Subject – Math AA(Standard Level)
Topic - Number and Algebra
Year - May 2021 – Nov 2024
Paper -2
Answers

Question 1

(a) $\left(1+\frac{5.5}{4\times100}\right)^4$ ***(M1)(A1)***

1.056 ***A1***

[3 marks]

(b) **EITHER**

$2P = P\times\left(1+\frac{5.5}{100\times4}\right)^{4n}$ **OR** $2P = P\times(\text{their } (a))^m$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into loan payment formula. Award ***(A1)*** for correct substitution.

OR

PV $= \pm1$
FV $= \mp2$
I% $= 5.5$
P/Y $= 4$
C/Y $= 4$
$n = 50.756\ldots$ ***(M1)(A1)***

OR

PV $= \pm1$
FV $= \mp2$
I% $= 100(\text{their } (a)-1)$
P/Y $= 1$
C/Y $= 1$ ***(M1)(A1)***

THEN

$\Rightarrow$ 12.7 years
Laurie will have double the amount she invested during 2032 ***A1***

[3 marks]

Total [6 marks]

Question 2

(a) $\frac{4}{3}\pi(12.7)^3$ (or equivalent) **A1**

8580.24 **(A1)**

$V = 8.58 \times 10^3$ **A1**

[3 marks]

(b) recognising volume of the cone is same as volume of **their** sphere **(M1)**

$\frac{1}{3}\pi r^2(14.8) = 8580.24$ (or equivalent) **A1**

$r = 23.529$

$r = 24\,(\text{cm})$ correct to 2 significant figures **A1**

[3 marks]

Total [6 marks]

Question 3

(a) (i) **EITHER**

$9000 \times \left(1 + \frac{7}{100}\right)^5$ **(A1)**

$12622.965...$ **(A1)**

OR

$n = 5$

$\text{I}\% = 7$

$\text{PV} = \mp 9000$

$\text{P/Y} = 1$

$\text{C/Y} = 1$ **(A1)**

$\pm 12622.965...$ **(A1)**

THEN

(\$) 12600 **A1**

(ii) **EITHER**

$9000\left(1+\frac{7}{100}\right)^{x}=20000$ ***(A1)***

OR

$\mathrm{I}\%=7$
$\mathrm{PV}=\mp 9000$
$\mathrm{FV}=\pm 20000$
$\mathrm{P/Y}=1$
$\mathrm{C/Y}=1$ ***(A1)***

THEN

$=12$ (years) ***A1***

[5 marks]

(b) **METHOD 1**
attempt to substitute into compound interest formula (condone absence of compounding periods) ***(M1)***

$9000\left(1+\frac{r}{100\times 12}\right)^{12\times 10}=20000$

$8.01170...$ ***(A1)***

$r=8.02\,(\%)$ ***A1***

METHOD 2

$n=10$
$\mathrm{PV}=\pm 9000$
$\mathrm{FV}=\mp 20000$
$\mathrm{P/Y}=1$
$\mathrm{C/Y}=12$
$r=8.01170...$ ***(M1)(A1)***

$r=8.02\,(\%)$ ***A1***

(c) (i) recognising geometric series (seen anywhere) ***(M1)***

$r = \frac{4500}{9000} \quad \left(= \frac{1}{2}\right)$ ***(A1)***

EITHER

considering S_∞ ***(M1)***

$\frac{9000}{1-0.5} \left(= 18000\right)$ ***A1***

correct reasoning that $18000 < 20000$ ***R1***

THEN

Therefore, Chris will never reach the target. ***AG***

(ii) recognising geometric sum ***M1***

$\frac{u_1\left(1-0.5^5\right)}{0.5} = 20000$ ***(A1)***

$10322.58...$

$(\$)\, 10323$ ***A1***

[8 marks]

Total [16 marks]

