Subject – Math AA(Standard Level)
Topic - Geometry and Trigonometry
Year - May 2021 – Nov 2022
Paper -2
Questions

Question 1

[Maximum mark: 13]

Consider a function f, such that $f(x) = 5.8\sin\left(\frac{\pi}{6}(x+1)\right) + b$, $0 \le x \le 10$, $b \in \mathbb{R}$.

(a) Find the period of f. [2]

The function f has a local maximum at the point $(2, 21.8)$, and a local minimum at $(8, 10.2)$.

(b) (i) Find the value of b.

(ii) Hence, find the value of $f(6)$. [4]

A second function g is given by $g(x) = p\sin\left(\frac{2\pi}{9}(x - 3.75)\right) + q$, $0 \le x \le 10$; $p, q \in \mathbb{R}$.

The function g passes through the points $(3, 2.5)$ and $(6, 15.1)$.

(c) Find the value of p and the value of q. [5]

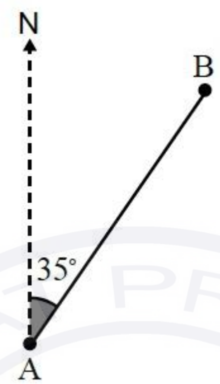
(d) Find the value of x for which the functions have the greatest difference. [2]

Question 2

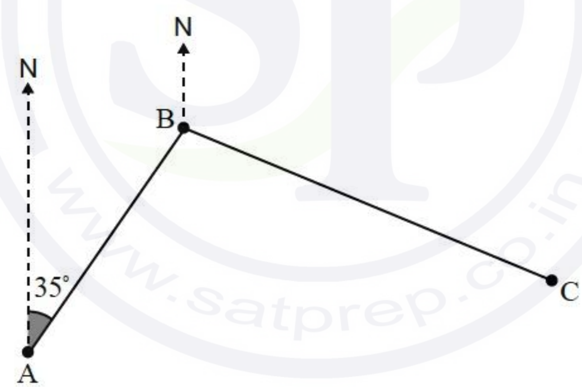
[Maximum mark: 16]

Adam sets out for a hike from his camp at point A. He hikes at an average speed of $4.2\,\mathrm{km/h}$ for 45 minutes, on a bearing of 035° from the camp, until he stops for a break at point B.

(a) Find the distance from point A to point B. [2]



Adam leaves point B on a bearing of 114° and continues to hike for a distance of $4.6\,\mathrm{km}$ until he reaches point C.



(b) (i) Show that $\mathrm{A\hat{B}C}$ is 101°.

(ii) Find the distance from the camp to point C. [5]

(c) Find $\mathrm{B\hat{C}A}$. [3]

Adam's friend Jacob wants to hike directly from the camp to meet Adam at point C.

(d) Find the bearing that Jacob must take to point C. [3]

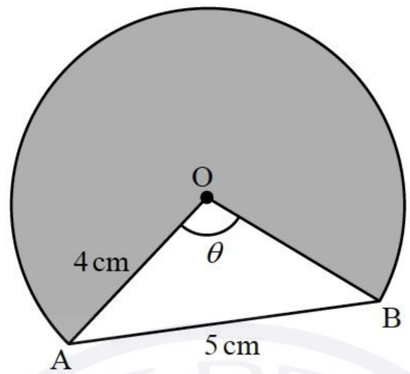
Jacob hikes at an average speed of $3.9\,\mathrm{km/h}$.

(e) Find, to the nearest minute, the time it takes for Jacob to reach point C. [3]

Question 3

[Maximum mark: 6]

The following diagram shows part of a circle with centre O and radius $4\,\mathrm{cm}$.



Chord AB has a length of $5\,\mathrm{cm}$ and $\mathrm{A\hat{O}B} = \theta$.

