Subject – Math AA(Standard Level)
Topic - Statistics and Probability
Year - May 2021 – Nov 2024
Paper -2
Answers

Question 1

(a) attempt to use the symmetry of the normal curve *(M1)*

eg diagram, $0.5-0.1446$

$\mathrm{P}(24.15 < X < 25) = 0.3554$ ***A1***

[2 marks]

(b) (i) use of inverse normal to find z score *(M1)*

$z = -1.0598$

correct substitution $\dfrac{24.15-25}{\sigma} = -1.0598$ *(A1)*

$\sigma = 0.802$ ***A1***

(ii) $\mathrm{P}(X > 26) = 0.106$ *(M1)****A1***

[5 marks]

(c) recognizing binomial probability *(M1)*

$\mathrm{E}(Y) = 10 \times 0.10621$ *(A1)*

$= 1.06$ ***A1***

[3 marks]

(d) $\mathrm{P}(Y = 3)$ *(M1)*

$= 0.0655$ ***A1***

[2 marks]

(e) recognizing conditional probability *(M1)*

correct substitution ***A1***

$\dfrac{0.3554}{1-0.10621}$

$= 0.398$ ***A1***

[3 marks]

Total [15 marks]

Question 2

(a) $a = 1.29$ and $b = -10.4$ ***A1A1***

[2 marks]

(b) recognising both lines pass through the mean point *(M1)*

$p = 28.7$, $q = 30.3$ ***A2***

[3 marks]

Total [5 marks]

Question 3

(a) recognition of binomial *(M1)*

$X \sim \mathrm{B}(5,\ 0.7)$

attempt to find $\mathrm{P}(X \le 3)$ ***M1***

$= 0.472 (= 0.47178)$ ***A1***

[3 marks]

(b) recognition of 2 sixes in 4 tosses *(M1)*

$\mathrm{P}(\text{3rd six on the 5th toss}) = \left[\binom{4}{2} \times (0.7)^2 \times (0.3)^2\right] \times 0.7 (= 0.2646 \times 0.7)$ ***A1***

$= 0.185 (= 0.18522)$ ***A1***

[3 marks]

Total [6 marks]

Question 4

Let X = mass of a bag of sugar

(a) evidence of identifying the correct area *(M1)*

$\mathrm{P}(X < 995) = 0.0765637...$

$= 0.0766$ ***A1***

[2 marks]

(b) 0.0766×100

≈ 8 ***A1***

[1 mark]

(c) recognition that $\mathrm{P}(X > 1005 | X \ge 995)$ is required *(M1)*

$$\frac{\mathrm{P}(X \ge 995 \cap X > 1005)}{\mathrm{P}(X \ge 995)}$$

$$\frac{\mathrm{P}(X > 1005)}{\mathrm{P}(X \ge 995)}$$ *(A1)*

$$\frac{0.0765637\ldots}{1 - 0.0765637\ldots}\left(= \frac{0.0765637\ldots}{0.923436\ldots}\right)$$

$= 0.0829$ ***A1***

[3 marks]

Total [6 marks]

Question 5

(a) $a = 0.433156\ldots$, $b = 4.50265\ldots$

$a = 0.433$, $b = 4.50$ **A1A1**

[2 marks]

(b) attempt to substitute $x = 18$ into their equation ***(M1)***

$y = 0.433 \times 18 + 4.50$

$= 12.2994\ldots$

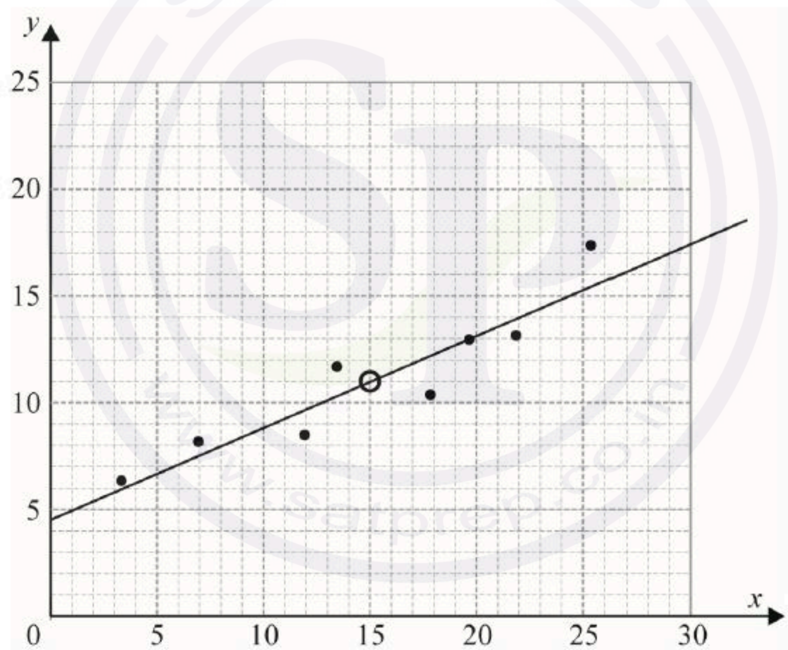
$= 12.3$ ***A1***

[2 marks]

(c) $\bar{x} = 15$, $\bar{y} = 11$ ***A1***

[1 mark]

(d)



A1A1

Note: Award marks as follows:

A1 for a straight line going through $(15,11)$

A1 for intercepting the y-axis between their $b \pm 1.5$ (when their line is extended), which includes all the data for $3.3 \le x \le 25.3$.

If the candidate does not use a ruler, award ***A0A1*** where appropriate.

[2 marks]

Total [7 marks]

Question 6

(a) considering that sum of probabilities is 1 *(M1)*

$0.85 + c + 0.03 + 0.002 + 0.0001 = 1$

0.1179 ***A1***

[2 marks]

(b) valid attempt to find $\mathrm{E}(D)$ *(M1)*

$\mathrm{E}(D) = (0 \times 0.85) + (2 \times 0.1179) + (10 \times 0.03) + (50 \times 0.002) + (1000 \times 0.0001)$

$\mathrm{E}(D) = 0.7358$ ***A1***

No, not a fair game ***A1***

for a fair game, $\mathrm{E}(D)$ would be \$2 OR players expected winnings are 1.264 ***R1***

[4 marks]

(c) recognition of GP with $r = 2$ *(M1)*

$1000 \times 2^{n-1}$ OR $500(2^n)$ ***A1***

[2 marks]

(d) recognizing $\mathrm{E}(D) > 2$ *(M1)*

correct expression for w^{th} week (or n^{th} week) *(A1)*

$(0\times 0.85)+(2\times 0.1179)+(10\times 0.03)+(50\times 0.002)+(1000\times 2^{w-1}\times 0.0001)$

correct inequality (accept equation) *(A1)*

$0.6358+(1000\times 2^{w-1}\times 0.0001) > 2$ OR $2^{n-1} > 13.642$

EITHER

$n-1 > 3.76998$ OR $w = 4.76998...$ *(A1)*

OR

$\mathrm{E}(D) = 1.4358$ in week 4 or $\mathrm{E}(D) = 2.2358$ in week 5 *(A1)*

THEN

$w = 5$ ***A1***

expected profit per ticket $=$ their $\mathrm{E}(D) - 2$ *(M1)*

$= 0.2358$ ***A1***

[7 marks]

Total [15 marks]

Question 7

(a) use of inverse normal to find z-score ***(M1)***

$z = 2.0537...$

$2.0537... = \frac{82-75}{\sigma}$ ***(A1)***

$\sigma = 3.408401...$

$\sigma = 3.41$ ***A1***

[3 marks]

(b) evidence of identifying the correct area under the normal curve ***(M1)***

$\mathrm{P}(T > 80) = 0.071193...$

$\mathrm{P}(T > 80) = 0.0712$ ***A1***

[2 marks]

(c) recognition that $\mathrm{P}(80 < T < 82)$ is required ***(M1)***

$P(T < 82 | T > 80) = \frac{\mathrm{P}(80 < T < 82)}{\mathrm{P}(T > 80)} = \left(\frac{0.051193...}{0.071193...}\right)$ ***(M1)(A1)***

$= 0.719075...$

$= 0.719$ ***A1***

[4 marks]

(d) recognition of binomial probability *(M1)*

$X \sim \mathrm{B}(64, 0.071193...)$ or $\mathrm{E}(X) = 64 \times 0.071193...$ *(A1)*

$\mathrm{E}(X) = 4.556353...$

$\mathrm{E}(X) = 4.56$ (flights) ***A1***

[3 marks]

(e) $\mathrm{P}(X > 6) = P(X \geq 7) = 1 - \mathrm{P}(X \leq 6)$ *(M1)*

$= 1 - 0.83088...$ *(A1)*

$= 0.1691196...$

$= 0.169$ ***A1***

[3 marks]

Total [15 marks]

Question 8

(a) **EITHER**

$\mathrm{P}(S) + \mathrm{P}(T) + \mathrm{P}(S' \cap T') - \mathrm{P}(S \cap T) = 1$ OR $\mathrm{P}(S \cup T) = \mathrm{P}((S' \cap T')')$ *(M1)*

$0.7 + 0.2 + 0.18 - \mathrm{P}(S \cap T) = 1$ OR $\mathrm{P}(S \cup T) = 1 - 0.18$

OR

a clearly labelled Venn diagram *(M1)*

THEN

$\mathrm{P}(S \cap T) = 0.08$ (accept 8%) ***A1***

Note: To obtain the ***M1*** for the Venn diagram all labels must be correct and in the correct sections. For example, do not accept 0.7 in the area corresponding to $S \cap T'$.