Question 4

METHOD 1

product of a binomial coefficient, a power of 3 (and a power of x^2) seen **(M1)**

evidence of correct term chosen **(A1)**

$${}^{n+1}C_2 \times 3^{n+1-2} \times \left(x^2\right)^2 \left(= \frac{n(n+1)}{2} \times 3^{n-1} \times x^4 \right) \text{ OR } n - r = 1$$

equating their coefficient to 20412 or their term to $20412x^4$ **(M1)**

EITHER

$${}^{n+1}C_2 \times 3^{n-1} = 20412$$ **(A1)**

OR

$${}^{r+2}C_r \times 3^r = 20412 \Rightarrow r = 6$$ **(A1)**

THEN

$n = 7$ **A1**

METHOD 2

$$3^{n+1}\left(1 + \frac{x^2}{3}\right)^{n+1}$$

product of a binomial coefficient, and a power of $\frac{x^2}{3}$ OR $\frac{1}{3}$ seen **(M1)**

evidence of correct term chosen **(A1)**

$$3^{n+1} \times \frac{n(n+1)}{2!} \times \left(\frac{x^2}{3}\right)^2 \left(= \frac{3^{n-1}}{2} n(n+1)x^4 \right)$$

equating their coefficient to 20412 or their term to $20412x^4$ **(M1)**

$$3^{n-1} \times \frac{n(n+1)}{2} = 20412$$ **(A1)**

$n = 7$ **A1**

Total [5 marks]

Question 5

(a) attempt to use $u_1+(n-1)d=0$ ***(M1)***

$60-2.5(k-1)=0$

$k=25$ ***A1***

[2 marks]

(b) **METHOD 1**

attempting to express S_n in terms of n ***(M1)***

use of a graph or a table to attempt to find the maximum sum ***(M1)***

$=750$ ***A1***

METHOD 2

EITHER

recognizing maximum occurs at $n=25$ ***(M1)***

$S_{25}=\frac{25}{2}(60+0),\ S_{25}=\frac{25}{2}(2\times 60+24\times -2.5)$ ***(A1)***

OR

attempting to calculate S_{24} ***(M1)***

$S_{24}=\frac{24}{2}(2\times 60+23\times -2.5)$ ***(A1)***

THEN

$=750$ ***A1***

[3 marks]

Total [5 marks]

Question 6

(a) $u_1 = S_1 = \frac{2}{3} \times \frac{7}{8}$ ***(M1)***

$= \frac{14}{24} \left(= \frac{7}{12} = 0.583333\ldots \right)$ ***A1***

[2 marks]

(b) $r = \frac{7}{8} \left(= 0.875 \right)$ ***(A1)***

substituting their values for u_1 and r into $S_\infty = \frac{u_1}{1-r}$ ***(M1)***

$= \frac{14}{3} \left(= 4.66666\ldots \right)$ ***A1***

[3 marks]

(c) attempt to substitute their values into the inequality or formula for S_n ***(M1)***

$\frac{14}{3} - \sum_{r=1}^{n} \frac{2}{3} \left(\frac{7}{8} \right)^r < 0.001$ OR $S_n = \frac{\frac{7}{12}\left(1 - \left(\frac{7}{8}\right)^n\right)}{\left(1 - \frac{7}{8}\right)}$

attempt to solve their inequality using a table, graph or logarithms (must be exponential) ***(M1)***

Note: Award ***(M0)*** if the candidate attempts to solve $S_\infty - u_n < 0.001$.

correct critical value or at least one correct crossover value ***(A1)***

$63.2675\ldots$ OR $S_\infty - S_{63} = 0.001036\ldots$ OR $S_\infty - S_{64} = 0.000906\ldots$

OR $S_\infty - S_{63} - 0.001 = 0.0000363683\ldots$ OR $S_\infty - S_{64} - 0.001 = -0.0000931777\ldots$

least value is $n = 64$ ***A1***

[4 marks]

Total [9 marks]

Question 7

(a) **EITHER**

$N = 10$	OR	$N = 20$
$I\% = 2.74$		$I\% = 2.74$
$PV = (\mp)1700$		$PV = (\mp)1700$
$P/Y = 1$		$P/Y = 2$
$C/Y = 2$		$C/Y = 2$

(M1)(A1)

Note: Award ***(M1)*** for an attempt to use a financial app in their technology with at least two entries seen, and award ***(A1)*** for all entries correct. Accept a positive or negative value for PV.

