Subject – Math AI(Higher Level)
Topic - Calculus
Year - May 2021 – Nov 2022
Paper -1
Answers

Question 1

(a) (i) $a = 33$ ***A1***

(ii) $\frac{1}{\sqrt[3]{0.08}} = 2.32$ ***M1A1***

[3 marks]

(b) volume within outer dome

$\frac{2}{3}\pi \times 16^3 + \pi \times 16^2 \times 17 = 22\,250.85$ ***M1A1***

volume within inner dome

$\pi\int_0^{33}\left(\frac{33-y}{0.08}\right)^{\frac{2}{3}}\mathrm{d}y = 3446.92$ ***M1A1***

volume between $= 22\,250.85 - 3446.92 = 18\,803.93\,\mathrm{m}^3$ ***A1***

[5 marks]

Total [8 marks]

Question 2

(a) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{16-20}{24-20}$ ***M1***

$= -1$ ***A1***

[2 marks]

(b) asymptote of trajectory along $\boldsymbol{r} = k\begin{pmatrix}2\\1\end{pmatrix}$ ***M1A1***

Note: Award ***M1A0*** if asymptote along $\begin{pmatrix}1\\2\end{pmatrix}$.

trajectory begins at $(8, 10)$ with negative gradient ***A1A1***

[4 marks]

Total [6 marks]

Question 3

(a) $E = 5(2\sin t)^2 \left(= 20\sin^2 t\right)$ ***A1***

[1 mark]

(b) $\frac{\mathrm{d}E}{\mathrm{d}t} = 40\sin t\cos t$ ***(M1)A1***

[2 marks]

(c) $t = 0.126$ ***(M1)A1***

[2 marks]

Total [5 marks]

Question 4

(a) $\sin(x+y) = 0$ ***A1***

$\Rightarrow x + y = 0$ ***(M1)***

(the equation of L_1 is) $y = -x$ ***A1***

[3 marks]

(b) $x + y = \pi$ **OR** $y = -x + \pi$ ***(M1)A1***

[2 marks]

Total [5 marks]

Question 5

(a) $\frac{dV}{dt} = -kV^{\frac{1}{2}}$

use of separation of variables **(M1)**

$\Rightarrow \int V^{-\frac{1}{2}}\ dV = \int -k\ dt$ **A1**

$2V^{\frac{1}{2}} = -kt\ (+c)$ **A1**

considering initial conditions $40 = c$ **A1**

$2\sqrt{324} = -10k + 40$

$\Rightarrow k = 0.4$ **A1**

$2\sqrt{V} = -0.4t + 40$

$\Rightarrow \sqrt{V} = 20 - 0.2t$ **A1**

Note: Award ***A1*** for any correct intermediate step that leads to the ***AG***.

$\Rightarrow V = \left(20 - \frac{t}{5}\right)^2$ ***AG***

Note: Do not award the final ***A1*** if the ***AG*** line is not stated.

[6 marks]

(b) $0 = \left(20 - \frac{t}{5}\right)^2 \Rightarrow t = 100$ minutes ***(M1)A1***

[2 marks]

Total [8 marks]

Question 6

(a) (i) $A=\frac{1}{2}\times 6\times q+\frac{1}{2}\times 8\times p+48$ **OR** $A=\frac{1}{2}(p+6)(q+8)$ **OR**

$A=3q+4p+48$ ***A1***

(ii) valid attempt to link p and q, using tangents, similar triangles or other method ***(M1)***

eg. $\tan\theta=\frac{8}{p}$ and $\tan\theta=\frac{q}{6}$ **OR** $\tan\theta=\frac{p}{8}$ and $\tan\theta=\frac{6}{q}$ **OR** $\frac{8}{p}=\frac{q}{6}$

correct equation linking p and q ***A1***

eg. $pq=48$ **OR** $p=\frac{48}{q}$ **OR** $q=\frac{48}{p}$

substitute $p=\frac{48}{q}$ into a correct area expression ***M1***

eg. $(A=)\frac{1}{2}\times 6\times q+\frac{1}{2}\times 8\times\frac{48}{q}+48$ **OR** $(A=)\frac{1}{2}\left(\frac{48}{q}+6\right)(q+8)$

$A=3q+\frac{192}{q}+48$ ***AG***

Note:	The ***AG*** line must be seen with no incorrect, intermediate working, for the final ***M1*** to be awarded.