[2 marks]

(b) **EITHER**

$\mathrm{P}(T \cap S') = \mathrm{P}(T) - \mathrm{P}(T \cap S) (= 0.2 - 0.08)$ OR

$\mathrm{P}(T \cap S') = \mathrm{P}(T \cup S) - \mathrm{P}(S) (= 0.82 - 0.7)$

(M1)

OR

a clearly labelled Venn diagram including $\mathrm{P}(S)$, $\mathrm{P}(T)$ and $\mathrm{P}(S \cap T)$ ***(M1)***

THEN

$= 0.12$ (accept 12%) ***A1***

[2 marks]

(c) $\mathrm{P}(G \cap T) = \mathrm{P}(T / G)\mathrm{P}(G)$ (0.25×0.48) ***(M1)***

$= 0.12$ ***A1***

[2 marks]

(d) **METHOD 1**

$\mathrm{P}(G) \times \mathrm{P}(T) (= 0.48 \times 0.2) = 0.096$ ***A1***

$\mathrm{P}(G) \times \mathrm{P}(T) \neq \mathrm{P}(G \cap T) \Rightarrow$ G and T are not independent ***R1***

METHOD 2

$\mathrm{P}(T \mid G) = 0.25$ ***A1***

$\mathrm{P}(T \mid G) \neq \mathrm{P}(T) \Rightarrow$ G and T are not independent ***R1***

Note: Do not award ***A0R1.***

[2 marks]

Total [8 marks]

Question 9

(a) (i) $a = 0.805084...$ and $b = 2.88135...$

$a = 0.805$ and $b = 2.88$ ***A1A1***

(ii) $r = 0.97777...$

$r = 0.978$ ***A1***

[3 marks]

(b) a represents the (average) increase in waiting time (0.805 mins) per additional customer (waiting to receive their coffee) ***R1***

[1 mark]

(c) attempt to substitute $x = 7$ into their equation ***(M1)***

$8.51693...$

8.52 (mins) ***A1***

[2 marks]

Total [6 marks]

Question 10

(a) $\mathrm{P}\left(\frac{\mu - 1.5\sigma - \mu}{\sigma} < \frac{X - \mu}{\sigma} < \frac{\mu + 1.5\sigma - \mu}{\sigma}\right)$ ***(M1)***

$P(-1.5 < Z < 1.5)$ OR $1 - 2 \times \mathrm{P}(Z < -1.5)$ ***(A1)***

$\mathrm{P}(-1.5 < Z < 1.5) = 0.866385...$

$\mathrm{P}(\mu - 1.5\sigma < X < \mu + 1.5\sigma) = 0.866$ ***A1***

Note: Do not award any marks for use of their answers from part (b).

[3 marks]

(b) $z_1 = -1.75068...$ and $z_2 = 1.30468...$ (seen anywhere) ***(A1)***

correct equations ***(A1)(A1)***

$\frac{106.2 - \mu}{\sigma} = -1.75068...$, $\mu + 1.30468...\sigma = 182.6$

attempt to solve their equations involving **z** values ***(M1)***

$\mu = 149.976...$, $\sigma = 25.0051...$

$\mu = 150$, $\sigma = 25.0$ ***A1***

[5 marks]

(c) (i) new sample space is 96% (may be seen in (ii) or (iii)) ***(M1)***

$\mathrm{P}(\text{medium}|\text{not small})$ OR $\frac{0.576}{0.96}$

$\mathrm{P}(\text{Medium}) = 0.6$ ***A1***

(ii) $\mathrm{P}(\text{Large}) = 0.3$ ***A1***

(iii) $\mathrm{P}(\text{Premium}) = 0.1$ ***A1***

[4 marks]

(d) attempt to express revenue from avocados ***(M1)***

$1.1 \times 0.6 + 1.29 \times 0.3 + 1.96 \times 0.1$ OR $1.243n$

correct inequality or equation for net profit in terms of n ***(A1)***

$1.1 \times 0.6n + 1.29 \times 0.3n + 1.96 \times 0.1n - 200 \geq 438$ OR $1.243n - 200 = 438$

attempt to solve the inequality ***(M1)***

sketch OR $n = 513.274...$

$n = 514$ ***A1***

> **Note:** Only award follow through in part (d) for 3 probabilities which add up to 1. FT of probabilities from c) that do not add up to 1 should only be awarded ***M*** marks, where appropriate, in d).

[4 marks]

Total [16 marks]

Question 11

(a) recognize that the variable has a Binomial distribution ***(M1)***

$X \sim \mathrm{B}(30, 0.05)$

attempt to find $\mathrm{P}(X \geq 1)$ ***(M1)***

$1 - \mathrm{P}(X = 0)$ OR $1 - 0.95^{30}$ OR $1 - 0.214638\ldots$ OR $0.785361\ldots$

Note: The two ***M*** marks are independent of each other.

$\mathrm{P}(X \geq 1) = 0.785$ ***A1***

[3 marks]

(b) recognition of conditional probability ***(M1)***

$\mathrm{P}(X \leq 2 \mid X \geq 1)$ OR $\mathrm{P}(\text{at most 2 defective} \mid \text{at least 1 defective})$

Note: Recognition must be shown in context either in words or symbols but not just $\mathrm{P}(A \mid B)$.

$\dfrac{\mathrm{P}(1 \leq X \leq 2)}{\mathrm{P}(X \geq 1)}$ OR $\dfrac{\mathrm{P}(X=1) + \mathrm{P}(X=2)}{\mathrm{P}(X \geq 1)}$ ***(A1)***

$\dfrac{0.597540\ldots}{0.785361\ldots}$ OR $\dfrac{0.812178\ldots - 0.214638\ldots}{0.785361\ldots}$ OR $\dfrac{0.338903\ldots + 0.258636\ldots}{0.785361\ldots}$ ***(A1)***

$= 0.760847\ldots$

$\mathrm{P}(X \leq 2 \mid X \geq 1) = 0.761$ ***A1***

[4 marks]

Total [7 marks]

Question 12

(a) use of GDC to give *(M1)*

$r = 0.883529...$

$r = 0.884$ **A1**

> **Note:** Award the ***(M1)*** for any correct value of r, a, b or $r^2 = 0.780624...$ seen in part (a) or part (b).

[2 marks]

(b) $a = 1.36609...$, $b = 64.5171...$

$a = 1.37$, $b = 64.5$ **A1**

[1 mark]

(c) attempt to find their difference *(M1)*

$5 \times 1.36609...$ OR $1.36609...(h+5) + 64.5171... - (1.36609...h + 64.5171...)$

$6.83045...$

$= 6.83$ $(6.85 \text{ from } 1.37)$

the student could have expected her score to increase by 7 marks. **A1**

> **Note:** Accept an increase of 6, 6.83 or 6.85.

[2 marks]

Total [5 marks]

Question13

(a) $P(C < 61)$ ***(M1)***

$= 0.365112...$

$= 0.365$ ***A1***

[2 marks]

(b) recognition of binomial eg $X \sim B(12, 0.365...)$ ***(M1)***

$P(X = 5) = 0.213666...$

$= 0.214$ ***A1***

[2 marks]

(c) (i) Let CM represent 'chocolate muffin' and BM represent 'banana muffin'

$P(B < 61) = 0.0197555...$ ***(A1)***

EITHER

$P(CM) \times P(C < 61 | CM) + P(BM) \times P(B < 61 | BM)$ (or equivalent in words) ***(M1)***

OR

tree diagram showing two ways to have a muffin weigh < 61 ***(M1)***

THEN

$(0.6 \times 0.365...) + (0.4 \times 0.0197...)$ ***(A1)***

$= 0.226969...$

$= 0.227$ ***A1***

(ii) recognizing conditional probability ***(M1)***

Note: Recognition must be shown in context either in words or symbols, not just $P(A \mid B)$.