OR

$$1700\left(1+\frac{0.0274}{2}\right)^{2\times10}$$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into compound interest formula.
Award ***(A1)*** for correct substitution.

THEN

$2231.71 ***A1***

[3 marks]

(b) **EITHER**

$N = 10$

$PV = \mp 1700$

$FV = \pm 2231.71...$

$P/Y = 1$

$C/Y = 1$ ***(M1)***

Note: Award ***(M1)*** for an attempt to use a financial app in their technology with at least two entries seen.

OR

$1700\left(1+\frac{r}{100}\right)^{10} = 2231.71...$ ***(M1)***

THEN

$r = 2.75876\ldots$

$r = 2.76$ ***A1***

Note: Ignore omission of opposite signs for PV and FV if $r = 2.76$ is obtained.

[2 marks]

(c) $531.71 ***A1***

[1 mark]

Total [6 marks]

Question 8

(a) **METHOD 1**

using geometric sequence with $r = 1.02$ ***(M1)***

correct expression or listing terms correctly ***(A1)***

45000×1.02^{10} OR $45000 \times 1.02^{11-1}$ OR listing terms

Gemma's salary is \$54855 (must be to the nearest dollar) ***A1***

[3 marks]

METHOD 2

$N = 10$

$\text{PV} = \mp 45000$

$\text{I}\% = 2$

$\text{P/Y} = 1$

$\text{C/Y} = 1$

$FV = \pm 54854.7489...$ ***(M1)(A1)***

Gemma's salary is \$54855 (must be to the nearest dollar) ***A1***

[3 marks]

(b) finds $a = 1096.89...$ and $b = -2160753.8...$ (accept $b = -2.16 \times 10^6$) ***(A1)(A1)***

Note: Award ***(A1)(A1)*** for $S = 1096.89...x + 33028.49...$, or $S = 1096.89...x + 43997.4...$, or $S = 1096.89...x + 45094.3...$

Kaia's salary in 2021 is \$56063.21 (accept \$56817.09 from $b = -2.16 \times 10^6$) ***A1***

Kaia had a higher salary than Gemma in 2021 ***AG***

[3 marks]

Total [6 marks]

Question 9

valid approach for expansion (must be the product of a binomial coefficient with $n=9$ and a power of ax) ***(M1)***

$^9C_r(ax)^{9-r}(1)^r$ OR $^9C_{9-r}(ax)^r(1)^{9-r}$ OR $^9C_0(ax)^0(1)^9+{}^9C_1(ax)^1(1)^8+\ldots$

recognizing that the term in x^6 is needed ***(M1)***

$\frac{\text{Term in } x^6}{21x^2}=kx^4$ OR $r=6$ OR $r=3$ OR $9-r=6$

correct term or coefficient in binomial expansion (seen anywhere) ***(A1)***

$^9C_6(ax)^6(1)^3$ OR $^9C_3\,a^6x^6$ OR $84\left(a^6x^6\right)(1)$ OR $84a^6$

EITHER

correct term in x^4 or coefficient (may be seen in equation) ***(A1)***

$\frac{^9C_6}{21}a^6x^4$ OR $4a^6x^4$ OR $4a^6$

Set their term in x^4 or coefficient of x^4 equal to $\frac{8}{7}a^5x^4$ or $\frac{8}{7}a^5$ (do not accept other powers of *x*) ***(M1)***

$\frac{^9C_3}{21}a^6x^4=\frac{8}{7}a^5x^4$ OR $4a^6=\frac{8}{7}a^5$

correct term in x^6 or coefficient of x^6 (may be seen in equation) ***(A1)***

$84a^6x^6$ OR $84a^6$

set their term in x^6 or coefficient of x^6 equal to $24a^5x^6$ or $24a^5$ (do not accept other powers of *x*) ***(M1)***

