Subject - Math AA(Standard Level) Topic - Statistics and Probability Year - May 2021 - Nov 2022 Paper -2 Answers

Question 1

Ques	CHOIL I		
(a)	attempt to use the symmetry of the normal curve	(M1)	
	eg diagram, $0.5-0.1446$		
	P(24.15 < X < 25) = 0.3554	A1	
			[2 marks]
(b)	(i) use of inverse normal to find z score	(M1)	
(b)	(i) use of inverse normal to find z score $z = -1.0598$	(IVI I)	
	correct substitution $\frac{24.15-25}{}=-1.0598$		
	correct substitution $\frac{1}{\sigma} = -1.0598$	(A1)	
	$\sigma = 0.802$	A1	
	(ii) $P(X > 26) = 0.106$	(M1)A1	
			[5 marks]
(c)	recognizing binomial probability	(M1)	
. ,	$E(Y) = 10 \times 0.10621$	(A1)	
	= 1.06	A1	
			[3 marks]
(d)	P(Y=3)	(M1)	
(4)	= 0.0655	A1	
	- 0.0033	A	[2 marks]
	3	-0'/	-
(e)	recognizing conditional probability correct substitution	(M1) A1	
	0.3554	Al	
	1-0.10621		
	= 0.398	A1	
			[3 marks]
		Total	[15 marks]
•		Total	To marks ₁
Ques	stion 2		
(a)	a = 1.29 and $b = -10.4$	A1A1	[2 marks]
			[2 marks]
(b)	recognising both lines pass through the mean point	(M1)	
	p = 28.7, q = 30.3	A2	
		1	[3 marks]
		Total	[5 marks]
			(A)

(M1)

 $X \sim B(5, 0.7)$ attempt to find $P(X \le 3)$ = 0.472(= 0.47178)

M1 A1

[3 marks]

(b) recognition of 2 sixes in 4 tosses

P(3rd six on the 5th toss) =
$$\left[\binom{4}{2} \times (0.7)^2 \times (0.3)^2 \right] \times 0.7 (= 0.2646 \times 0.7)$$

(M1) A1

= 0.185 (= 0.18522)

A1

[3 marks]

Total [6 marks]

Question 4

Let X = mass of a bag of sugar

(a) evidence of identifying the correct area

(M1)

$$P(X < 995) = 0.0765637...$$

$$=0.0766$$

A1

[2 marks]

(b) 0.0766×100 ≈ 8

A1

[1 mark]

(c) recognition that $P(X>1005|X\ge 995)$ is required

(M1)

$$\frac{P(X \ge 995 \cap X > 1005)}{P(X \ge 995)}$$

 $P(X \ge 995)$

$$\frac{0.0765637...}{1-0.0765637...} \left(= \frac{0.0765637...}{0.923436...} \right)$$

$$=0.0829$$

AI

[3 marks] Total [6 marks]

(a) a = 0.433156..., b = 4.50265...

$$a = 0.433$$
, $b = 4.50$

A1A1

[2 marks]

(b) attempt to substitute x = 18 into their equation

(M1)

$$y = 0.433 \times 18 + 4.50$$

=12.2994...

$$=12.3$$

A1

[2 marks]

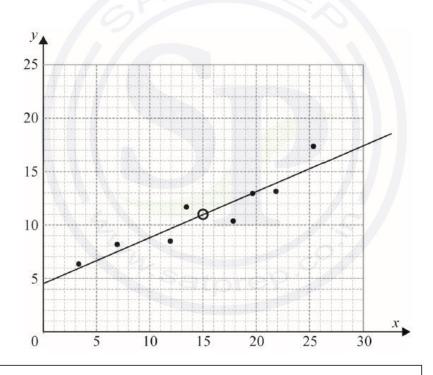
$$\overline{x} = 15$$
 , $\overline{y} = 11$

A1

[1 mark]

(d)

(c)



A1A1

Note: Award marks as follows:

A1 for a straight line going through (15,11)

A1 for intercepting the y-axis between their $b\pm 1.5$ (when their line is extended), which includes all the data for $3.3 \le x \le 25.3$.

If the candidate does not use a ruler, award AOA1 where appropriate.