[4 marks]

(b) $\frac{-192}{q^2}+3$ ***A1A1***

Note:	Award ***A1*** for $\frac{-192}{q^2}$, ***A1*** for 3. Award ***A1A0*** if extra terms are seen.

[2 marks]

(c) (i) $\frac{-192}{q^2}+3=0$ ***A1***

(ii) $q=8\text{ cm}$ ***A1***

[2 marks]

Total [8 marks]

Question 7

(a) $x_n = x_{n-1} + h\,f(x_{n-1}, t_{n-1})$

$h = 0.1,\ f(x, t) = x\cos t\,(\mathrm{e}^{-\sin t})$

$x_n = x_{n-1} + 0.1x_{n-1}\cos t_{n-1}(\mathrm{e}^{-\sin t_{n-1}})$ ***(M1)***

Note: Award ***M1*** for a valid start.

n	t_n	x_n
0	0	0.367879
1	0.1	0.404667
2	0.2	0.441106
3	0.3	0.476548

(A1)

Note: Award ***A1*** for a correct x value when $n = 1$.

$x(0.3) \approx 0.477\ \ (0.476548...)$ ***A1***

[3 marks]

(b) **EITHER**

$\int \frac{\mathrm{d}x}{x} = \int \cos t(\mathrm{e}^{-\sin t})\,\mathrm{d}t\ \ (+c)$ ***M1***

$\ln x = -\mathrm{e}^{-\sin t} + c$ ***A1***

$t = 0,\ x = \frac{1}{\mathrm{e}} \Rightarrow c = 0$ ***M1***

$x = \mathrm{e}^{(-\mathrm{e}^{-\sin t})}$

$x(0.3) \approx 0.475140...$ ***A1***

OR

$\int_{1/\mathrm{e}}^{x} \frac{\mathrm{d}u}{u} = \int_{0}^{0.3} \cos t(\mathrm{e}^{-\sin t})\,\mathrm{d}t$ ***M1***

$[\ln u]_{1/\mathrm{e}}^{x} = 0.255855...$ (from GDC) ***A1***

$\ln x + 1 = 0.255855...$

$\ln x = -0.744145...$ ***A1***

$x = \mathrm{e}^{-0.744145} = 0.475140....$ ***A1***

THEN

percentage error $= \left|\frac{0.476548... - 0.475140...}{0.475140...}\right| \times 100 = 0.296\%\ \ (2.96192...)$ ***A1***

Note: If candidates do not attempt to find c, they may score ***M1A0M0A1A1***.

[5 marks]

Total [8 marks]

Question 8

(a) attempt to use $V = \pi\int_a^b x^2\,dy$ **(M1)**

$x = e^{\frac{y}{6}}$ or any reasonable attempt to find x in terms of y **(M1)**

$V = \pi\int_0^h e^{\frac{y}{3}}\,dy$ **A1**

Note: Correct limits must be seen for the ***A1*** to be awarded.

$= \pi\left[3e^{\frac{y}{3}}\right]_0^h$ **(A1)**

Note: Condone the absence of limits for this ***A1*** mark.

$= 3\pi\left[e^{\frac{h}{3}} - e^0\right]$ **A1**

$= 3\pi\left[e^{\frac{h}{3}} - 1\right]$ **AG**

Note: If the variable used in the integral is x instead of y (i.e. $V = \pi\int_0^h e^{\frac{x}{3}}dx$) and the candidate has not stated that they are interchanging x and y then award at most ***M1M1A0A1A1AG***.

[5 marks]

(b) maximum volume when $h = 9$ cm **(M1)**

max volume $= 180$ cm^3 **A1**

[2 marks]

Total: [7 marks]

Question 9

(a) $15 = 3 + 4r + 2r\theta$ **M1**

$12 = 2r(2 + \theta)$ **A1**

Note: Award ***A1*** for any reasonable working leading to expected result e,g, factorizing r.