$\dfrac{0.6 \times 0.365112...}{0.226969...}$ ***(A1)***

$= 0.965183...$

$= 0.965$ ***A1***

[7 marks]

(d) **METHOD 1**

$\mathrm{P}(CM)\times\mathrm{P}(C<61\,|\,CM)+\mathrm{P}(BM)\times\mathrm{P}(B<61\,|\,BM)=0.157$ ***(M1)***

$\left(0.6\times\mathrm{P}(C<61)\right)+(0.4\times 0.0197555...)=0.157$

$\mathrm{P}(C<61)=0.248496...$ ***(A1)***

attempt to solve for σ using GDC ***(M1)***

Note: Award ***(M1)*** for a graph or table of values to show their $\mathrm{P}\left(C<61\right)$ with a variable standard deviation.

$\sigma=1.47225...$

$\sigma=1.47$ (g) ***A2***

METHOD 2

$\mathrm{P}(CM)\times\mathrm{P}(C<61\,|\,CM)+\mathrm{P}(BM)\times\mathrm{P}(B<61\,|\,BM)=0.157$ ***(M1)***

$\left(0.6\times\mathrm{P}(C<61)\right)+(0.4\times 0.0197555...)=0.157$

$\mathrm{P}(C<61)=0.248496...$ ***(A1)***

use of inverse normal to find z score of their $\mathrm{P}(C<61)$ ***(M1)***

$z=-0.679229...$

correct substitution ***(A1)***

$$\frac{61-62}{\sigma}=-0.679229...$$

$\sigma=1.47225...$

$\sigma=1.47$ (g) ***A1***

[5 marks]

Total [16 marks]

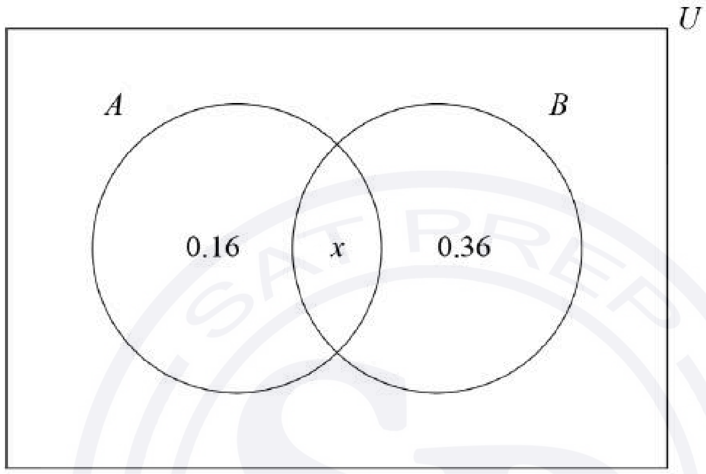
Question 14

(a) **METHOD 1**

EITHER

one of $\mathrm{P}(A) = x + 0.16$ OR $\mathrm{P}(B) = x + 0.36$ ***A1***

OR



A1

THEN

attempt to equate their $\mathrm{P}(A \cap B)$ with their expression for $\mathrm{P}(A) \times \mathrm{P}(B)$ ***M1***

$\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B) \Rightarrow x = (x + 0.16) \times (x + 0.36)$ ***A1***

$x = 0.24$ ***A1***

METHOD 2

attempt to form at least one equation in $\mathrm{P}(A)$ and $\mathrm{P}(B)$ using independence ***M1***

$\left(\mathrm{P}(A \cap B') = \mathrm{P}(A) \times \mathrm{P}(B') \Rightarrow\right)$ $\mathrm{P}(A) \times \left(1 - \mathrm{P}(B)\right) = 0.16$ OR

$\left(\mathrm{P}(A' \cap B) = \mathrm{P}(A') \times \mathrm{P}(B) \Rightarrow\right)$ $\left(1 - \mathrm{P}(A)\right) \times \mathrm{P}(B) = 0.36$

$\mathrm{P}(A) = 0.4$ AND $\mathrm{P}(B) = 0.6$ ***A1***

$\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B) = 0.4 \times 0.6$ ***(A1)***

$x = 0.24$ ***A1***

[4 marks]

(b) **METHOD 1**

recognising $\mathrm{P}(A' \mid B') = \mathrm{P}(A')$ ***(M1)***

$= 1 - 0.16 - 0.24$

$= 0.6$ ***A1***

METHOD 2

$\mathrm{P}(B) = 0.36 + 0.24 (= 0.6)$

$\mathrm{P}(A' \mid B') = \dfrac{\mathrm{P}(A' \cap B')}{\mathrm{P}(B')} \quad \left(= \dfrac{0.24}{0.4}\right)$ ***(A1)***

$= 0.6$ ***A1***

[2 marks]

Total [6 marks]

Question 15

(a) $0.41 + k - 0.28 + 0.46 + 0.29 - 2k^2 = 1$ OR $k - 2k^2 + 0.01 = 0.13$ (or equivalent) ***A1***

$2k^2 - k + 0.12 = 0$ ***AG***

[1 mark]

(b) one of 0.2 OR 0.3 ***(M1)***

$k = 0.3$ ***A1***

reasoning to reject $k = 0.2$ eg $\mathrm{P}(1) = k - 0.28 \geq 0$ therefore $k \neq 0.2$ ***R1***

[3 marks]

(c) attempting to use the expected value formula ***(M1)***

$\mathrm{E}(X) = 0 \times 0.41 + 1 \times (0.3 - 0.28) + 2 \times 0.46 + 3 \times (0.29 - 2 \times 0.3^2)$

$= 1.27$ ***A1***

[2 marks]

Total [6 marks]

Question 16

(a) **EITHER**

recognising that half the total frequency is 10 (may be seen in an ordered list or indicated on the frequency table) ***(A1)***

OR

$5+1+4=3+x$ ***(A1)***

OR

$\sum f = 20$ ***(A1)***

THEN

$x=7$ ***A1***

[2 marks]

(b) **METHOD 1**

$1.58429\ldots$

1.58 ***A2***

METHOD 2

EITHER

$$\sigma^2 = \frac{5\times(2-4.3)^2+1\times(3-4.3)^2+4\times(4-4.3)^2+3\times(5-4.3)^2+7\times(6-4.3)^2}{20} \quad (=2.51)$$ ***(A1)***

OR

$$\sigma^2 = \frac{5\times2^2+1\times3^2+4\times4^2+3\times5^2+7\times6^2}{20}-4.3^2 \quad (=2.51)$$ ***(A1)***

THEN

$\sigma = \sqrt{2.51} = 1.58429\ldots$

$= 1.58$ ***A1***

[2 marks]

Total [4 marks]

Question 17

(a) **METHOD 1**

$T \sim \mathrm{N}(35, \sigma^2)$

$\mathrm{P}(T > 40) = 0.25$ or $\mathrm{P}(T < 40) = 0.75$ ***(M1)***

attempt to solve for σ graphically or numerically using the GDC ***(M1)***

graph of normal curve $T \sim \mathrm{N}(35, \sigma^2)$ for $\mathrm{P}(T > 40)$ and $y = 0.25$ OR $\mathrm{P}(T < 40)$ and $y = 0.75$ OR table of values for $\mathrm{P}(T < 40)$ or $\mathrm{P}(T > 40)$

$\sigma = 7.413011...$

$\sigma = 7.41$ (min) ***A2***

METHOD 2

$T \sim \mathrm{N}(35, \sigma^2)$

$\mathrm{P}(T > 40) = 0.25$ or $\mathrm{P}(T < 40) = 0.75$ ***(M1)***

$z = 0.674489...$ ***(A1)***

valid equation using their z-score (clearly identified as z-score and not a probability) ***(M1)***

$\frac{40-35}{\sigma} = 0.674489...$ OR $5 = 0.674489...\sigma$

$7.413011\ldots$

$\sigma = 7.41$ (min) ***A1***

[4 marks]

(b) $\mathrm{P}(T > 45)$ ***(M1)***

$= 0.0886718...$

$= 0.0887$ ***A1***

[2 marks]

(c) recognizing binomial probability *(M1)*

$L \sim \mathrm{B}(5, 0.0886718...)$

$\mathrm{P}(L \geq 1) = 1 - \mathrm{P}(L = 0)$ OR

$\mathrm{P}(L \geq 1) = \mathrm{P}(L = 1) + \mathrm{P}(L = 2) + \mathrm{P}(L = 3) + \mathrm{P}(L = 4) + \mathrm{P}(L = 5)$ *(M1)*

$0.371400\ldots$

$\mathrm{P}(L \geq 1) = 0.371$ ***A1***

[3 marks]

(d) recognizing conditional probability in context *(M1)*

finding $\{L < 3\} \cap \{L \geq 1\} = \{L = 1, L = 2\}$ (may be seen in conditional probability) *(A1)*