[2 marks] Total [7 marks]

(a) considering that sum of probabilities is 1 (M1) 0.85+c+0.03+0.002+0.0001=1

0.1179

[2 marks]

A1

(b) valid attempt to find $\mathrm{E}(D)$

$$E(D) = (0 \times 0.85) + (2 \times 0.1179) + (10 \times 0.03) + (50 \times 0.002) + (1000 \times 0.0001)$$

$$E(D) = 0.7358$$

No, not a fair game

for a fair game, $\mathrm{E}(D)$ would be \$2 OR players expected winnings are 1.264

[4 marks]

(c) recognition of GP with r=2 (M1)

 $1000 \times 2^{n-1}$ OR $500(2^n)$

[2 marks]

(d) recognizing
$$E(D) > 2$$
 (M1)

correct expression for
$$w^{\text{th}}$$
 week (or n^{th} week) (A1)

$$(0 \times 0.85) + (2 \times 0.1179) + (10 \times 0.03) + (50 \times 0.002) + (1000 \times 2^{w-1} \times 0.0001)$$

$$0.6358 + (1000 \times 2^{w-1} \times 0.0001) > 2 \text{ OR } 2^{n-1} > 13.642$$

EITHER

$$n-1>3.76998 \text{ OR } w=4.76998...$$
 (A1)

OR

$$E(D) = 1.4358$$
 in week 4 or $E(D) = 2.2358$ in week 5 (A1)

THEN

$$w=5$$

expected profit per ticket = their
$$E(D)-2$$
 (M1)

$$=0.2358$$

[7 marks] Total [15 marks]

(a) use of inverse normal to find z-score (M1)

$$z = 2.0537...$$

$$2.0537... = \frac{82 - 75}{\sigma}$$
 (A1)

$$\sigma = 3.408401...$$

$$\sigma = 3.41$$

[3 marks]

(b) evidence of identifying the correct area under the normal curve (M1)

$$P(T > 80) = 0.071193...$$

$$P(T > 80) = 0.0712$$

[2 marks]

(c) recognition that P(80 < T < 82) is required (M1)

$$P(T < 82 | T > 80) = \frac{P(80 < T < 82)}{P(T > 80)} = \left(\frac{0.051193...}{0.071193...}\right)$$
 (M1)(A1)

$$= 0.719075...$$

(M1)

$$X \sim B(64, 0.071193...)$$
 or $E(X) = 64 \times 0.071193...$

(A1)

$$E(X) = 4.556353...$$

$$E(X) = 4.56$$
 (flights)

A1

[3 marks]

(e)
$$P(X > 6) = P(X \ge 7) = 1 - P(X \le 6)$$

(M1)

$$=1-0.83088...$$

(A1)

$$=0.1691196...$$

$$=0.169$$

A1

[3 marks]

Total [15 marks]

Question 8

(a) EITHER

$$P(S) + P(T) + P(S' \cap T') - P(S \cap T) = 1 \text{ OR } P(S \cup T) = P((S' \cap T')')$$

(M1)

$$0.7 + 0.2 + 0.18 - P(S \cap T) = 1 \text{ OR } P(S \cup T) = 1 - 0.18$$

OR

a clearly labelled Venn diagram

(M1)

THEN

$$P(S \cap T) = 0.08$$
 (accept 8%)

A1

Note: To obtain the *M1* for the Venn diagram all labels must be correct and in the correct sections. For example, do not accept 0.7 in the area corresponding to $S \cap T'$.

[2 marks]

(b) EITHER

$$P(T \cap S') = P(T) - P(T \cap S) (= 0.2 - 0.08) \text{ OR}$$

 $P(T \cap S') = P(T \cup S) - P(S) (= 0.82 - 0.7)$

(M1)

OR

a clearly labelled Venn diagram including
$$P(S)$$
, $P(T)$ and $P(S \cap T)$ (M1)

THEN

$$=0.12$$
 (accept 12%)

[2 marks]

(c)
$$P(G \cap T) = P(T/G)P(G) (0.25 \times 0.48)$$
 (M1)
= 0.12

[2 marks]

(d) METHOD 1

$$P(G) \times P(T) (= 0.48 \times 0.2) = 0.096$$
 A1
 $P(G) \times P(T) \neq P(G \cap T) \Rightarrow G \text{ and } T \text{ are not independent}$ R1

METHOD 2

$$P(T | G) = 0.25$$
 A1 $P(T | G) \neq P(T) \Rightarrow G$ and T are not independent R1

Note: Do not award AOR1.