(a) Find the value of θ, giving your answer in radians. [3]

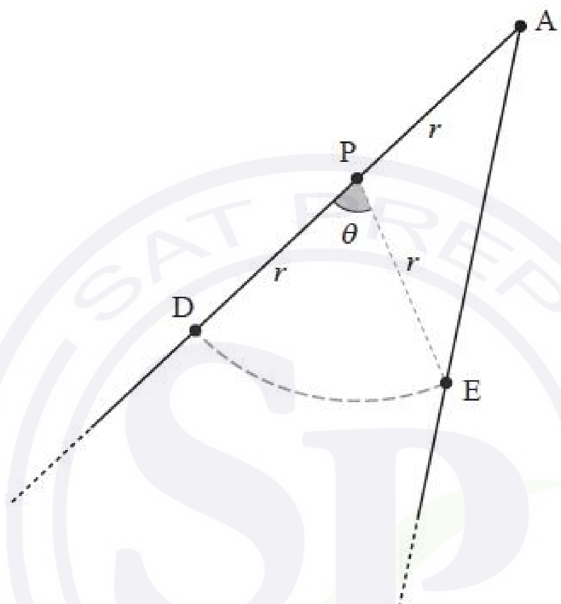
(b) Find the area of the shaded region. [3]

Question 4

[Maximum mark: 14]

Two straight fences meet at point A and a field lies between them.

A horse is tied to a post, P, by a rope of length r metres. Point D is on one fence and point E is on the other, such that $\mathrm{PD} = \mathrm{PE} = \mathrm{PA} = r$ and $\mathrm{D\hat{P}E} = \theta$ radians. This is shown in the following diagram.



The length of the arc DE shown in the diagram is $28\,\mathrm{m}$.

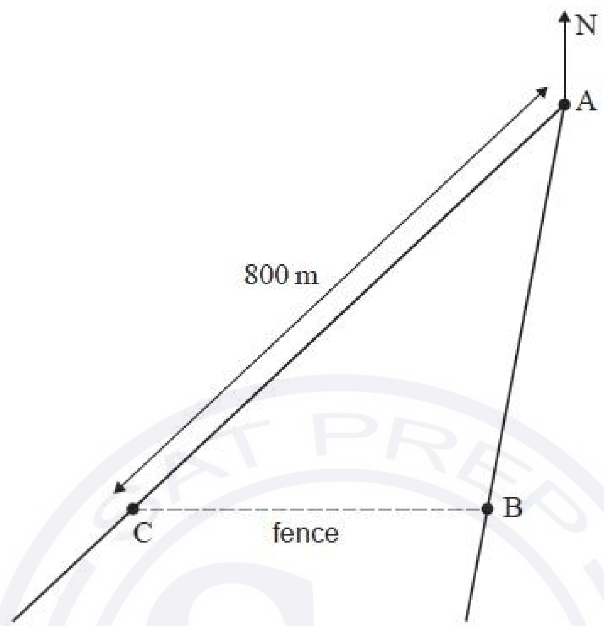
(a) Write down an expression for r in terms of θ. [1]

(b) Show that the area of the field that the horse can reach is $\dfrac{392}{\theta^2}(\theta + \sin\theta)$. [4]

(c) The area of field that the horse can reach is $460\,\mathrm{m}^2$. Find the value of θ. [2]

(d) Hence, find the size of $\mathrm{D\hat{A}E}$. [2]

A new fence is to be constructed between points B and C which will enclose the field, as shown in the following diagram.



Point C is due west of B and $AC = 800\,\text{m}$. The bearing of B from A is $195°$.

(e) (i) Find the size of $A\hat{B}C$.

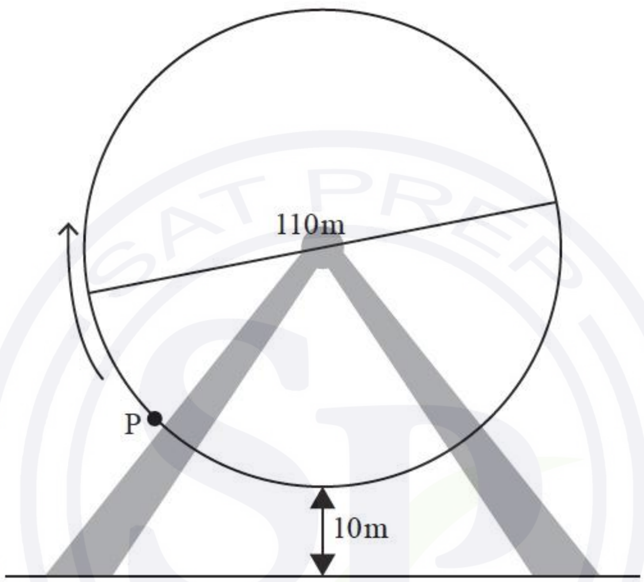
(ii) Find the length of new fence required. [5]