$r = \frac{6}{2+\theta}$ ***AG***

[2 marks]

(b) (i) attempt to use sector area to find volume ***(M1)***

volume $= \frac{1}{2} r^2 \theta \times 1$

$= \frac{1}{2} \times \frac{36}{(2+\theta)^2} \times \theta \quad \left(= \frac{18\theta}{(2+\theta)^2} \right)$ ***A1***

(ii) $\frac{\mathrm{d}V}{\mathrm{d}\theta} = \frac{(2+\theta)^2 \times 18 - 36\theta(2+\theta)}{(2+\theta)^4}$ ***M1A1A1***

$\frac{\mathrm{d}V}{\mathrm{d}\theta} = \frac{36 - 18\theta}{(2+\theta)^3}$

(iii) $\frac{\mathrm{d}V}{\mathrm{d}\theta} = \frac{36 - 18\theta}{(2+\theta)^3} = 0$ ***M1***

Note: Award this ***M1*** for simplified version equated to zero. The simplified version may have been seen in part (b)(ii).

$\theta = 2$ ***A1***

[7 marks]

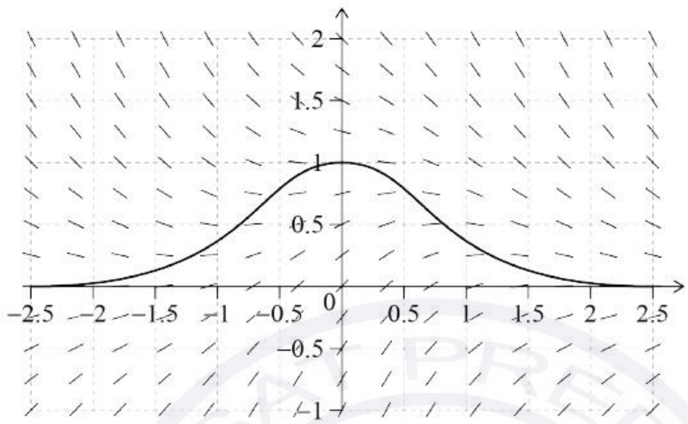
Total: [9 marks]

Question 10

(a) $\left(\frac{dy}{dx} = e^0 - 1\right) = 0$ **A1**

[1 mark]

(b)

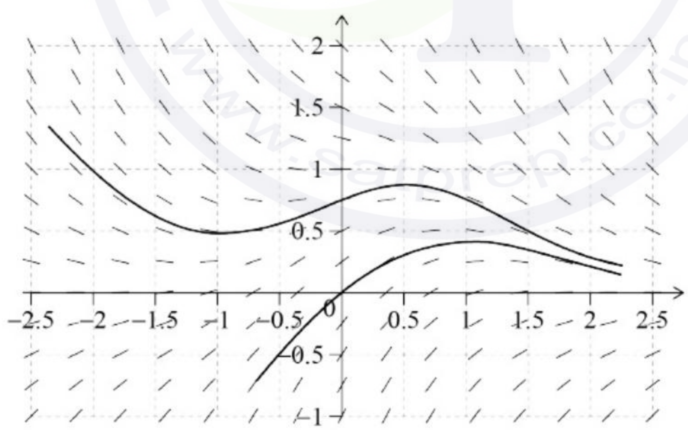


gradient $= 0$ at $(0, 1)$ **A1**
correct shape **A1**

Note:	Award second ***A1*** for horizontal asymptote of $y = 0$, and general symmetry about the y-axis.

[2 marks]

(c)

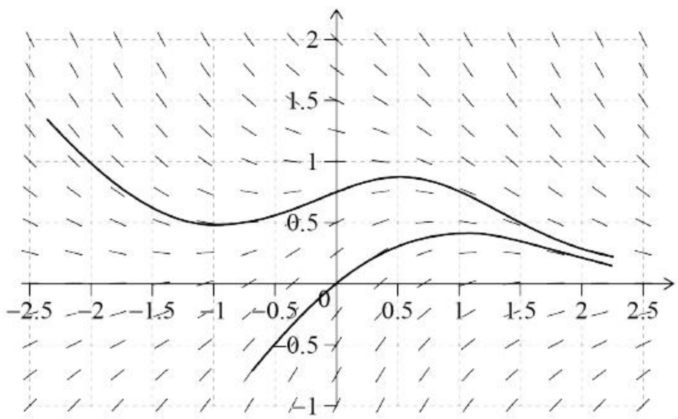


(i) positive gradient at origin **A1**
correct shape **A1**

Note:	Award second ***A1*** for a single maximum in 1st quadrant and tending toward an asymptote.