$\mathrm{P}(L = 1) + \mathrm{P}(L = 2) = 0.36532...$ (may be seen in conditional probability) *(A1)*

$\mathrm{P}(L < 3 \mid L \geq 1) = \dfrac{0.36532...}{0.37140...}$ *(A1)*

$0.983636\ldots$

0.984 ***A1***

[5 marks]

(e) **METHOD 1**

recognizing that Suzi can be late no more than once (in the remaining six days) *(M1)*

$\mathrm{X} \sim \mathrm{B}(6, 0.0886718...)$, where X is the number of days late *(A1)*

$\mathrm{P}(X \leq 1) = \mathrm{P}(X = 0) + \mathrm{P}(X = 1)$ *(M1)*

$= 0.907294...$

$\mathrm{P}(\text{Suzi gets a bonus}) = 0.907$ ***A1***

METHOD 2

recognizing that Suzi must be on time at least five times (of the remaining six days) *(M1)*

$\mathrm{X} \sim \mathrm{B}(6, 0.911328...)$, where X is the number of days on time *(A1)*

$\mathrm{P}(X \geq 5) = 1 - \mathrm{P}(X \leq 4)$ OR $1 - 0.0927052...$ OR $\mathrm{P}(X = 5) + (X = 6)$ OR $0.334434... + 0.572860...$ *(M1)*

$= 0.907294...$

$\mathrm{P}(\text{Suzi gets a bonus}) = 0.907$ ***A1***

[4 marks]

Total [18 marks]

Question 18

(a) 0.28 (s) **_A1_**

[1 mark]

(b) $\text{IQR} = 0.35 - 0.27\ (= 0.08)$ (s) **_(A1)_**

substituting **their** IQR into correct expression for upper fence **_(A1)_**

$0.35 + 1.5 \times 0.08\ (= 0.47)$ (s)

$0.46 < 0.47$ **_R1_**

so 0.46 (s) is not an outlier **_AG_**

[3 marks]

Question 19

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B) = 0.68$

substitution of $\mathrm{P}(A) \cdot \mathrm{P}(B)$ for $\mathrm{P}(A \cap B)$ in $\mathrm{P}(A \cup B)$ **_(M1)_**

$\mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A)\mathrm{P}(B)\ (= 0.68)$

substitution of $3\mathrm{P}(B)$ for $\mathrm{P}(A)$ **_(M1)_**

$3\mathrm{P}(B) + \mathrm{P}(B) - 3\mathrm{P}(B)\mathrm{P}(B) = 0.68$ (or equivalent) **_(A1)_**

Note: The first two ***M*** marks are independent of each other.

attempts to solve their quadratic equation **_(M1)_**

$\mathrm{P}(B) = 0.2, 1.133\ldots \left(\frac{1}{5}, \frac{17}{15}\right)$

$\mathrm{P}(B) = 0.2 \left(= \frac{1}{5}\right)$ **_A2_**

Note: Award ***A1*** if both answers are given as final answers for $\mathrm{P}(B)$.

[6 marks]

Question 20

(a) recognising to find $\mathrm{P}(T > 40)$ **(M1)**

$\mathrm{P}(T > 40) = 0.574136...$

$\mathrm{P}(T > 40) = 0.574$ **A1**

[2 marks]

(b) attempt to multiply four independent probabilities using their $\mathrm{P}(T > 40)$ and $\mathrm{P}(T < 40)$ **(M1)**

$(1-p)^3 \cdot p$ OR $(1-0.574136...)^3 \cdot 0.574136...$ OR $(0.425863...)^3 \cdot 0.574136...$ **(A1)**

0.0443430...

0.0443 , 0.0444 from 3 sf values **A1**

[3 marks]

(c) (i) recognizing conditional probability **(M1)**

$\mathrm{P}(T < 55 \mid T > 40)$

$\dfrac{\mathrm{P}(40 < T < 55)}{\mathrm{P}(T > 40)}$ **(A1)**

$\dfrac{0.461944...}{0.574136...}$ **(A1)**

$\mathrm{P}(T < 55 \mid T > 40) = 0.804590...$

$= 0.805$ **A1**

(ii) recognizing binomial probability **(M1)**

$X \sim \mathrm{B}(n, p)$

$n = 10$ and $p = 0.804589...$ **(A1)**

$0.0242111..., 0.0240188...$ using $p = 0.805$

$\mathrm{P}(X = 5) = 0.0242$ **A1**

[7 marks]

(d) Let $\mathrm{P}(T < a) = x$

recognition that probabilities sum to 1 (seen anywhere) ***(M1)***

EITHER

expressing the three regions in one variable ***(M1)***

$x + 0.904 + 2x$ OR $\mathrm{P}(T < a) + 0.904 + 2\mathrm{P}(T < a)$ OR $\frac{1}{2}\mathrm{P}(T > b) + 0.904 + \mathrm{P}(T > b)$

OR x and $2x$ correctly indicated on labelled bell diagram

$\mathrm{P}(T < a) + 0.904 + 2\mathrm{P}(T < a) = 1$ OR $\frac{1}{2}\mathrm{P}(T > b) + 0.904 + \mathrm{P}(T > b) = 1$ (or equivalent) ***(A1)***

OR

expressing either $\mathrm{P}(T < a)$ or $\mathrm{P}(T > b)$ only in terms of $\mathrm{P}(a \le T \le b)$ ***(M1)***

$(\mathrm{P}(T < a) =)\frac{1}{3}(1 - \mathrm{P}(a \le T \le b))$ OR $(\mathrm{P}(T > b) =)\frac{2}{3} \cdot (1 - \mathrm{P}(a \le T \le b))$

$x = \frac{1}{3}(1 - 0.904)(= 0.032)$ OR $\mathrm{P}(T > b) = \frac{2}{3}(1 - 0.904)(= 0.064)$ ***(A1)***

THEN

$\mathrm{P}(T < a) = 0.032$

$a = 22.18167...$

$a = 22.2$ accept 22.1 ***A1***

[4 marks]

Total [16 marks]

Question 21

(a) 1.01206…, 2.45230…

$a = 1.01$, $b = 2.45$ $(1.01x + 2.45)$ ***A1A1***

[2 marks]

(b) 0.981464…

$r = 0.981$ ***A1***

Note: A common error is to enter the data incorrectly into the GDC, and obtain the answers $a = 1.01700...$, $b = 2.09814...$ and $r = 0.980888...$ Some candidates may write the 3 sf answers, ie. $a = 1.02$, $b = 2.10$ and $r = 0.981$ or 2 sf answers, ie. $a = 1.0$, $b = 2.1$ and $r = 0.98$. In these cases award ***A0A0*** for part (a) and ***A0*** for part (b). Even though some values round to an accepted answer, they come from incorrect working.

[1 mark]

(c) correct substitution of 78 into **their** regression equation ***(M1)***

81.3930… 81.23 from 3 sf answer

81 ***A1***

[2 marks]

Total [5 marks]

Question 22

(a) (i) $p = 12$ **A1**

(ii) $q = 100$ **A1**

[2 marks]

(b) $\mathrm{P(Adult)} = \dfrac{100}{160}$ $(= 0.625)$ (seen anywhere) ***(A1)***

> **Note:** Award ***A1*** for $(X \sim)\,\mathrm{B}(3, 0.625)$ or $\left(\dfrac{100}{160}\right)^3$ but no further marks.

recognition that choice of adults is without replacement (may be seen in tree diagram) ***(M1)***

$\dfrac{100}{160} \times \dfrac{99}{159} \times \dfrac{98}{158}$ ***(A1)***

$0.241372\ldots$

0.241 ***A1***

[4 marks]

(c) (i) $\dfrac{x}{48 + x}\left(= \dfrac{1}{3}\right)$ OR $\dfrac{\frac{x}{160}}{\frac{48 + x}{160}}$ ***(A1)(A1)***

> **Note:** Award ***A1*** for correct numerator and ***A1*** for correct denominator.

$x = 24$ ***A1***

(ii) $\mathrm{P}(A \cap M) = \dfrac{24}{160}\left(= \dfrac{3}{20}\right)$ ***A1***

[4 marks]

(d) **METHOD 1 (using** $\mathrm{P}(A \mid B) = \mathrm{P}(A)$ **)**

recognition that A and M are independent if $\mathrm{P}(A \mid M) = \mathrm{P}(A)$ ***(M1)***

$\frac{1}{3} \neq \frac{100}{160}$ ***R1***

so they are not independent ***A1***

METHOD 2 (using $\mathrm{P}(A) \cdot \mathrm{P}(B) = \mathrm{P}(A \cap B)$**)**

attempt to find the product $\mathrm{P}(A) \times \mathrm{P}(M)$ OR $\mathrm{P}(A) \times \mathrm{P}(D)$ ***(M1)***

$\frac{100}{160} \times \frac{72}{160} \neq \frac{24}{160}$ $\left(\frac{9}{32} \neq \frac{3}{20}\right)$ OR $\frac{100}{160} \times \frac{88}{160} \neq \frac{76}{160}$ $\left(\frac{11}{32} \neq \frac{19}{40}\right)$ ***R1***

so they are not independent ***A1***

Note: Do not award ***R0A1***.