$84a^6x^6=24a^5x^6$ OR $84a=24$

THEN

$a=\frac{2}{7}\approx 0.286\left(0.285714\ldots\right)$ ***A1***

Total [6 marks]

Question 10

$86.4 = 50r^3$ **(A1)**

$r = 1.2\left(= \sqrt[3]{\frac{86.4}{50}}\right)$ seen anywhere **(A1)**

$\frac{50(1.2^n - 1)}{0.2} > 33500$ OR $250(1.2^n - 1) = 33500$ **(A1)**

attempt to solve their geometric S_n inequality or equation **(M1)**

sketch OR $n > 26.9045$, $n = 26.9$ OR $S_{26} = 28368.8$ OR $S_{27} = 34092.6$ OR algebraic manipulation involving logarithms

$n = 27$ accept $n \geq 27$ **A1**

Total [5 marks]

Question 11

product of a binomial coefficient, a power of ax^3 and a power of b seen **(M1)**

evidence of correct term chosen

for $n = 8$: $r = 2$ (or $r = 6$) OR for $n = 10$: $r = 2$ (or $r = 8$) **(A1)**

correct equations (may include powers of x) **A1A1**

${}^8C_2 a^2 b^6 = 448$ $\left(28a^2b^6 = 448 \Rightarrow a^2b^6 = 16\right)$, ${}^{10}C_2 a^2 b^8 = 2880$ $\left(45a^2b^8 = 2880 \Rightarrow a^2b^8 = 64\right)$

attempt to solve their system in a and b algebraically or graphically **(M1)**

$b = 2$; $a = \frac{1}{2}$ **A1A1**

[7 marks]

Question 12

(a) 9% (accept 0.09) ***A1***

[1 mark]

(b) $t = 5$ (seen anywhere) ***(A1)***

$24961.28...$

25000 (dollars) ***A1***

[2 marks]

(c) **EITHER**

$n = 5$

$\mathrm{I}\% = 3$

$\mathrm{PV} = (\mp)15000$

$\mathrm{P/Y} = 1$

$\mathrm{C/Y} = 1$ ***(A1)***

Note: Award ***(A1)*** for use of a financial app in their technology with all entries correct.

$(\Rightarrow \mathrm{FV} = (\pm)17389.11...)$

OR

$15000\left(1+\frac{3}{100}\right)^5 (=17389.11...)$ ***(A1)***

THEN

subtracting their value from their answer to part (b) ***(M1)***

$7572.17\ldots$

7570 (dollars) ***A1***

[3 marks]

Total [6 marks]

Question 13

(a) attempt to use the binomial expansion of $(x+h)^8$ ***(M1)***

$^8C_0x^8h^0 + {}^8C_1x^7h^1 + {}^8C_2x^6h^2 + \ldots$

(i) $a = 8h$ (accept $^8C_1\,h$) ***A1***

(ii) $b = 28h^2$ (accept $^8C_2\,h^2$) ***A1***

(iii) $d = 70h^4$ (accept $^8C_4\,h^4$) ***A1***

[4 marks]

(b) recognition that there is a common ratio between their terms ***(M1)***

$8h \times r = 28h^2$ **OR** $28h^2 \times r = 70h^4$ **OR** $8h \times r^2 = 70h^4$

correct equation in terms of h ***(A1)***

$\frac{28h^2}{8h} = \frac{70h^4}{28h^2}$ (or equivalent)

$h = 1.4$ ***A1***

[3 marks]

Total [7 marks]

Question 14

(a) (i) 4200×36 ***(A1)***

$=151200$

$=(\$)151000$ ***A1***

(ii) recognizing sum of a geometric sequence is required ***(M1)***

$\frac{1500\left(1-1.04^{36}\right)}{1-1.04}$ ***(A1)***

$=116397.4707...$

$=(\$)116397.47$ ***A1***

[5 marks]