(ii) positive gradient at $(0, 0.75)$ **A1**
correct shape **A1**

(c)



(i) positive gradient at origin **A1**
correct shape **A1**

> **Note:** Award second ***A1*** for a single maximum in 1st quadrant and tending toward an asymptote.

(ii) positive gradient at $(0, 0.75)$ **A1**
correct shape **A1**

> **Note:** Award second ***A1*** for a single minimum in 2nd quadrant, single maximum in 1st quadrant and tending toward an asymptote.

[4 marks]
Total: [7 marks]

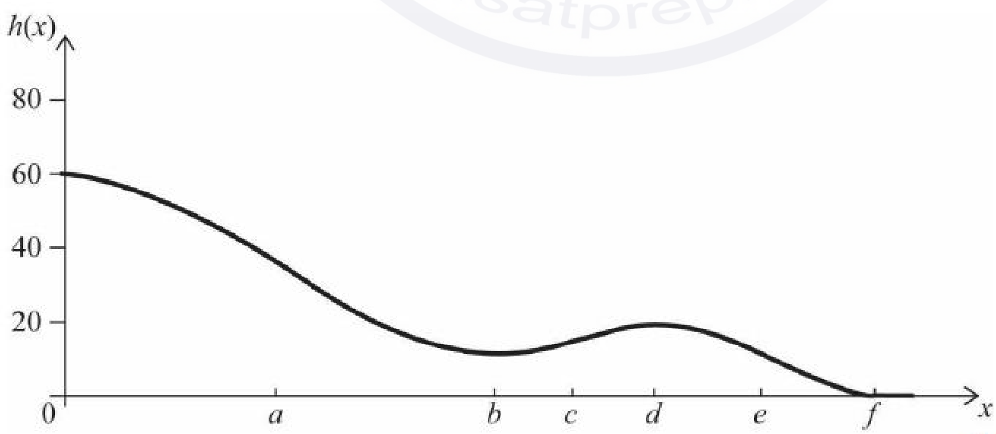
Question 11

(a) (i) a **A1**

(ii) the hill is at its steepest / largest slope of hill **A1**

[2 marks]

(b)



A1A1A1

[3 marks]
Total: [5 marks]

Question 12

substitute coordinates of A

$f(0) = p\,e^{q\cos(0)} = 6.5$

$6.5 = p\,e^{q}$ ***(A1)***

substitute coordinates of B

$f(5.2) = pe^{q\cos(5.2r)} = 0.2$

EITHER

$f'(t) = -pqr\sin(rt)\,e^{q\cos(rt)}$ ***(M1)***

minimum occurs when $-pqr\sin(5.2r)\,e^{q\cos(5.2r)} = 0$

$\sin(rt) = 0$

$r \times 5.2 = \pi$ ***(A1)***

OR

minimum value occurs when $\cos(rt) = -1$ ***(M1)***

$r \times 5.2 = \pi$ ***(A1)***

OR

period = $2 \times 5.2 = 10.4$ ***(A1)***

$r = \frac{2\pi}{10.4}$ ***(M1)***

THEN

$r = \frac{\pi}{5.2} = 0.604152... \ (0.604)$ ***A1***

$0.2 = p\,e^{-q}$ ***(A1)***

eliminate p or q ***(M1)***

$e^{2q} = \frac{6.5}{0.2}$ **OR** $0.2 = \frac{p^2}{6.5}$

$q = 1.74\ (1.74062...)$ ***A1***

$p = 1.14017...\ (1.14)$ ***A1***

[Total 8 marks]

Question 13

(a) use of power rule *(M1)*

$\frac{dW}{dv} = -1.1848v^{-0.84}$ **OR** $-1.18v^{-0.84}$ ***A1***

[2 marks]