[3 marks]

(e) P(dark chocolate) $= \frac{88}{160}$ $(= 0.55)$ (maybe seen in part (d)) ***(A1)***

recognize that the variable has a Binomial distribution ***(M1)***

$X \sim \mathrm{B}(10, 0.55)$

recognition that $\mathrm{P}(X \geq 5)$ or $1 - \mathrm{P}(X \leq 4)$ is required ***(M1)***

Note: These two M marks are independent of each other.

0.738437…

0.738 ***A1***

[4 marks]

Total [17 marks]

Question 23

METHOD 1

$Q_1 = 31.86$ OR $Q_3 = 32.14$ ***(A1)***

recognition that the area under the normal curve below Q_1 or above Q_3 is 0.25 OR the area between Q_1 and Q_3 is 0.5 (seen anywhere including on a diagram) ***(M1)***

EITHER

equating an appropriate correct normal CDF function to its correct probability (0.25 or 0.5 or 0.75) ***(A2)***

OR

$z = -0.674489...$ OR $z = 0.674489...$ (seen anywhere) ***(A1)***

$-0.674489... = \frac{31.86 - 32}{\sigma}$ OR $0.674489... = \frac{32.14 - 32}{\sigma}$ ***(A1)***

THEN

$0.207564...$

$\sigma = 0.208$ (mm) ***A1***

METHOD 2

recognition that the area under the normal curve below Q_1 or above Q_3 is 0.25 OR the area between Q_1 and Q_3 is 0.5 (seen anywhere including on a diagram) ***(M1)***

$z = -0.674489...$ OR $z = 0.674489...$ ***(A1)***

$(Q_1 =)32 - 0.674489...\sigma$ OR $(Q_3 =)32 + 0.674489...\sigma$ ***(A1)***

$(Q_3 - Q_1 =)2 \times 0.674489...\sigma$

$2 \times 0.674489...\sigma = 0.28$ ***(A1)***

$0.207564...$

$\sigma = 0.208$ (mm) ***A1***

Total [5 marks]

Question 24

(a) recognising to find $y(25)$ **(M1)**

$y(25) = -0.6 \times 25^2 + 23 \times 25 + 110$

$= 310$ (children) **A1**

[2 marks]

(b) recognizing x on y is required **(M1)**

$0.0935114\ldots$ and $7.43053\ldots$ **(A1)**

$x = 0.0935y + 7.43$ **A1**

[3 marks]

(c) attempt to substitute their answer to part (a) into their regression equation for either x or y **(M1)**

$x = 0.0935114\ldots \times 310 + 7.43053\ldots (= 36.4190\ldots)$

36 (accept 37 or 36.4) **A1**

> **Note:** Award ***(M1)A1FT*** for $x = 37$ found from using $y = 9.39x - 41.5$.
>
> Award ***(M1)A0FT*** for a correct ***FT*** answer that lies outside $[15, 46]$.

[2 marks]

Total [7 marks]

Question 25

(a) recognizing probabilities sum to 1 ***(M1)***

$0.213 + \mathrm{P}(82.4 < X < 87.3) + 0.409 = 1$

$\mathrm{P}(82.4 < X < 87.3) = 0.378$ ***A1***

[2 marks]

(b) **METHOD 1**

recognizing the need to use inverse normal with 0.213, $(1 - 0.409)$ or 0.409 ***(M1)***

$m + \mathrm{invNorm}(0.213)s = 82.4$, $m + \mathrm{invNorm}(1 - 0.409)s = 87.3$ (or equivalent) ***(A1)(A1)***

attempt to solve their equations in two variables using the GDC (that involve either z-values or 'invNorm' rather than probabilities) ***(M1)***

$\mu = 86.2011...,\ \sigma = 4.77502...$

$\mu = 86.2,\ \sigma = 4.78$ ***A1***

METHOD 2

use of inverse normal to find at least one z-score for $P(Z < z) = 0.213$, or $\mathrm{P}(Z < z) = 1 - 0.409$ ***(M1)***

$z_1 = -0.796055...$ OR $z_2 = 0.230118...$

$\dfrac{82.4 - \mu}{\sigma} = -0.796055...,\quad \dfrac{87.3 - \mu}{\sigma} = 0.230118...$ (or equivalent) ***(A1)(A1)***

attempt to solve their equations (that involve z-values rather than probabilities) ***(M1)***

$m = 86.2011...,\ s = 4.77502...$

$m = 86.2,\ s = 4.78$ ***A1***

[5 marks]

(c) (i) recognition of Binomial distribution ***(M1)***

$X \sim \mathrm{B}(100, 0.409)$

$\mathrm{P}(X = 32) = 0.0157931...$

$= 0.0158$ ***A1***

(ii) $\mathrm{P}(X < 44) = 0.702975...$(seen anywhere) ***(A1)***

recognition of conditional probability ***(M1)***

$\left(\mathrm{P}(X=32 \mid X<44)=\right) \dfrac{\mathrm{P}(X=32)}{\mathrm{P}(X<44)}$ OR $\dfrac{\mathrm{P}(X=32)}{\mathrm{P}(X\leq 43)}$ $\left(=\dfrac{0.0157931\ldots}{0.702975\ldots}\right)$ ***(A1)***

$= 0.0224661\ldots$

$\mathrm{P}(X=32 \mid X<44)=0.0225$ ***A1***

[6 marks]

(d) $\mathrm{Q}_1 = 90.54$ OR $\mathrm{Q}_3 = 95.06$ (may be seen on a labelled diagram with areas indicated) ***(A1)***

$\mathrm{P}(90.54 < F < 95.06) = 0.5$ OR $\mathrm{P}(F < 90.54) = 0.25$ OR $\mathrm{P}(F < 95.06) = 0.75$ (or equivalent)

EITHER

attempt to find d using graph or table ***(M1)***

OR

$1-2\mathrm{P}\left(Z<-\dfrac{2.26}{d}\right)=0.5$ OR $\mathrm{P}(Z<-\dfrac{2.26}{d})=0.25$ OR $\mathrm{P}(Z<\dfrac{2.26}{d})=0.75$

OR $\mathrm{P}(-\dfrac{2.26}{d}<Z<\dfrac{2.26}{d})=0.5$ (or equivalent) ***(M1)***

$-\dfrac{2.26}{d} = -0.674489\ldots$ OR $\dfrac{2.26}{d} = 0.674489\ldots$

THEN

$3.35068\ldots$

$d = 3.35$ ***A1***

[3 marks]

Total [16 marks]

Question 26

$\mathrm{E}(X) = k + 2k^2 + 3a + 4k^3 = 2.6$ ***(A1)***

$k + k^2 + a + k^3 = 1$ ***(A1)***

EITHER (finding intersections of functions)

attempt to make a the subject in both of their equations ***(M1)***

$a = 1 - k - k^2 - k^3$ and $a = \frac{1}{3}\left(2.6 - k - 2k^2 - 4k^3\right)$

use of graph or table to attempt to find intersection ***(M1)***

OR (solving algebraically)

attempt to solve their equations algebraically to find a cubic in k ***(M1)***

$k^3 - k^2 - 2k + 0.4 = 0$ OR $3\left(1 - k - k^2 - k^3\right) = 2.6 - k - 2k^2 - 4k^3$ (or equivalent)

attempt to solve their cubic in k ***(M1)***

THEN

$a = 0.773073...$ OR $k = 0.185928...$

(Other solutions to the cubic are: $k = 1.92921...,\quad k = -1.11514...$)

$a = 0.773$ ***A1***

Total [5 marks]

Question 27

(a) (i) $\dfrac{10}{n}$ ***A1***

(ii) multiplying probabilities for *GG* ***(M1)***

$$\mathrm{P}(GG) = \frac{10}{n} \times \frac{9}{n-1}$$

$$\mathrm{P}(GG) = \frac{90}{n^2 - n} \left(\text{accept } \frac{90}{n(n-1)}\right)$$ ***A1***

[3 marks]

(b) $\mathrm{P}(\text{First red}) = \dfrac{15}{25}$ and $\mathrm{P}(\text{Second red}) = \dfrac{14}{24}$ (seen anywhere) ***(A1)***

$\mathrm{P}(RR) = \dfrac{15}{25} \times \dfrac{14}{24}$ (or equivalent) ***A1***

$= 0.35$ ***AG***

[2 marks]