Question 5

[Maximum mark: 5]

A Ferris wheel with diameter 110 metres rotates at a constant speed. The lowest point on the wheel is 10 metres above the ground, as shown on the following diagram. P is a point on the wheel. The wheel starts moving with P at the lowest point and completes one revolution in 20 minutes.

diagram not to scale



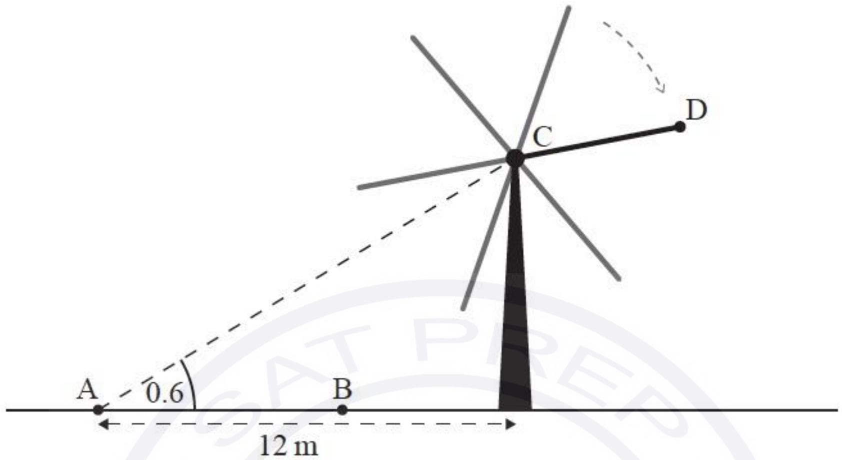
The height, h metres, of P above the ground after t minutes is given by $h(t) = a\cos(bt) + c$, where a, b, $c \in \mathbb{R}$.

Find the values of a, b and c.

Question 6

[Maximum mark: 13]

The six blades of a windmill rotate around a centre point C. Points A and B and the base of the windmill are on level ground, as shown in the following diagram.



From point A the angle of elevation of point C is 0.6 radians.

(a) Given that point A is 12 metres from the base of the windmill, find the height of point C above the ground. [2]

An observer walks 7 metres from point A to point B.

(b) Find the angle of elevation of point C from point B. [2]

The observer keeps walking until he is standing directly under point C. The observer has a height of 1.8 metres, and as the blades of the windmill rotate, the end of each blade passes 2.5 metres over his head.

(c) Find the length of each blade of the windmill. [2]

One of the blades is painted a different colour than the others. The end of this blade is labelled point D. The height h, in metres, of point D above the ground can be modelled by the function $h(t) = p\cos\left(\frac{3\pi}{10}t\right) + q$, where t is in seconds and $p, q \in \mathbb{R}$. When $t = 0$, point D is at its maximum height.

(d) Find the value of p and the value of q. [4]

If the observer stands directly under point C for one minute, point D will pass over his head n times.

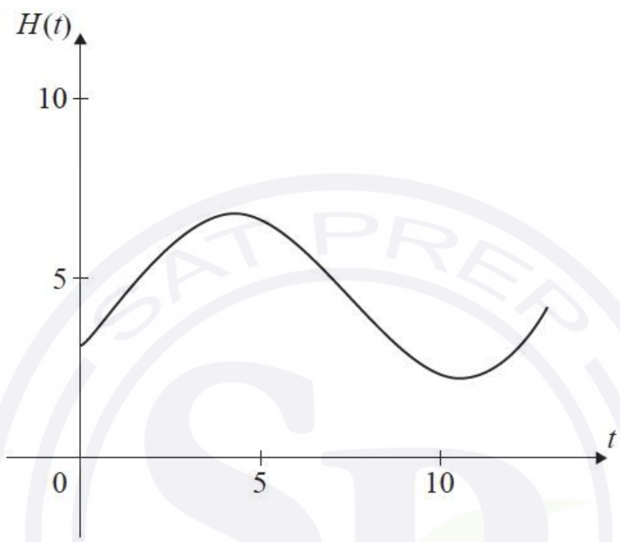
(e) Find the value of n. [3]

Question 7

[Maximum mark: 13]

The height of water, in metres, in Dungeness harbour is modelled by the function $H(t) = a\sin(b(t-c)) + d$, where t is the number of hours after midnight, and a, b, c and d are constants, where $a > 0$, $b > 0$ and $c > 0$.