(b) $\frac{dv}{dt} = 5$ *(A1)*

$\frac{dW}{dt} = \frac{dv}{dt} \times \frac{dW}{dv}$ *(M1)*

$\left(\frac{dW}{dt} = -5 \times 1.1848v^{-0.84}\right)$

when $v = 10$

$\frac{dW}{dt} = -5 \times 1.1848 \times 10^{-0.84}$ *(M1)*

-0.856 $(-0.856278\ldots)\,^{\circ}\mathrm{C\,min}^{-1}$ ***A2***

Note:	Accept a negative answer communicated in words, "decreasing at a rate of…". Accept a final answer of $-0.852809\ldots\,^{\circ}\mathrm{C\,min}^{-1}$ from use of -1.18. Accept 51.4 (or 51.2) $^{\circ}\mathrm{C\,hour}^{-1}$.

[5 marks]
[Total 7 marks]

Question 14

(a) (i) $\frac{1}{u^2} + \frac{2}{u} + 1$ ***A1***

(ii) $\int \left(\frac{1}{(x+2)} + 1\right)^2 dx$

$= \int \left(\frac{1}{(x+2)^2} + \frac{2}{x+2} + 1\right) dx$ **OR** $\int \left(\frac{1}{u^2} + \frac{2}{u} + 1\right) du$ *(M1)*

$= -\frac{1}{(x+2)} + 2\ln|x+2| + x\,(+c)$ ***A1A1***

[4 marks]

(b) volume $= \pi \left[-\frac{1}{(x+2)} + 2\ln(x+2) + x \right]_0$ **M1**

$= \pi \left(-\frac{1}{4} + 2\ln(4) + 2 + \frac{1}{2} - 2\ln 2 \right)$ **A1**

$= \pi \left(\frac{9}{4} + 2\ln(4) - 2\ln 2 \right)$

use of log laws seen, for example **M1**

$\pi \left(\frac{9}{4} + 4\ln(2) - 2\ln 2 \right)$ **OR** $\pi \left(\frac{9}{4} + 2\ln\left(\frac{4}{2}\right) \right)$

$= \frac{\pi}{4}(9 + 8\ln(2))$ **OR** $a = 9$, $b = 8$ and $c = 2$ **A1**

[4 marks]
Total [8 marks]

Question 15

(a) (i) use of product rule *(M1)*

$$\frac{dy}{dx} = 2\left(4 - e^x\right) + 2x\left(-e^x\right)$$ **A1**

$$= 8 - 2e^x - 2xe^x$$

(ii) use of product rule *(M1)*

$$\frac{d^2y}{dx^2} = -2e^x - 2e^x - 2xe^x$$ **A1**

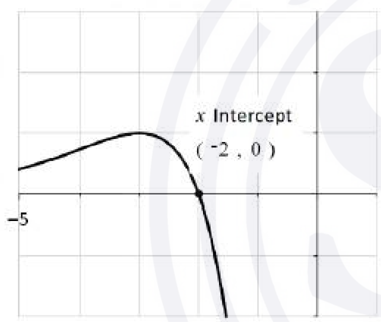
$$= -4e^x - 2xe^x$$

$$= -2(2+x)e^x$$

[4 marks]

(b) $-2(2+a)e^a = 0$ **OR** sketch of $\frac{d^2y}{dx^2}$ with x-intercept indicated

OR finding the local maximum of $\frac{dy}{dx}$ at $(-2,\ 8.27)$ *(M1)*



$(a =)\ -2$ **A1**

[2 marks]
[Total 6 marks]

