Question 1

[Maximum mark: 6]

On 1st January 2020, Laurie invests P in an account that pays a nominal annual interest rate of 5.5%, compounded **quarterly**.

The amount of money in Laurie's account **at the end of each year** follows a geometric sequence with common ratio, r.

(a)   Find the value of r, giving your answer to four significant figures.   [3]

Laurie makes no further deposits to or withdrawals from the account.

(b)   Find the year in which the amount of money in Laurie's account will become double the amount she invested.   [3]

Question 2

[Maximum mark: 6]

A metal sphere has a radius $12.7\,\text{cm}$.

(a)   Find the volume of the sphere expressing your answer in the form $a \times 10^k$, $1 \le a < 10$ and $k \in \mathbb{Z}$.   [3]

The sphere is to be melted down and remoulded into the shape of a cone with a height of $14.8\,\text{cm}$.

(b)   Find the radius of the base of the cone, correct to 2 significant figures.   [3]

Question 3

[Maximum mark: 16]

Two friends Amelia and Bill, each set themselves a target of saving $20 000. They each have $9000 to invest.

(a)   Amelia invests her $9000 in an account that offers an interest rate of 7% per annum compounded **annually**.

     (i)   Find the value of Amelia's investment after 5 years to the nearest hundred dollars.

     (ii)   Determine the number of years required for Amelia's investment to reach the target. [5]

(b)   Bill invests his $9000 in an account that offers an interest rate of r% per annum compounded **monthly**, where r is set to two decimal places.

     Find the minimum value of r needed for Bill to reach the target after 10 years. [3]

(c)   A third friend Chris also wants to reach the $20 000 target. He puts his money in a safe where he does not earn any interest. His system is to add more money to this safe each year. Each year he will add half the amount added in the previous year.

     (i)   Show that Chris will never reach the target if his initial deposit is $9000.

     (ii)   Find the amount Chris needs to deposit initially in order to reach the target after 5 years. Give your answer to the nearest dollar. [8]

Question 4

[Maximum mark: 5]

Consider the expansion of $(3 + x^2)^{n+1}$, where $n \in \mathbb{Z}^+$.

Given that the coefficient of x^4 is $20\,412$, find the value of n.

Question 5

[Maximum mark: 5]

An arithmetic sequence has first term 60 and common difference -2.5.

(a)   Given that the kth term of the sequence is zero, find the value of k. [2]

Let S_n denote the sum of the first n terms of the sequence.

(b)   Find the maximum value of S_n. [3]

Question 6

[Maximum mark: 9]

The sum of the first n terms of a geometric sequence is given by $S_n = \sum_{r=1}^{n} \frac{2}{3}\left(\frac{7}{8}\right)^r$.

(a)   Find the first term of the sequence, u_1.   [2]

(b)   Find S_∞.   [3]

(c)   Find the least value of n such that $S_\infty - S_n < 0.001$.   [4]

Question 7

[Maximum mark: 6]

In this question, give all answers correct to two decimal places.

Sam invests $\$1700$ in a savings account that pays a nominal annual rate of interest of 2.74%, compounded half-yearly. Sam makes no further payments to, or withdrawals from, this account.

(a)   Find the amount that Sam will have in his account after 10 years.   [3]

David also invests $\$1700$ in a savings account that pays an annual rate of interest of $r\%$, compounded yearly. David makes no further payments or withdrawals from this account.

(b)   Find the value of r required so that the amount in David's account after 10 years will be equal to the amount in Sam's account.   [2]

(c)   Find the interest David will earn over the 10 years.   [1]

Question 8

[Maximum mark: 6]

Gemma and Kaia started working for different companies on January 1st 2011.

Gemma's starting annual salary was $45\,000$, and her annual salary increases 2% on January 1st each year after 2011.

(a) Find Gemma's annual salary for the year 2021, to the nearest dollar.   [3]

Kaia's annual salary is based on a yearly performance review. Her salary for the years 2011, 2013, 2014, 2018, and 2022 is shown in the following table.

year (x)	2011	2013	2014	2018	2022
annual salary ($\$S$)	45 000	47 200	48 500	53 000	57 000

(b) Assuming Kaia's annual salary can be approximately modelled by the equation $S = ax + b$, show that Kaia had a higher salary than Gemma in the year 2021, according to the model.   [3]

Question 9

[Maximum mark: 6]

Consider the expansion of $\dfrac{(ax+1)^9}{21x^2}$, where $a \neq 0$. The coefficient of the term in x^4 is $\dfrac{8}{7}a^5$.

Find the value of a.

Question 10

[Maximum mark: 5]

A geometric sequence has a first term of 50 and a fourth term of 86.4.

The sum of the first n terms of the sequence is S_n.

Find the smallest value of n such that $S_n > 33\,500$.