(b) Sorin's future value after n years = $160000\left(1+\frac{5}{100\times 12}\right)^{12n}$ ***A1***

[1 mark]

(c) (i) Sorin's total = $160000\left(1+\frac{5}{100\times 12}\right)^{6}\left(=164041.89...\right)$ ***(A1)***

Daniela's total = $\frac{1500\left(1-1.04^{6}\right)}{1-1.04}\left(=9949.46...\right)$ ***(A1)***

total value = $(\$)173991.36$ ***A1***

(ii)

EITHER (finding number of months, m)

$160000\left(1+\frac{5}{100\times 12}\right)^{m}+\frac{1500\left(1-1.04^{m}\right)}{1-1.04}\ \left(\geq 257000\right)$ ***(A1)***

$m\geq 28.4412...$ OR $\left(m=28\Rightarrow\right)\ 254707$ AND $\left(m=29\Rightarrow\right)\ 259954$ ***(A1)***

OR (finding number of years, n)

$$160000\left(1+\frac{5}{100\times 12}\right)^{12\times n}+\frac{1500\left(1-1.04^{12\times n}\right)}{1-1.04}\ \left(\geq 257000\right)$$ ***(A1)***

$n \geq 2.37010...$ (years) ***(A1)***

THEN

$m = 29$ (months) ***A1***

[6 marks]

(d) **EITHER**

$N = 24$	OR	$N = 6$
$PV = \mp 30000$		$PV = \mp 30000$
$PMT = 0$		$PMT = 0$
$FV = \pm 41000$		$FV = \pm 41000$
$P/Y = 4$		$P/Y = 1$
$C/Y = 4$		$C/Y = 4$

(M1)(A1)

OR

$$30000\left(1+\frac{r}{100\times 4}\right)^{6\times 4}=41000$$ ***(M1)(A1)***

THEN

5.24027…

$(r =)\ 5.24\ (\%)$ ***A1***

[3 marks]

Total [15 marks]

Question 15

(a) (i) 5500×36 ***(A1)***

$= (\$)\,198000$ ***A1***

(ii) recognizing sum of a geometric sequence is required ***(M1)***

$$\frac{2000\left(1-1.06^{36}\right)}{1-1.06}$$ ***(A1)***

$= 238241.7333...$

$= (\$)238241.73$ ***A1***

[5 marks]

(b) Sorin's future value after n years = $120000\left(1+\frac{4}{100\times 12}\right)^{12n}$ ***A1***

[1 mark]

(c) (i) Sorin's total $= 120000\left(1+\frac{4}{100\times 12}\right)^{6}\ (=122420.09)$ ***(A1)***

Daniela's total $= \frac{2000\left(1-1.06^{6}\right)}{1-1.06}\ (=13950.64)$ ***(A1)***

total value = ($)136370.73 ***A1***

(ii) **EITHER** (finding number of months, m)

$120000\left(1+\frac{4}{100\times 12}\right)^{m}+\frac{2000\left(1-1.06^{m}\right)}{1-1.06}\ (\geq 250000)$ ***(A1)***

$m \geq 26.0905...$ OR $(m=26\Rightarrow)\ 249157...$ AND $(m=27\Rightarrow)\ 258692...$ ***(A1)***

Note: Condone use of an equation or strict inequality.

OR (finding number of years, n)

$120000\left(1+\frac{4}{100\times 12}\right)^{12\times n}+\frac{2000\left(1-1.06^{12\times n}\right)}{1-1.06}\ (\geq 250000)$ ***(A1)***

$n \geq 2.17421...$ (years) ***(A1)***

Note: Condone use of an equation or strict inequality.

THEN

$m=27$ (months) ***A1***

[6 marks]

(d) **EITHER**

$N = 24$	OR	$N = 6$
$PV = \mp 40000$		$PV = \mp 40000$
$PMT = 0$		$PMT = 0$
$FV = \pm 53000$		$FV = \pm 53000$
$P/Y = 4$		$P/Y = 1$
$C/Y = 4$		$C/Y = 4$

(M1)(A1)

: Award ***(M1)*** for an attempt to use a financial app in their technology with at least two entries seen, and award ***(A1)*** for all entries correct. PV and FV must have opposite signs.