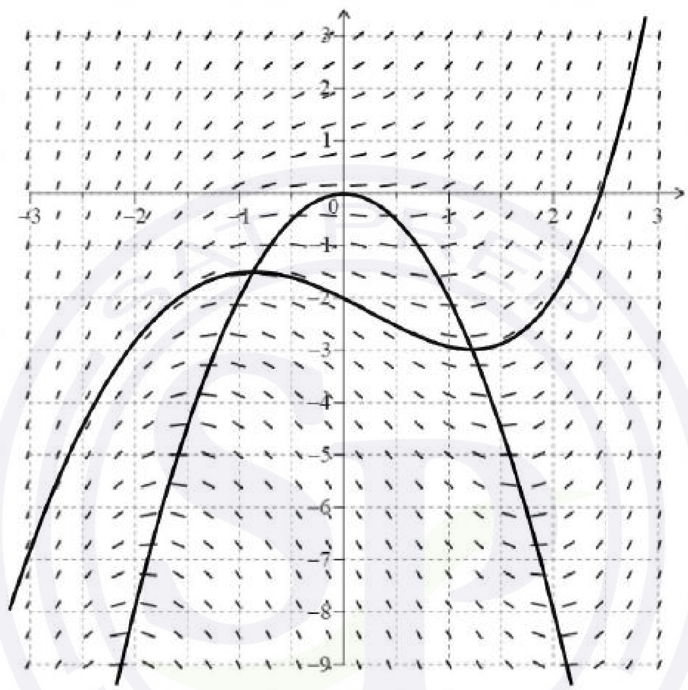
Question 16

(a) (i) $x^2 + \frac{y}{2} = 0 \ (y = -2x^2)$ **A1**

(ii) $y = -2x^2$ drawn on diagram (correct shape with a maximum at $(0, 0)$) **A1**

[2 marks]

(b)



correct shape with a local maximum and minimum, passing through $(0, -2)$ **A1**

local maximum and minimum on the graph of $y = -2x^2$ **A1**

[2 marks]
[Total 4 marks]

Question 17

(a) use of chain rule **(M1)**

$$\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt}$$

attempt to find $\frac{dy}{dx}$ at $x=1$ **(M1)**

$$0.2 = 0.04 \times \frac{dx}{dt}$$

$$\left(\frac{dx}{dt} = \right) 5 \text{ m h}^{-1}$$ ***A1***

[3 marks]

(b) (i) if the position of the snail is (X, Y)

from part (a) $\frac{dX}{dt} = \frac{1}{0.04X}\frac{dY}{dt}$

since speed is 1:

finding modulus of velocity vector and equating to 1 **(M1)**

$$1 = \sqrt{\left(\frac{\dot{Y}}{0.04X}\right)^2 + \dot{Y}^2} \quad \textbf{OR} \quad 1 = \sqrt{\dot{X}^2 + 0.0016X^2\dot{X}^2}$$

$$1 = \dot{Y}^2\left(\frac{1}{0.0016X^2} + 1\right) \quad \textbf{OR} \quad 1 = \dot{X}^2(1 + 0.0016X^2)$$

$$\dot{Y} = \sqrt{\frac{1}{\frac{1}{0.08Y} + 1}} \quad \textbf{OR} \quad \dot{X} = \sqrt{\frac{1}{1 + 0.0016X^2}}$$ **(A1)**

$$\int_{0.02}^{2} \sqrt{\frac{1}{0.08Y} + 1}\, dY = \int_0^T dt \quad \textbf{OR} \quad \int_1^{10} \sqrt{1 + 0.0016X^2}\, dX = \int_0^T dt$$ **(M1)**

$T = 9.26$ hours ***A1***

(ii) **EITHER**

time for water to reach top is $\frac{2}{0.2} = 10$ hours (seen anywhere) ***A1***

OR

or at time $t = 9.26$, height of water is $0.2 \times 9.26 = 1.852$ ***A1***

THEN

so the water will not reach the snail ***AG***

[5 marks]

Total [8 marks]

Question 18

(a) attempt at chain rule **(M1)**

$\left(v = \frac{\mathrm{d}OP}{\mathrm{d}t} =\right) \begin{pmatrix} 2t\cos t^2 \\ -2t\sin t^2 \end{pmatrix}$ **A1**

[2 marks]

(b) attempt at product rule **(M1)**

$a = \begin{pmatrix} 2\cos t^2 - 4t^2\sin t^2 \\ -2\sin t^2 - 4t^2\cos t^2 \end{pmatrix}$ **A1**

METHOD 1

let $S = \sin t^2$ and $C = \cos t^2$

finding cos θ using

$a \bullet \overrightarrow{\mathrm{OP}} = 2SC - 4t^2S^2 - 2SC - 4t^2C^2 = -4t^2$ **M1**