The following graph shows the height of the water for 13 hours, starting at midnight.



The first high tide occurs at 04:30 and the next high tide occurs 12 hours later. Throughout the day, the height of the water fluctuates between $2.2\,\text{m}$ and $6.8\,\text{m}$.

All heights are given correct to one decimal place.

(a) Show that $b = \frac{\pi}{6}$. [1]

(b) Find the value of a. [2]

(c) Find the value of d. [2]

(d) Find the smallest possible value of c. [3]

(e) Find the height of the water at 12:00. [2]

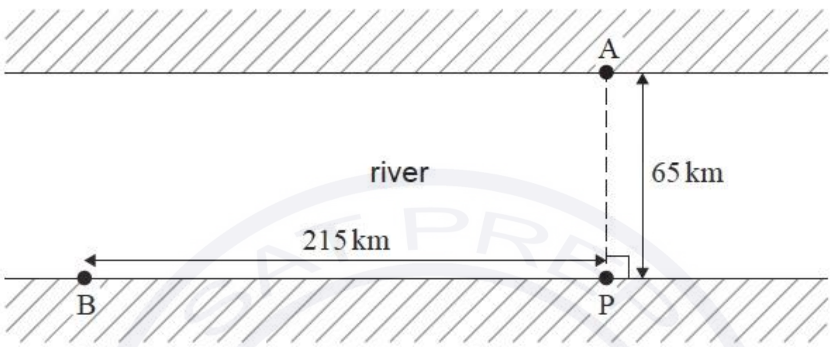
(f) Determine the number of hours, over a 24-hour period, for which the tide is higher than 5 metres. [3]

Question 8

[Maximum mark: 14]

Points A and P lie on opposite banks of a river, such that AP is the shortest distance across the river. Point B represents the centre of a city which is located on the riverbank. $\mathrm{PB} = 215\,\mathrm{km}$, $\mathrm{AP} = 65\,\mathrm{km}$ and $\mathrm{A\hat{P}B} = 90°$.

The following diagram shows this information.



A boat travels at an average speed of $42\,\mathrm{km\,h^{-1}}$. A bus travels along the straight road between P and B at an average speed of $84\,\mathrm{km\,h^{-1}}$.

(a) Find the travel time, in hours, from A to B given that

(i) the boat is taken from A to P, and the bus from P to B;

(ii) the boat travels directly to B. [4]

There is a point D, which lies on the road from P to B, such that $\mathrm{BD} = x\,\mathrm{km}$. The boat travels from A to D, and the bus travels from D to B.

(b) (i) Find an expression, in terms of x for the travel time T, from A to B, passing through D.

(ii) Find the value of x so that T is a minimum.

(iii) Write down the minimum value of T. [6]

(c) An excursion involves renting the boat and the bus. The cost to rent the boat is $\$200$ per hour, and the cost to rent the bus is $\$150$ per hour.

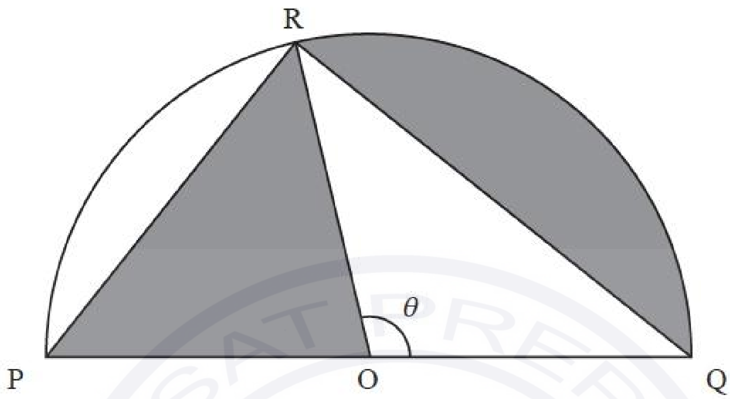
(i) Find the new value of x so that the total cost C to travel from A to B via D is a minimum.

(ii) Write down the minimum total cost for this journey. [4]

Question 9

[Maximum mark: 6]

The following diagram shows a semicircle with centre O and radius r. Points P, Q and R lie on the circumference of the circle, such that $\mathrm{PQ} = 2r$ and $\mathrm{R\hat{O}Q} = \theta$, where $0 < \theta < \pi$.