[2 marks] Total [8 marks]

(a) (i) a = 0.805084... and b = 2.88135... a = 0.805 and b = 2.88

A1A1

(ii) r = 0.97777...r = 0.978

A1

[3 marks]

(b) a represents the (average) increase in waiting time ($0.805\,$ mins) per additional customer (waiting to receive their coffee)

R1

[1 mark]

(c) attempt to substitute x = 7 into their equation

(M1)

8.51693...

8.52 (mins)

A1

[2 marks]

Total [6 marks]

Question 10

(a)
$$P\left(\frac{\mu-1.5\sigma-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{\mu+1.5\sigma-\mu}{\sigma}\right)$$

(M1)

$$P(-1.5 < Z < 1.5)$$
 OR $1-2 \times P(Z < -1.5)$

(A1)

$$P(-1.5 < Z < 1.5) = 0.866385...$$

$$P(\mu-1.5\sigma < X < \mu+1.5\sigma) = 0.866$$

A1

Note: Do not award any marks for use of their answers from part (b).

[3 marks]

(b)
$$z_1 = -1.75068...$$
 and $z_2 = 1.30468...$ (seen anywhere) (A1) correct equations (A1)(A1) $\frac{106.2 - \mu}{\sigma} = -1.75068...$, $\mu + 1.30468...$ $\sigma = 182.6$ attempt to solve their equations involving z values (M1) $\mu = 149.976...$, $\sigma = 25.0051...$ $\mu = 150$, $\sigma = 25.0$ A1 [5 marks] (c) (i) new sample space is 96% (may be seen in (ii) or (iii)) P(medium|not small) OR $\frac{0.576}{0.96}$ P(Medium) = 0.6 A1 (iii) P(Large) = 0.3 A1 (iiii) P(Premium) = 0.1 A1 [4 marks] (d) attempt to express revenue from avocados $1.1 \times 0.6 + 1.29 \times 0.3 + 1.96 \times 0.1$ Or $1.243n$ correct inequality or equation for net profit in terms of n (A1) $1.1 \times 0.6n + 1.29 \times 0.3n + 1.96 \times 0.1n - 200 \ge 438$ attempt to solve the inequality sketch OR $n = 513.274...$ $n = 514$ A1 Note: Only award follow through in part (d) for 3 probabilities which add up to 1. FT of probabilities from c) that do not add up to 1 should only be awarded M marks, where appropriate, in d).

Total [16 marks]

(a) recognize that the variable has a Binomial distribution

(M1)

$$X \sim B(30, 0.05)$$

attempt to find $P(X \ge 1)$

(M1)

$$1-P(X=0)$$
 OR $1-0.95^{30}$ OR $1-0.214638...$ OR $0.785361...$

Note: The two M marks are independent of each other.

$$P(X \ge 1) = 0.785$$

A1

[3 marks]

(b) recognition of conditional probability

(M1)

 $P(X \le 2 \mid X \ge 1)$ OR $P(\text{at most 2 defective} \mid \text{at least 1 defective})$

Note: Recognition must be shown in context either in words or symbols but not just P(A | B).

$$\frac{P(1 \le X \le 2)}{P(X \ge 1)}$$
 OR $\frac{P(X=1) + P(X=2)}{P(X \ge 1)}$ (A1)

$$\frac{0.597540...}{0.785361...} \text{ OR } \frac{0.812178...-0.214638...}{0.785361...} \text{ OR } \frac{0.338903...+0.258636...}{0.785361...} \tag{A1)}$$

=0.760847...

$$P(X \le 2 | X \ge 1) = 0.761$$

A1

[4 marks] Total [7 marks]

(a) use of GDC to give

(M1)

r = 0.883529...

r = 0.884

A1

Note: Award the *(M1)* for any correct value of r, a, b or $r^2 = 0.780624...$ seen in part (a) or part (b).

[2 marks]

(b) a = 1.36609..., b = 64.5171...

$$a=1.37$$
, $b=64.5$

A1

[1 mark]

(c) attempt to find their difference

(M1)

 $5 \times 1.36609...$ OR 1.36609...(h+5)+64.5171...-(1.36609...h+64.5171...)

6.83045...