(c) $\dfrac{15}{25} \times \dfrac{14}{24} \times \dfrac{13}{23}$ OR $0.35 \times \dfrac{13}{23}$ ***(A1)***

$0.197826\ldots$

$\mathrm{P}(\text{three red}) = 0.198$ (exact answer is $\dfrac{91}{460}$) ***A1***

[2 marks]

(d) $\text{P(at least one green)} = 1 - \text{P(three red)}$ OR

$\text{P(at least one G)=P(one G)+P(two G)+P(three G)}$ ***(M1)***

$1-\left(\frac{15}{25}\times\frac{14}{24}\times\frac{13}{23}\right)$ OR $3\left(\frac{10}{25}\times\frac{15}{24}\times\frac{14}{23}\right)+3\left(\frac{10}{25}\times\frac{9}{24}\times\frac{15}{23}\right)+\left(\frac{10}{25}\times\frac{9}{24}\times\frac{8}{23}\right)$

$0.802173...$

$\text{P(at least one green)} = 0.802$ (exact answer is $\frac{369}{460}$) ***A1***

[2 marks]

(e) $\text{P(first green on third draw)} = \frac{15}{25}\times\frac{14}{24}\times\frac{10}{23}\times\frac{22}{22}\left(=\frac{7}{46}=0.152173...\right)$ ***(A1)***

$\text{P}\left(\text{first green on fourth draw}\right) = \frac{15}{25}\times\frac{14}{24}\times\frac{13}{23}\times\frac{10}{22}\left(=\frac{91}{1012}=0.0899209...\right)$ ***(A1)***

Note: The first two ***(A1)*** are independent.

attempt to substitute their probabilities into expected value formula ***(M1)***

expected points per game $=10\times\frac{7}{46}+50\times\frac{91}{1012}\left(=\frac{3045}{506}=6.01778...\right)$ ***(A1)***

setting up inequality or equation in k ***(M1)***

$\frac{3045}{506}k > 100$

$k > 16.6174...\left(=\frac{10120}{609}\right)$

Millie must play at least 17 times. ***A1***

[6 marks]

Total [15 marks]

Question 28

(a) $7.8 = \dfrac{2\pi}{\text{period}}$ ***(M1)***

$\dfrac{2\pi}{7.8} = 0.805536...$

$\text{period} = 0.806\left(= \dfrac{10\pi}{39}\right)$ ***A1***

[2 marks]

(b) **METHOD 1**

(i) $\text{amplitude} = \dfrac{\max - \min}{2}$ ***(M1)***

$\dfrac{1.8-1}{2}$

$a = -0.4$ ***A1***

(ii) $b = 1.4$ ***A1***

METHOD 2

attempt to form two simultaneous equations in a and b ***(M1)***

$H(0) = 1 \Rightarrow a + b = 1$, $H\left(\dfrac{\pi}{7.8}\right) = 1.8 \Rightarrow -a + b = 1.8$

$a = -0.4, b = 1.4$ ***A1A1***

[3 marks]

(c) **EITHER**

$\dfrac{5}{\text{period}} = 6.207... < 6\tfrac{1}{2}$ ***(A1)***

OR

consideration of number of maximums on graph in first 5 seconds ***(A1)***

OR

maximums when $t = 0.403,\ 1.21,\ 2.01,\ 2.82,\ 3.62,\ 4.43$ ***(A1)***

THEN

6 times ***A1***

[2 marks]

(d) recognizing that $H(t) = 1.5$ ***(M1)***

$-0.4\cos(7.8t) + 1.4 = 1.5$

$0.233779...$

$t = 0.234$ (seconds) ***A1***

[2 marks]

(e) finding second time height is 1.5 metres ***(M1)***

$t = 0.571757...$

in each period, height is greater than 1.5 metres for 0.337978… seconds ***(A1)***

Note: Award ***(M1)(A1)*** for total time $2.02787...$ seen.

multiplying their value by 6 and dividing by 5 ***(M1)***

$\dfrac{0.337978...\times 6}{5}$ OR $\dfrac{2.02787...}{5}$

$= 0.405574...$

$\text{P(height is greater than 1.5 m)} = 0.406$ ***A1***

[4 marks]

Total [13 marks]

Question 29

(a) evidence of attempting to find correct area under normal curve ***(M1)***

$\mathrm{P}(W > 210)$ OR sketch

$\mathrm{P}(W > 210) = 0.115069...$

$\mathrm{P}(W > 210) = 0.115$ ***A1***

[2 marks]

(b) recognizing $\mathrm{P}(W < w) = 1 - \mathrm{P}(w < W < 210) - \mathrm{P}(W > 210)$ ***(M1)***

$\mathrm{P}(W < w) = 1 - 0.8 - 0.115069...$

$\mathrm{P}(W < w) = 0.084930...$

$\mathrm{P}(W < w) = 0.0849$ ***A1***

[2 marks]

(c) evidence of attempting to use inverse normal function ***(M1)***

$w = 197.136...$

$w = 197$ (grams) ***A1***

[2 marks]

(d) recognition of binomial distribution ***(M1)***

$X \sim \mathrm{B}(10, 0.0849302...)$

$\mathrm{P}(X = 1) = 0.382076...$

$\mathrm{P}(X = 1) = 0.382$ ***A1***

[2 marks]

Total [8 marks]

Question 30

(a) $a = 1.93258...$, $b = 7.21662...$

$a = 1.93$, $b = 7.22$ ***A1A1***

[2 marks]

(b) $r = 0.991087...$

$r = 0.991$ ***A1***

[1 mark]

(c) attempt to substitute $d = 20$ into their equation ***(M1)***

$\text{height} = 45.8683...$

$\text{height} = 45.9$ (cm) ***A1***

[2 marks]

Total [5 marks]

Question 31

(a) recognizing probabilities sum to 1 **(M1)**

$0.288 + \mathrm{P}(94.6 < X < 98.1) + 0.434 = 1$

$\mathrm{P}(94.6 < X < 98.1) = 0.278$ **A1**

[2 marks]

(b) **METHOD 1**

recognizing the need to use inverse normal with 0.288, $(1-0.434)$ or 0.434 **(M1)**

$\mu + \text{invNorm}(0.288)\sigma = 94.6, \mu + \text{invNorm}(1-0.434)\sigma = 98.1$ (or equivalent) **(A1)(A1)**

attempt to solve their equations in two variables using the GDC (that involve either z-values or 'invNorm' rather than probabilities) **(M1)**

$\mu = 97.2981...,\ \sigma = 4.82468...$

$\mu = 97.3,\ \sigma = 4.82$ **A1**

METHOD 2

use of inverse normal to find at least one z-score for $\mathrm{P}(Z < z) = 0.288$ or $\mathrm{P}(Z < z) = 1 - 0.434$ **(M1)**

$z_1 = -0.559236...$ OR $z_2 = 0.166199...$

$\frac{94.6-\mu}{\sigma} = -0.559236...,\ \frac{98.1-\mu}{\sigma} = 0.166199...$ (or equivalent) **(A1)(A1)**

attempt to solve their equations (that involve z-values rather than probabilities) **(M1)**

$\mu = 97.2981...,\ \sigma = 4.82468...$

$\mu = 97.3,\ \sigma = 4.82$ **A1**

[5 marks]

(c) (i) recognition of Binomial distribution *(M1)*

$X \sim \mathrm{B}(100, 0.434)$

$\mathrm{P}(X = 34) = 0.0133198...$

$= 0.0133$ ***A1***

(ii) $\mathrm{P}(X < 49) = 0.848218...$ (seen anywhere) ***(A1)***

recognition of conditional probability ***(M1)***

(d) $\mathrm{Q}_1 = 96.19$ OR $\mathrm{Q}_3 = 101.01$ (may be seen on a labelled diagram with areas indicated) ***(A1)***

$\mathrm{P}(96.19 < F < 101.01) = 0.5$ OR $\mathrm{P}(F < 96.19) = 0.25$ OR $\mathrm{P}(F < 101.01) = 0.75$ (or equivalent)

EITHER

attempt to find d using graph or table ***(M1)***

OR

$1 - 2\mathrm{P}\left(Z < -\frac{2.41}{d}\right) = 0.5$ OR $\mathrm{P}(Z < -\frac{2.41}{d}) = 0.25$ OR $\mathrm{P}(Z < \frac{2.41}{d}) = 0.75$

OR $\mathrm{P}(-\frac{2.41}{d} < Z < \frac{2.41}{d}) = 0.5$ (or equivalent) ***(M1)***

$-\frac{2.41}{d} = -0.674489...$ OR $\frac{2.41}{d} = 0.674489...$

THEN

$3.57307...$

$d = 3.57$ ***A1***

[3 marks]