(a) Given that the areas of the two shaded regions are equal, show that $\theta = 2\sin\theta$. [5]

(b) Hence determine the value of θ. [1]

Question 10

[Maximum mark: 5]

Consider a triangle ABC, where $\mathrm{AC} = 12$, $\mathrm{CB} = 7$ and $\mathrm{B\hat{A}C} = 25°$.

Find the smallest possible perimeter of triangle ABC.

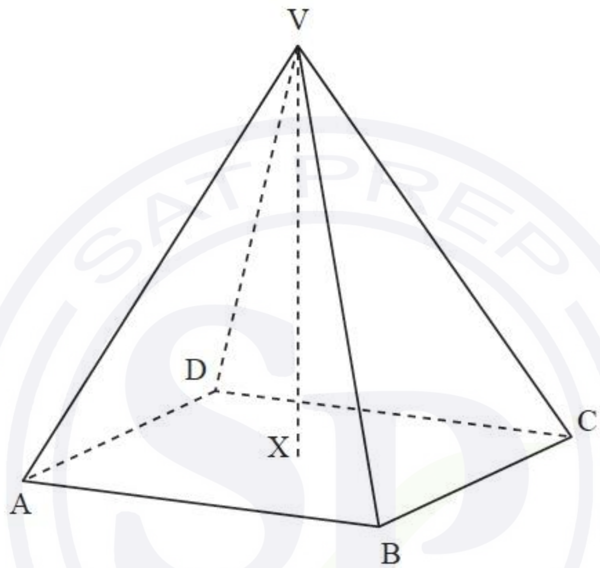
Question 11

[Maximum mark: 13]

All lengths in this question are in centimetres.

A solid metal ornament is in the shape of a right pyramid, with vertex V and square base ABCD. The centre of the base is X. Point V has coordinates $(1, 5, 0)$ and point A has coordinates $(-1, 1, 6)$.

diagram not to scale



(a) Find AV. [2]

(b) Given that $\mathrm{A\hat{V}B} = 40^\circ$, find AB. [3]

The volume of the pyramid is $57.2\,\mathrm{cm}^3$, correct to three significant figures.

(c) Find the height of the pyramid, VX. [3]

A second ornament is in the shape of a cuboid with a rectangular base of length $2x\,\mathrm{cm}$, width $x\,\mathrm{cm}$ and height $y\,\mathrm{cm}$. The cuboid has the same volume as the pyramid.

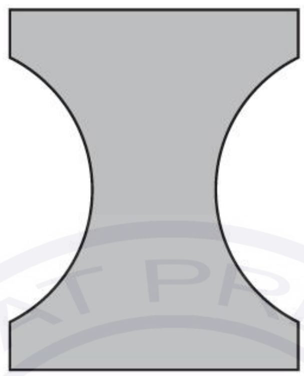
(d) The cuboid has a minimum surface area of $S\,\mathrm{cm}^2$. Find the value of S. [5]

Question 12

[Maximum mark: 6]

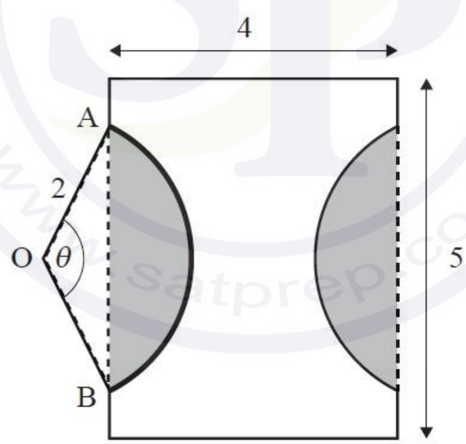
A company is designing a new logo. The logo is created by removing two equal segments from a rectangle, as shown in the following diagram.

diagram not to scale



The rectangle measures $5\,\text{cm}$ by $4\,\text{cm}$. The points A and B lie on a circle, with centre O and radius $2\,\text{cm}$, such that $\text{A}\hat{\text{O}}\text{B} = \theta$, where $0 < \theta < \pi$. This information is shown in the following diagram.

diagram not to scale



(a) Find the area of one of the shaded segments in terms of θ. [3]

(b) Given that the area of the logo is $13.4\,\text{cm}^2$, find the value of θ. [3]

Question 13

[Maximum mark: 12]

A scientist conducted a nine-week experiment on two plants, A and B, of the same species. He wanted to determine the effect of using a new plant fertilizer. Plant A was given fertilizer regularly, while Plant B was not.