=6.83 (6.85 from 1.37)

the student could have expected her score to increase by 7 marks.

A1

Note: Accept an increase of 6, 6.83 or 6.85.

[2 marks]

Total [5 marks]

(a)
$$P(C < 61)$$
 (M1) = 0.365112... = 0.365 A1 [2 marks]

(b) recognition of binomial eg $X \sim B(12,0.365...)$ (M1) $P(X = 5) = 0.213666...$ = 0.214 A1 [2 marks]

(c) (i) Let CM represent 'chocolate muffin' and BM represent 'banana muffin' $P(B < 61) = 0.0197555...$ (A1) EITHER $P(CM) \times P(C < 61 | CM) + P(BM) \times P(B < 61 | BM)$ (or equivalent in words) (M1) OR tree diagram showing two ways to have a muffin weigh < 61 (M1) THEN (0.6 \times 0.365...) + (0.4 \times 0.0197...) (A1) = 0.226969... = 0.227 A1

(ii) recognizing conditional probability (M1) Note: Recognition must be shown in context either in words or symbols, not just $P(A|B)$.

[7 marks]

(d) METHOD 1

$$P(CM) \times P(C < 61 | CM) + P(BM) \times P(B < 61 | BM) = 0.157$$
 (M1)

 $(0.6 \times P(C < 61)) + (0.4 \times 0.0197555...) = 0.157$

$$P(C < 61) = 0.248496...$$
 (A1)

attempt to solve for σ using GDC (M1)

Note: Award *(M1)* for a graph or table of values to show their P(C < 61) with a variable standard deviation.

$$\sigma = 1.47225...$$

$$\sigma = 1.47$$
 (g)

METHOD 2

$$P(CM) \times P(C < 61 | CM) + P(BM) \times P(B < 61 | BM) = 0.157$$
 (M1)

 $(0.6 \times P(C < 61)) + (0.4 \times 0.0197555...) = 0.157$

$$P(C < 61) = 0.248496...$$
 (A1)

use of inverse normal to find z score of their P(C < 61) (M1)

$$z = -0.679229...$$

$$\frac{61-62}{\sigma} = -0.679229...$$

$$\sigma = 1.47225...$$

$$\sigma$$
=1.47 (g)

[5 marks]

Total [16 marks]

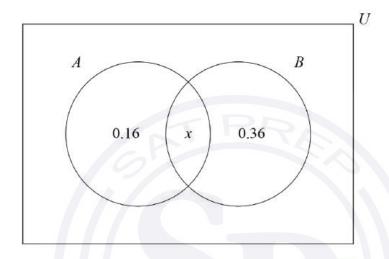
(a) METHOD 1

EITHER

one of P(A) = x + 0.16 OR P(B) = x + 0.36

A1

OR



A1

THEN

attempt to equate their $P(A \cap B)$ with their expression for $P(A) \times P(B)$

M1

$$P(A \cap B) = P(A) \times P(B) \Rightarrow x = (x + 0.16) \times (x + 0.36)$$

A1

$$x = 0.24$$

A1

METHOD 2

attempt to form at least one equation in P(A) and P(B) using independence

M1

$$(P(A \cap B') = P(A) \times P(B') \Rightarrow) P(A) \times (1 - P(B)) = 0.16 \text{ OR}$$

$$(P(A' \cap B) = P(A') \times P(B) \Rightarrow) (1 - P(A)) \times P(B) = 0.36$$

$$P(A) = 0.4 \text{ AND } P(B) = 0.6$$

A1

$$P(A \cap B) = P(A) \times P(B) = 0.4 \times 0.6$$

(A1)

$$x = 0.24$$

A1

[4 marks]

(b) METHOD 1

recognising
$$P(A'|B') = P(A')$$
 (M1)
= 1-0.16-0.24
= 0.6

METHOD 2

$$P(B) = 0.36 + 0.24 (= 0.6)$$

$$P(A' | B') = \frac{P(A' \cap B')}{P(B')} \left(= \frac{0.24}{0.4} \right)$$
 (A1)

$$=0.6$$

[2 marks]

Total [6 marks]

Question 15

(a)
$$0.41+k-0.28+0.46+0.29-2k^2=1$$
 OR $k-2k^2+0.01=0.13$ (or equivalent) **A1** $2k^2-k+0.12=0$