OR

$$40000\left(1+\frac{r}{100\times 4}\right)^{6\times 4} = 53000$$ ***(M1)(A1)***

: Award ***(M1)*** for attempting to substitute into compound interest formula,

award ***(A1)*** for correct equation.

THEN

$4.71781\ldots$

$(r =) 4.72\ (\%)$ ***A1***

[3 marks]

Total [15 marks]

Question 16

(a) recognition that a 15% loss leaves 85% OR finding 15% and subtracting from original ***(M1)***

0.85×35000 OR $35000-0.15\times 35000$

$=(\$)29750$ ***A1***

Note: Accept $(\$)29800$.

[2 marks]

(b) **EITHER**

29750×0.89^{9} ***(A1)***

OR

$\mathrm{N}=9$

$\mathrm{I}\%=-11$

$\mathrm{PV}=\mp 29750$

(A1)

THEN

value $(FV)=(\$)10423$ ***A1***

Note: For this ***A1*** the answer must be rounded to the nearest dollar. Accept $(\$)10441$ from using 3 sf answer from part (a).

[2 marks]

(c) **METHOD 1**

attempt to solve the inequality (or equation) $29750\times0.89^{n-1}<3500$ OR table of values ***(M1)***

$19.3643...$ OR $(n=19\Rightarrow)\,3651.80...$ OR $(n=20\Rightarrow)\,3250.10...$ ***(A1)***

Note: For candidates using $(\$)29800$, $n>19.3787...$, $(n=19\Rightarrow)\,3657.93...$, $(n=20\Rightarrow)\,3255.56...$.

$n=20$ ***A1***

[3 marks]

METHOD 2

use of the finance app with $\mathrm{I\%}=-11,\ \mathrm{PV}=\mp29750,\ \mathrm{FV}=\pm3500$

OR $29750\times0.89^{N}<3500$ (condone the use of n or x) ***(M1)***

$(N=)\,18.3643...$ ***(A1)***

Note: For candidates using $(\$)29800$, $N=18.3787...$

$n=20$ ***A1***

[3 marks]

Total [7 marks]

Question 17

EITHER

attempt to form a product of binomial coefficient, a power of $2x$ and a power of -5 seen ***(M1)***

${}^{9}C_{3}(2x)^{6}(-5)^{3}$ OR ${}^{9}C_{6}(2x)^{6}(-5)^{3}$ OR $84\times(2x)^{6}(-5)^{3}$ ***(A1)(A1)***

Note: Award ***A1*** for ${}^{9}C_{6}$ or ${}^{9}C_{3}$ or 84, ***A1*** for $(2x)^{6}(-5)^{3}$.

OR

attempt to use the general term ***(M1)***

${}^{9}C_{r}(2x)^{9-r}(-5)^{r}$ and $r=3$ ***(A1)(A1)***

THEN

-672000 (exact) ***A1***

Note: Award ***A0*** for a final answer of $-672000x^{6}$.

[4 marks]

Question 18

EITHER

attempt to form a product of binomial coefficient, a power of $2x$ and a power of -5 seen ***(M1)***

$^{11}C_8(2x)^8(-5)^3$ OR $^{11}C_3(2x)^8(-5)^3$ OR $165\times(2x)^8(-5)^3$ ***(A1)(A1)***

Note: Award ***A1*** for $^{11}C_8$ or $^{11}C_3$ or 165, ***A1*** for $(2x)^8(-5)^3$.

OR

attempt to use the general term ***(M1)***

$^{11}C_r(2x)^{11-r}(-5)^r$ and $r=3$ ***(A1)(A1)***

THEN

-5280000 (exact) ***A1***

Note: Award ***A0*** for a final answer of $-5280000x^8$.

[4 marks]