The scientist found that the height of Plant A, h_A cm, at time t weeks can be modelled by the function $h_A(t) = \sin(2t+6) + 9t + 27$, where $0 \le t \le 9$.

The scientist found that the height of Plant B, h_B cm, at time t weeks can be modelled by the function $h_B(t) = 8t + 32$, where $0 \le t \le 9$.

(a) Use the scientist's models to find the initial height of

(i) Plant B;

(ii) Plant A correct to three significant figures. [3]

(b) Find the values of t when $h_A(t) = h_B(t)$. [3]

(c) For $0 \le t \le 9$, find the total amount of time when the rate of growth of Plant B was greater than the rate of growth of Plant A. [6]

Question 14

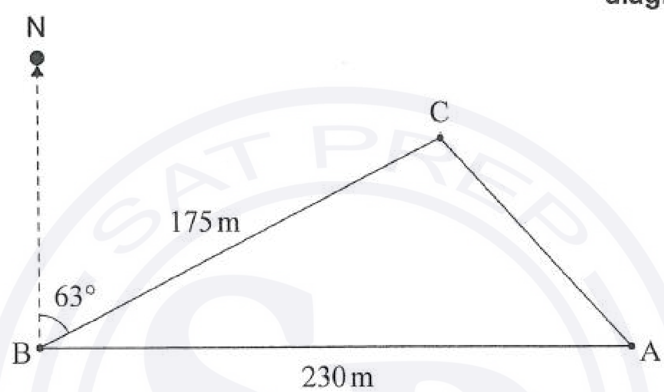
[Maximum mark: 14]

A farmer is placing posts at points A, B, and C in the ground to mark the boundaries of a triangular piece of land on his property.

From point A, he walks due west 230 metres to point B.
From point B, he walks 175 metres on a bearing of $063°$ to reach point C.

This is shown in the following diagram.

diagram not to scale



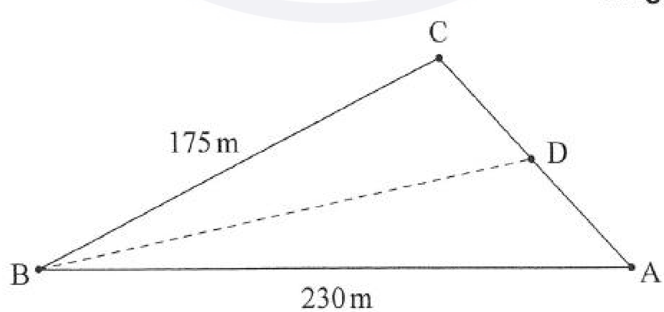
(a) Find the distance from point A to point C. [4]

(b) Find the area of this piece of land. [2]

(c) Find $\mathrm{C\hat{A}B}$. [3]

The farmer wants to divide the piece of land into two sections. He will put a post at point D, which is between A and C. He wants the boundary BD to divide the piece of land such that the sections have equal area. This is shown in the following diagram.

diagram not to scale



(d) Find the distance from point B to point D. [5]

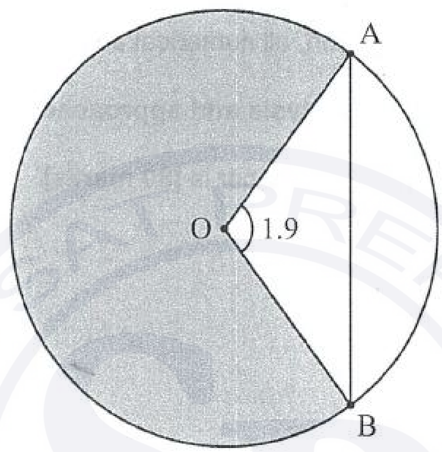
Question 15

[Maximum mark: 6]

The following diagram shows a circle with centre O and radius 5 metres.