AG [1 mark]

$$k = 0.3$$

reasoning to reject
$$k = 0.2$$
 eg $P(1) = k - 0.28 \ge 0$ therefore $k \ne 0.2$

[3 marks]

$$E(X) = 0 \times 0.41 + 1 \times (0.3 - 0.28) + 2 \times 0.46 + 3 \times (0.29 - 2 \times 0.3^{2})$$

[2 marks]

Total [6 marks]

(a) EITHER

recognising that half the total frequency is 10 (may be seen in an ordered list or indicated on the frequency table)

OR

$$5+1+4=3+x$$
 (A1)

OR

$$\sum f = 20 \tag{A1}$$

THEN

$$x = 7$$

[2 marks]

(A1)

(b) METHOD 1

1.58429...

METHOD 2

EITHER

$$\sigma^2 = \frac{5 \times (2 - 4.3)^2 + 1 \times (3 - 4.3)^2 + 4 \times (4 - 4.3)^2 + 3 \times (5 - 4.3)^2 + 7 \times (6 - 4.3)^2}{20} \quad (= 2.51)$$

OR

$$\sigma^2 = \frac{5 \times 2^2 + 1 \times 3^2 + 4 \times 4^2 + 3 \times 5^2 + 7 \times 6^2}{20} - 4.3^2 \quad (= 2.51)$$

THEN

$$\sigma = \sqrt{2.51} = 1.58429...$$
= 1.58

[2 marks]

Total [4 marks]

(a) METHOD 1

$$T \sim N(35, \sigma^2)$$

$$P(T > 40) = 0.25 \text{ or } P(T < 40) = 0.75$$
 (M1)

attempt to solve for σ graphically or numerically using the GDC

(M1)

graph of normal curve $T \sim N(35, \sigma^2)$ for P(T > 40) and y = 0.25 OR P(T < 40) and y = 0.75 OR table of values for P(T < 40) or P(T > 40)

$$\sigma = 7.413011...$$

$$\sigma = 7.41$$
 (min)

A2

METHOD 2

$$T \sim N(35, \sigma^2)$$

$$P(T > 40) = 0.25 \text{ or } P(T < 40) = 0.75$$
 (M1)

$$z = 0.674489...$$
 (A1)

valid equation using their z-score (clearly identified as z-score and not a probability) (M1)

$$\frac{40-35}{\sigma} = 0.674489... \text{ OR } 5 = 0.674489...\sigma$$

7.413011...

$$\sigma = 7.41$$
 (min)

A1

[4 marks]

(b)
$$P(T > 45)$$

= 0.0886718...

$$=0.0887$$

[2 marks]

A1

(c) recognizing binomial probability (M1) $L \sim B(5, 0.0886718...)$ $P(L \ge 1) = 1 - P(L = 0)$ OR $P(L \ge 1) = P(L = 1) + P(L = 2) + P(L = 3) + P(L = 4) + P(L = 5)$ (M1)0.371400... $P(L \ge 1) = 0.371$ A1 [3 marks] (d) recognizing conditional probability in context (M1)finding $\{L < 3\} \cap \{L \ge 1\} = \{L = 1, L = 2\}$ (may be seen in conditional probability) (A1)P(L=1)+P(L=2)=0.36532... (may be seen in conditional probability) (A1) $P(L < 3 | L \ge 1) = \frac{0.36532...}{0.37140...}$ (A1)0.983636... 0.984 A1 [5 marks] (e) **METHOD 1** recognizing that Suzi can be late no more than once (in the remaining six days) (M1) $X \sim B(6, 0.0886718...)$, where X is the number of days late (A1) $P(X \le 1) = P(X = 0) + P(X = 1)$ (M1)=0.907294...P(Suzi gets a bonus) = 0.907A1 **METHOD 2** recognizing that Suzi must be on time at least five times (of the remaining six days) (M1) $X \sim B(6, 0.911328...)$, where X is the number of days on time (A1) $P(X \ge 5) = 1 - P(X \le 4)$ OR 1 - 0.0927052... OR P(X = 5) + (X = 6) OR 0.334434...+0.572860...(M1)=0.907294...P(Suzi gets a bonus) = 0.907A1 [4 marks] Total [18 marks]

(a) 0.28 (s) A1

[1 mark]

