdot 17 \cdot AB \cdot \cos 113° \quad (AB^2 + 13.2848...AB - 287 = 0)$ OR

$17^2 = AB^2 + 24^2 - 2 \cdot 24 \cdot AB \cdot \cos 40.6943...° \quad (AB^2 - 36.3935...AB + 287 = 0)$ ***(A1)***

$11.5543\ldots$

$AB = 11.6$ ***A1***

METHOD 2 (cosine rule with $\hat{BCA}$)

attempt to use cosine rule ***(M1)***

correct substitution ***(A1)***

$AB^2 = 17^2 + 24^2 - 2 \cdot 17 \cdot 24 \cdot \cos 26.3056...°$ OR $AB^2 = 133.502...$

$11.5543\ldots$

$AB = 11.6$ ***A1***

METHOD 3 (sine rule)

attempt to use sine rule ***(M1)***

correct substitution ***(A1)***

$\frac{AB}{\sin 26.3056...°} = \frac{24}{\sin 113°} = \frac{17}{\sin 40.6943...°}$ OR $AB = \frac{24 \cdot \sin(180° - 113° - 40.6943...°)}{\sin 113°}$

$11.5543\ldots$

$AB = 11.6$ ***A1***

[3 marks]

Total [6 marks]

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