$|\overrightarrow{\mathrm{OP}}| = 1$

$|a| = \sqrt{\left(2C - 4t^2S\right)^2 + \left(-2S - 4t^2C\right)^2}$

$= \sqrt{4 + 16t^4} > 4t^2$

if θ is the angle between them, then

$\cos\theta = -\frac{4t^2}{\sqrt{4+16t^4}}$ **A1**

so $-1 < \cos\theta < 0$ therefore the vectors are never parallel **R1**

METHOD 2

solve

$\begin{pmatrix} 2\cos t^2 - 4t^2\sin t^2 \\ -2\sin t^2 - 4t^2\cos t^2 \end{pmatrix} = k\begin{pmatrix} \sin t^2 \\ \cos t^2 \end{pmatrix}$ **M1**

then

$(k =)\ \frac{2\cos t^2 - 4t^2\sin t^2}{\sin t^2} = \frac{-2\sin t^2 - 4t^2\cos t^2}{\cos t^2}$

Note: Condone candidates not excluding the division by zero case here. Some might go straight to the next line.

$2\cos^2 t^2 - 4t^2\cos t^2\sin t^2 = -2\sin^2 t^2 - 4t^2\cos t^2\sin t^2$

$2\cos^2 t^2 + 2\sin^2 t^2 = 0$

$2 = 0$ **A1**

this is never true so the two vectors are never parallel **R1**

$2\cos^2 t^2 - 4t^2 \cos t^2 \sin t^2 = -2\sin^2 t^2 - 4t^2 \cos t^2 \sin t^2$

$2\cos^2 t^2 + 2\sin^2 t^2 = 0$

$2 = 0$ ***A1***

this is never true so the two vectors are never parallel ***R1***

METHOD 3

embedding vectors in a 3d space and taking the cross product: ***M1***

$$\begin{pmatrix} \sin t^2 \\ \cos t^2 \\ 0 \end{pmatrix} \times \begin{pmatrix} 2\cos t^2 - 4t^2 \sin t^2 \\ -2\sin t^2 - 4t^2 \cos t^2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -2\sin^2 t^2 - 4t^2 \cos t^2 \sin t^2 - 2\cos^2 t^2 + 4t^2 \cos t^2 \sin t^2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix}$$ ***A1***

since the cross product is never zero, the two vectors are never parallel ***R1***

[5 marks]

Total [7 marks]

Question 19

$V = \pi \int_0^{10} y^2 \, dx$ **OR** $\pi \int_0^{10} x^2 \, dy$ ***(M1)***

$h = 2$

$\approx \pi \times \frac{1}{2} \times 2 \times \left(\left(4^2 + 5^2 \right) + 2 \times \left(6^2 + 8^2 + 7^2 + 3^2 \right) \right)$ ***M1A1***

$= 1120 \text{ cm}^3 \quad (1121.548\ldots)$ ***A1***

Note: Do not award the second ***M1*** If the terms are not squared.

Total [4 marks]

Question 20

(a) **EITHER**

attempt to substitute 3, 4 and 7 into area of a trapezoid formula ***(M1)***

$(A =)\ \frac{1}{2}(7+4)(3)$

OR

given area expressed as an integral ***(M1)***

$(A =)\ \int_{-1}^{2}(6-x)\ \mathrm{d}x$

OR

attempt to sum area of rectangle and area of triangle ***(M1)***

$(A =)\ 4 \times 3 + \frac{1}{2}\ (3)(3)$

THEN

16.5 (square units) ***A1***

[2 marks]

(b) (i) $(A =)\ \int_{-1}^{2} 1.5x^2 - 2.5x + 3\ \mathrm{d}x$ ***A1A1***

Note: Award ***A1*** for the limits $x=-1,\ x=2$ in correct location. Award ***A1*** for an integral of the quadratic function, $\mathrm{d}x$ must be included. Do not accept "y" in place of the function, given that two equations are in the question.