Points A and B lie on the circle and $A\hat{O}B = 1.9$ radians.

diagram not to scale



(a) Find the length of the chord [AB]. [3]

(b) Find the area of the shaded sector. [3]

Question 16

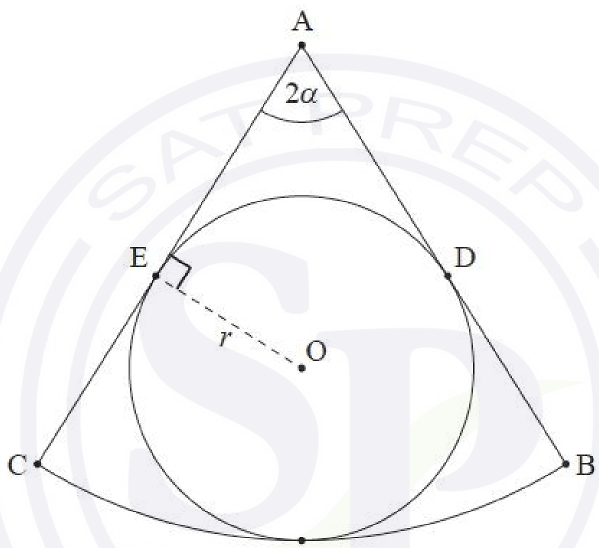
[Maximum mark: 13]

The following diagram shows a sector ABC of a circle with centre A. The angle $\mathrm{B\hat{A}C} = 2\alpha$, where $0 < \alpha < \frac{\pi}{2}$, and $\mathrm{O\hat{E}A} = \frac{\pi}{2}$.

A circle with centre O and radius r is inscribed in sector ABC.

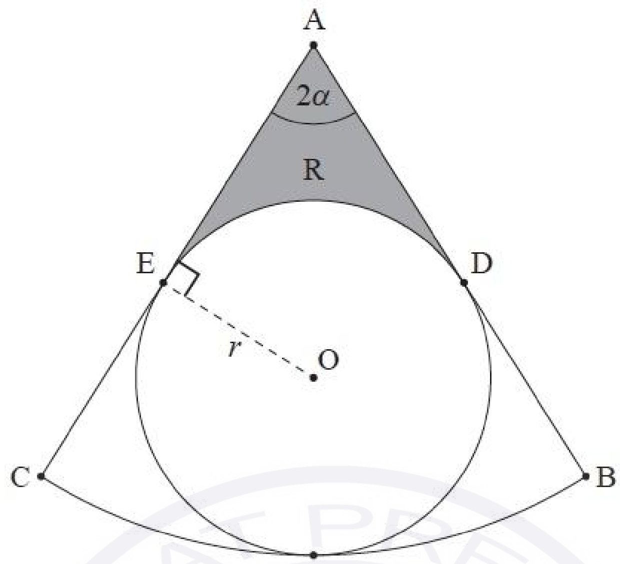
AB and AC are both tangent to the circle at points D and E respectively.

diagram not to scale



(a) Show that the area of the quadrilateral ADOE is $\dfrac{r^2}{\tan\alpha}$. [4]

R represents the shaded region shown in the following diagram.



(b) (i) Find $\hat{DOE}$ in terms of α.

(ii) Hence or otherwise, find an expression for the area of R. [5]

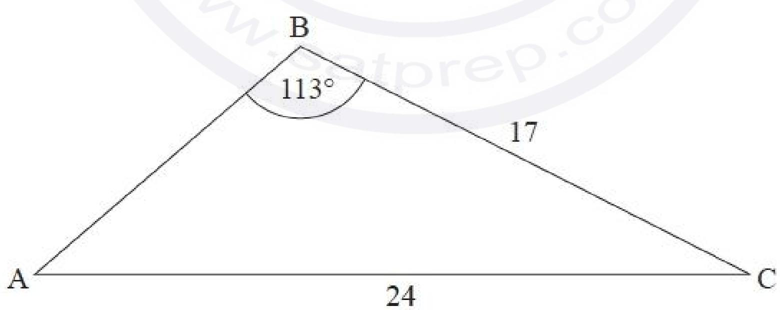
(c) Find the value of α for which the area of R is equal to the area of the circle of centre O and radius r. [4]

Question 17

[Maximum mark: 6]

The following diagram shows triangle ABC, with $AC = 24$, $BC = 17$, and $A\hat{B}C = 113°$.

diagram not to scale



(a) Find $B\hat{A}C$. [3]

(b) Find AB. [3]