(b)
$$IQR = 0.35 - 0.27 = 0.08$$
 (s) (A1)

substituting their IQR into correct expression for upper fence (A1)

$$0.35 + 1.5 \times 0.08 \ (= 0.47) \ (s)$$

so 0.46 (s) is not an outlier AG

[3 marks]

Question 19

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.68$$

substitution of
$$P(A) \cdot P(B)$$
 for $P(A \cap B)$ in $P(A \cup B)$

$$P(A)+P(B)-P(A)P(B) (= 0.68)$$

substitution of
$$3P(B)$$
 for $P(A)$ (M1)

$$3P(B)+P(B)-3P(B)P(B)=0.68 mtext{ (or equivalent)} mtext{ (A1)}$$

Note: The first two M marks are independent of each other.

attempts to solve their quadratic equation (M1)

$$P(B) = 0.2, 1.133...$$
 $\left(\frac{1}{5}, \frac{17}{15}\right)$

$$P(B) = 0.2 \left(= \frac{1}{5} \right)$$

Note: Award **A1** if both answers are given as final answers for P(B).

[6 marks]

Question 20 recognising to find P(T > 40)(M1)P(T > 40) = 0.574136...P(T > 40) = 0.574A1 [2 marks] (b) attempt to multiply four independent probabilities using their P(T > 40) and P(T < 40)(M1)

$$(1-p)^3 \cdot p$$
 OR $(1-0.574136...)^3 \cdot 0.574136...$ OR $(0.425863...)^3 \cdot 0.574136...$ (A1)

0.0443430...

0.0443, 0.0444 from 3 sf values

[3 marks]

A1

recognizing conditional probability (M1)(c)

P(T < 55 | T > 40)

$$\frac{P(40 < T < 55)}{P(T > 40)}$$
(A1)

P(T < 55 | T > 40) = 0.804590...

recognizing binomial probability (M1)

 $X \sim B(n, p)$

$$n = 10$$
 and $p = 0.804589...$ (A1)

0.0242111..., 0.0240188...using p = 0.805

$$P(X=5) = 0.0242$$

[7 marks]

(d) Let P(T < a) = x

recognition that probabilities sum to 1 (seen anywhere)

EITHER

expressing the three regions in one variable

(M1)

(M1)

$$x + 0.904 + 2x$$
 OR $P(T < a) + 0.904 + 2P(T < a)$ OR $\frac{1}{2}P(T > b) + 0.904 + P(T > b)$

OR x and 2x correctly indicated on labelled bell diagram

$$P(T < a) + 0.904 + 2P(T < a) = 1 \text{ OR } \frac{1}{2}P(T > b) + 0.904 + P(T > b) = 1 \text{ (or equivalent)}$$
 (A1)

OR

expressing either
$$P(T < a)$$
 or $P(T > b)$ only in terms of $P(a \le T \le b)$ (M1)

$$\left(P(T < a) = \right) \frac{1}{3} \left(1 - P\left(a \le T \le b \right) \right) \text{ OR } \left(P(T > b) = \right) \frac{2}{3} \cdot \left(1 - P\left(a \le T \le b \right) \right)$$

$$x = \frac{1}{3}(1 - 0.904)(= 0.032) \text{ OR } P(T > b) = \frac{2}{3}(1 - 0.904)(= 0.064)$$
 (A1)

THEN

$$P(T < a) = 0.032$$

a = 22.18167...

a = 22.2 accept 22.1

[4 marks]

Total [16 marks]

(a) 1.01206..., 2.45230...

$$a = 1.01$$
, $b = 2.45 (1.01x + 2.45)$

A1A1

[2 marks]

(b) 0.981464...

$$r = 0.981$$

A1

Note: A common error is to enter the data incorrectly into the GDC, and obtain the answers a=1.01700..., b=2.09814... and r=0.980888... Some candidates may write the 3 sf answers, ie. a=1.02, b=2.10 and r=0.981 or 2 sf answers, ie. a=1.0, b=2.1 and r=0.98. In these cases award **A0A0** for part (a) and **A0** for part (b). Even though some values round to an accepted answer, they come from incorrect working.

[1 mark]

(c) correct substitution of 78 into their regression equation

(M1)

81.3930... 81.23 from 3 sf answer

81

A1

[2 marks]

Total [5 marks]