(ii) 9.75 (square units) ***A1***

[3 marks]

(c) $16.5 - 9.75$ ***(M1)***

6.75 (square units) ***A1***

[2 marks]

Total [7 marks]

Question 21

EITHER

$$q_{n+1} = q_n + 0.1\left(\frac{\mathrm{d}q}{\mathrm{d}t}\right)_n$$

$$\left(\frac{\mathrm{d}q}{\mathrm{d}t}\right)_{n+1} = \left(\frac{\mathrm{d}q}{\mathrm{d}t}\right)_n + 0.1\left(\frac{\mathrm{d}^2q}{\mathrm{d}t^2}\right)_n$$ ***(M1)***

OR

let $\frac{\mathrm{d}q}{\mathrm{d}t} = y$

$$q_{n+1} = q_n + 0.1y_n$$

$$y_{n+1} = y_n + 0.1\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)_n$$ ***(M1)***

THEN

EITHER

$$\frac{\mathrm{d}y}{\mathrm{d}t} = 200 - 5y - 20q$$ ***(A1)***

OR

$$\frac{\mathrm{d}^2q}{\mathrm{d}t^2} = 200 - 5\frac{\mathrm{d}q}{\mathrm{d}t} - 20q$$ ***(A1)***

THEN

evidence of using Euler's method (e.g.) ***(M1)***

0	1	8	140
0.1	1.8	22	54

maximum charge $= 12.7$ (Coulombs, at $t = 0.7$) ***A2***

ote: Award ***A0A1*** for a final answer of 10.8, from reading the value at $t = 1$.

Total [5 marks]

Question 22

(a)	attempt to use product rule	*(M1)*
	$a = 2t^2 \cos(t^2) + \sin(t^2)$	***A1***
		[2 marks]
(b)	graph of a	*(M1)*
	$126\ (\text{m s}^{-2})\ (125.699...)$	***A1***
		[2 marks]
(c)	attempt at integration by substitution or inspection	*(M1)*
	$s = -\frac{1}{2}\cos(t^2)\ (+c)$	***A1***
	($s = 0$ when $t = 0$) $\Rightarrow c = \frac{1}{2}$	***A1***
	$\left(s = -\frac{1}{2}\cos(t^2) + \frac{1}{2}\right)$	
		[3 marks]
(d)	$\cos(t^2) \le 1$	***A1***
	$-\frac{1}{2}\cos(t^2) \ge -\frac{1}{2}$	
	so $\frac{1}{2} - \frac{1}{2}\cos(t^2) \ge 0$	***R1***
	hence the particle never has a negative displacement.	***AG***

Note: Do not accept reasoning based on a sketch of the graph.

[2 marks]
Total [9 marks]

Question 23

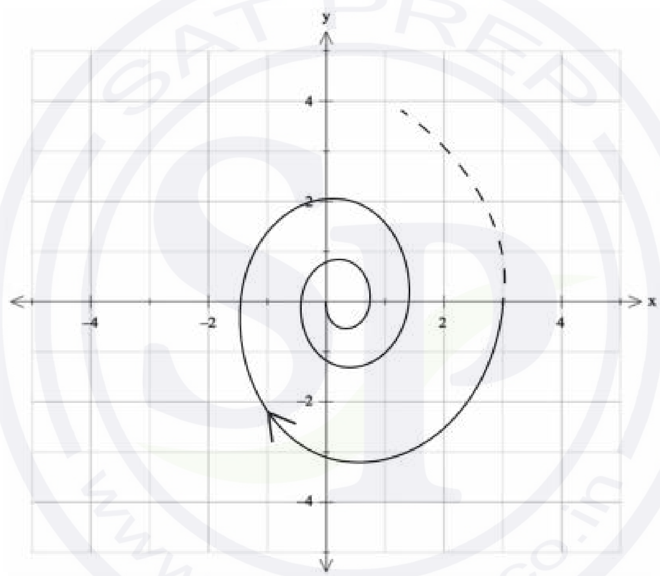
(a)

Description	Phase portrait
$\lambda_1 = 2$ with eigenvector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\lambda_2 = 3$ with eigenvector $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$	D
$\lambda_1 = 2$ with eigenvector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\lambda_2 = -3$ with eigenvector $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$	C
$\lambda_1 = -2$ with eigenvector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\lambda_2 = 3$ with eigenvector $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$	B

A1A1A1

[3 marks]

(b)



spiral (crossing x-axis at least twice), centre at origin ***A1***

arrow indicating clockwise, passing through or starting from $(3, 0)$ ***A1***

[2 marks]

Total [5 marks]