Total [16 marks]

Question 32

$\mathrm{E}(X) = k + 2k^2 + 3a + 4k^3 = 2.3$ ***(A1)***

$k + k^2 + a + k^3 = 1$ ***(A1)***

EITHER (finding intersections of functions)

attempt to make a the subject in both of their equations ***(M1)***

$a = 1 - k - k^2 - k^3$ and $a = \frac{1}{3}\left(2.3 - k - 2k^2 - 4k^3\right)$

use of graph or table to attempt to find intersection ***(M1)***

OR (solving algebraically)

attempt to solve their equations algebraically to find a cubic in k ***(M1)***

$k^3 - k^2 - 2k + 0.7 = 0$ OR $3\left(1 - k - k^2 - k^3\right) = 2.3 - k - 2k^2 - 4k^3$ (or equivalent)

attempt to solve their cubic in k ***(M1)***

THEN

$a = 0.552839...$ OR $k = 0.315870...$ (other solutions to cubic are $k = -1.18538..., 1.86951...$)

$a = 0.553$ ***A1***

Total [5 marks]

Question 33

(a) recognition to add μ and σ **(M1)**

49.8 (cm) ***A1***

[2 marks]

(b) $\mathrm{P}(L > 48)$ ***(M1)***

Note: Award ***(M1)*** for a clearly labelled diagram.

$= 0.283854\ldots$

$= 0.284$ ***A1***

[2 marks]

(c) $\mathrm{P}(L > x) = 0.99$ OR $\mathrm{P}(L < x) = 0.01$ ***(M1)***

Note: Award ***(M1)*** for a clearly labelled diagram or the use of inverse normal.

$x = 35.8293\ldots$

$x = 35.8$ (cm) ***A1***

[2 marks]

(d) (i) $\mathrm{P}(40 < L < 56) = 0.902149...$ (may be seen in part (ii)) *(A1)*

attempts to find $100 \times \mathrm{P}(40 < L < 56)$ with their probability *(M1)*

$= 90.2149...$

$= 90.2$ **A1**

Note: Accept 90 or 91.

(ii) recognizes binomial distribution *(M1)*

$X \sim \mathrm{B}(100, 0.902149...)$

$\mathrm{P}(X = 95) = 0.038105...$

$= 0.0381$ **A1**

[5 marks]

(e) $\mathrm{P}(45.55 \le L < 45.65)$ *(M1)(A1)*

Note: Award *(M1)* for any reasonable interval centred on 45.6, no wider than $\mathrm{P}(45.5 \le L < 45.7)$. Accept either of $\mathrm{P}(45.55 \le L \le 45.65)$ or $\mathrm{P}(45.55 < L < 45.65)$.

$= 0.009498...$

$= 0.00950$ **A1**

[3 marks]

Total [14 marks]

Question 34

METHOD 1

correct inequality or equation involving $\mathrm{P}(X=0)$ ***(A1)***

$1-\mathrm{P}(X=0)>0.99$ OR $\mathrm{P}(X=0)<0.01$ OR $1-\mathrm{P}(X=0)=0.99$ OR $\mathrm{P}(X=0)=0.01$

attempts to solve their inequality (equality) involving 0.75^n for n ***(M1)***

$1-0.75^n>0.99$ OR $0.75^n<0.01$ OR $0.75^n=0.01$ OR $1-0.75^n=0.99$

Note: Valid solving attempts include graphical, use of logarithms, tabular or trial and error.

EITHER

$n>16.0078...$ OR $n=16.0078...$ ***(A2)***

the least value of n is 17 ***A1***

OR

$\mathrm{P}(X=0)=0.010022...$ (>0.01) (corresponding to $n=16$) ***(A1)***

$\mathrm{P}(X=0)=0.0075169...$ (<0.01) ***(A1)***

corresponding to $n=17$ (which is the least value of n) ***A1***

METHOD 2 (TABLE ONLY APPROACH)

attempts to use binomial cdf to calculate a correct value of $\mathrm{P}(X \geq 1)$ for one value of n ***(M1)***

calculates correct values of $\mathrm{P}(X \geq 1)$ for at least one value of n ***(A1)***

$\mathrm{P}(X \geq 1) = 0.989977\ldots\ (< 0.99)$ (corresponding to $n = 16$) ***(A1)***

$\mathrm{P}(X \geq 1) = 0.992483\ldots\ (> 0.99)$ ***(A1)***

corresponding to $n = 17$ (which is the least value of n) ***A1***

[5 marks]

Question 35

3. (a) **EITHER**

$\overline{y} = 2.1875 \times 7 + 0.6875$ ***A1***

OR

$\overline{y} = 15.3125 + 0.6875$ ***A1***

THEN

$\overline{y} = 16$ ***AG***

[1 mark]

(b) attempts to use $16 = \frac{\sum y}{n}$ to form a linear equation in p and q ***(M1)***

$$16 = \frac{9+13+p+q+21}{5} \quad (80 = p+q+43 \Rightarrow p+q=37)$$ ***(A1)***

attempts to solve two linear equations simultaneously for p and q (one of which is $q = p+3$) ***(M1)***

$$16 = \frac{9+13+p+p+3+21}{5} \quad (80 = 2p+46)$$

$p = 17$ and $q = 20$ ***A1***

[4 marks]

Total [5 marks]

Question 36

(a) (i) 4.45 (hours) **A1**

(ii) one correct quartile either $Q_1 = 1.9$ or $Q_3 = 5.7$ ***(A1)***

$\text{IQR} = 5.7 - 1.9$

$= 3.8$ (hours) **A1**

[3 marks]

(b) attempts to find the upper fence value ***(M1)***

upper fence $= 11.4$ ***(A1)***

$11.7 > 11.4$ **R1**

$\Rightarrow 11.7$ is an outlier **AG**

[3 marks]

Total [6 marks]

Question 37

(a) $r = 0.901017\ldots$

$r = 0.901$ **A2**

[2 marks]

(b) Student 11 Test B: should not extrapolate **R1**

[1 mark]

(c) (i) Student 12 Test A: should not use line of y on x to predict x from y (or equivalent) **R1**

(ii) attempt to find the equation of the regression line of x on y ***(M1)***

$(x =)\, 0.987124\ldots y - 3.21970\ldots \ \left((x =)\ 0.987y - 3.22\right)$ **A1**

$(x =)\, 0.987124\ldots(90) - 3.21970\ldots \ (= 85.6214\ldots)$ **A1**

$= 86$ to nearest integer. **AG**

Note: Condone notation for x and y switched if values are correct.

[4 marks]

Total [7 marks]

Question 38

(a) recognition of $X > 13$ OR $Z > 1.5$ (could be seen in a diagram) ***(M1)***

$(\mathrm{P}(X > 13) =)$ $0.0668072...$

$= 0.0668$ ***A1***

[2 marks]

(b) **EITHER**

equating an appropriate correct normal CDF function to 0.1 or 0.9 ***(M1)***

$\mathrm{P}(X > 10 + 2k) = 0.1$ OR $\mathrm{P}(Z < k) = 0.9$ OR $\mathrm{P}(X < 10 - 2k) = 0.1$ OR $\mathrm{P}(Z < -k) = 0.1$

OR

recognising need to use inverse normal with 0.1 or 0.9 ***(M1)***

THEN

$1.28155...$

$k = 1.28$ ***A1***

Total [4 marks]

Question 39

(a) median = 75 *A1*

(upper quartile =) 84 OR (lower quartile =) 68 *(A1)*

Interquartile range = 16 *A1*

[3 marks]

(b)

Note: In this part, their reasoning and answer must be consistent with their values in part (a). In both part (i) and part (ii), award ***R0A1*** for a correct answer with no reasoning.

(i) $80 > 75$ OR Australia has a higher median OR Spain's median is lower ***R1***

Note: Award ***R1*** for correct reasoning based on a comparison of their medians.

in general (on average), rabbits in Australia have longer ears (than rabbits in Spain)

OR

in the top 50% of each distribution, some rabbits in Spain have smaller ears than those in Australia

OR

in the bottom 50% of each distribution, some rabbits in Australia have longer ears than those in Spain ***A1***

(ii) $16 > 11$ OR higher IQR in Spain OR lower IQR in Australia ***R1***

Note: Award ***R1*** for correct reasoning based on a comparison of their IQR's.

(the IQR's suggest that) there is more variation/spread in (the middle 50% of) rabbit ears in Spain (than those in Australia) (or equivalent) ***A1***

Note: Award ***A1*** for any correct answer which uses the IQR's to compare each distribution.

[4 marks]

Total [7 marks]

Question 40

(a) (i) $a = 0.358$ (exact); $b = 30.5$ (exact answer is 30.52) **A1A1**

Note: Award ***A1A0*** if the values of a and b are interchanged or not labeled.

(ii) a represents the (average) rate of increase (change) in population (0.358 millions of people per year). (or equivalent) **R1**

[3 marks]

(b) It is unreliable because 2030 is outside the range of data (extrapolation). **A1**

[1 mark]

(c) (i) attempt to find $B(100)$ ***(M1)***

$55.1633\ldots$

55.2 million OR $55{,}200{,}000$ **A1**

(ii) The annual growth rate of the population is 0.5%. **A1**

Note: Description must include some reference to annual rate.

[3 marks]

(d) $54.6094\ldots$

54.6 million OR $54{,}600{,}000$ **A1**

[1 mark]

(e) consideration of the difference function $C(t)-B(t)$ or $B(t)-C(t)$ or $\left|C(t)-B(t)\right|$ ***(M1)***

evidence of finding the maximum (or minimum) of this function. ***(M1)***

$t = 58.6283\ldots$

2058 (accept 2059) **A1**

[3 marks]

(f) (i) 0.242876…

$B'(75) = 0.243$ **A1**

(ii) 0.184941...

$C'(75) = 0.185$ **A1**

[2 marks]

(g) $B'(75) > C'(75)$ (or equivalent in words) **A1**

the population in Benoit's model is increasing at a faster rate than in Cecilia's model (in 2075) (or equivalent) **R1**

Note: Do not award ***A0R1***.

[2 marks]

Total [15 marks]

Question 41

(a) $22.4358\ldots$

$22.4(\%)$ **A1**

[1 mark]

(b) recognition of binomial ***(M1)***

$X \sim \mathrm{B}(1000, 0.224\ldots)$

attempt to find $\mathrm{P}(X < 200)$ OR $\mathrm{P}(X \le 199)$ ***(M1)***

$0.0284945\ldots$

$\mathrm{P}(X < 200) = 0.0285$ (accept 0.0303 from previous 3sf) ***A1***

[3 marks]

(c) recognition of conditional probability in context ***(M1)***

$\mathrm{P}(\text{age} \ge 65 \mid \text{age} \ge 45)$ OR $\dfrac{\mathrm{P}(\text{age} \ge 45 \cap \text{age} \ge 65)}{\mathrm{P}(\text{age} \ge 45)}$

Note: Recognition must be shown in context either in words or symbols, not just $\mathrm{P}(A \mid B)$

$\dfrac{\mathrm{P}(\text{age} \ge 65)}{\mathrm{P}(\text{age} \ge 45)}$ ***(A1)***

$\dfrac{3.7}{(3.7 + 17.3)}$ OR $\dfrac{0.0395}{0.224}$ ***(A1)***

$0.176190\ldots$

$0.176 \left(= \dfrac{37}{210}\right)$ ***A1***

Note: Condone use of "a" or "X" or any letter for age.

[4 marks]

(d) 15.5, 21.5, 35, 55, 75 **A1**

[1 mark]

(e) (i) 35.7280…

mean = 35.7 **A1**

(ii) variance is square of standard deviation ***(M1)***

$14.3159...^2$

204.945...

variance = 205 **A1**

[3 marks]

(f)

(i) Graph C **A1**

Note: Allow ***FT*** from incorrect values for mean and variance in (e).

(ii) MyLife **A1**

Note: Do not award ***A0A1***.

[2 marks]

Total [14 marks]

Question 42

(a) recognition of sum of probabilities equals 1 *(M1)*

$$\frac{3k}{20}+\frac{5k}{20}+\frac{8k}{20}+\frac{11k}{20}=1$$

$k=0.740740$

$k=0.741\left(=\frac{20}{27}\right)$ **A1**

[2 marks]

(b) correct probabilities: $\frac{3}{27},\frac{5}{27},\frac{8}{27},\frac{11}{27}$ OR $0.111,\ 0.185,\ 0.296,\ 0.407$ ***(A1)***

substitution of their probabilities into formula for expected value ***(M1)***

$3\times\frac{3}{27}+5\times\frac{5}{27}+8\times\frac{8}{27}+11\times\frac{11}{27}$ OR $\frac{219k}{20}$

$=8.11111\ldots$

$\mathrm{E}(X)=8.11\left(=\frac{219}{27}=\frac{73}{9}\right)$ (same 3sf from previous 3sf answer) ***A1***

[3 marks]

Total [5 marks]

Question 43

(a) (i) $a = 0.362$ (exact); $b = 30.5$ (exact) ***A1A1***

Note: Award ***A1A0*** if the values of a and b are interchanged or not labelled.

(ii) a represents the (average) rate of increase (change) in population (0.362 millions of people per year). (or equivalent) ***R1***

[3 marks]

(b) It is unreliable because 2030 is outside the range of data (extrapolation). ***A1***

[1 mark]

(c) (i) attempt to find $B(100)$ ***(M1)***

$61.4707\ldots$

61.5 million OR $61{,}500{,}000$ ***A1***

(ii) The annual growth rate of the population is 0.7%. ***A1***

Note: Description must include some reference to annual rate.

[3 marks]

(d) $58.1070\ldots$

58.1 million OR $58{,}100{,}000$ ***A1***

[1 mark]

(e) consideration of the difference function $C(t) - B(t)$ or $B(t) - C(t)$ or $|C(t) - B(t)|$ ***(M1)***

evidence of finding the maximum (or minimum) of this function. ***(M1)***

$t = 45.9583$

2045 (accept 2046) ***A1***

[3 marks]

(f) (i) 0.282151…

$B'(40) = 0.282$ **_A1_**

(ii) 0.325546...

$C'(40) = 0.326$ **_A1_**

[2 marks]

(g) $B'(40) < C'(40)$ (or equivalent in words) **_A1_**

the population in Cecilia's model is increasing at a faster rate than in Benoit's model (in 2040) (or equivalent) **_R1_**

Note:	Do not award ***A0R1***.

[2 marks]

Total [15 marks]

Question 44

(a) $16.7037\ldots$

$16.7(\%)$ **A1**

[1 mark]

(b) recognition of binomial **(M1)**

$X \sim \mathrm{B}(1000, 0.167\ldots)$

attempt to find $\mathrm{P}(X < 150)$ OR $\mathrm{P}(X \le 149)$ **(M1)**

$0.0669378\ldots$

$\mathrm{P}(X < 150) = 0.0669$ (accept 0.0673 from previous 3sf) **A1**

[3 marks]

(c) recognition of conditional probability in context **(M1)**

$\mathrm{P}(\text{age} \ge 75 \mid \text{age} \ge 55)$ OR $\dfrac{\mathrm{P}(\text{age} \ge 55 \cap \text{age} \ge 75)}{\mathrm{P}(\text{age} \ge 55)}$

Note: Recognition must be shown in context either in words or symbols, not just $\mathrm{P}(A \mid B)$

$\dfrac{\mathrm{P}(\text{age} \ge 75)}{\mathrm{P}(\text{age} \ge 55)}$ **(A1)**

$\dfrac{2.1}{(2.1+12.9)}$ OR $\dfrac{0.0233853\ldots}{0.167}$ **(A1)**

0.14 (exact) $\left(=\dfrac{7}{50}\right)$ **A1**

Note: Condone use of "a" or "X" or any letter for age.

[4 marks]

(d) $15, 26.5, 45, 65, 85$ **A1**

[1 mark]

(e) (i) 38.0740…

mean = 38.1 ***A1***

(ii) variance is square of standard deviation ***(M1)***

$16.3639...^2$

267.778...

variance = 268 ***A1***

[3 marks]

(f)

(i) Graph D ***A1***

Note: Allow ***FT*** from incorrect values for mean and variance in (e).

(ii) SmallTalk ***A1***

Note: Do not award ***A0A1***.

[2 marks]

Total [14 marks]

Question 45

(a) recognition of sum of probabilities equals 1 ***(M1)***

$$\frac{2k}{15}+\frac{4k}{15}+\frac{7k}{15}+\frac{10k}{15}=1$$

$k=0.652173...$

$k=0.652\left(=\frac{15}{23}\right)$ ***A1***

[2 marks]

(b) correct probabilities: $\frac{2}{23},\frac{4}{23},\frac{7}{23},\frac{10}{23}$ OR $0.0870,\ 0.174,\ 0.304,\ 0.435$ ***(A1)***

substitution of their probabilities into formula for expected value ***(M1)***

$2\times\frac{2}{23}+4\times\frac{4}{23}+7\times\frac{7}{23}+10\times\frac{10}{23}$ OR $\frac{169k}{15}$

$=7.34782...$

$\mathrm{E}(X)=7.35\left(=\frac{169}{23}\right)$ (same 3sf from previous 3sf answer) ***A1***

[3 marks]

Total [5 marks]