Subject – Math AI(Higher Level)
Topic - Function
Year - May 2021 – Nov 2024
Paper -1
Answers

Question 1

(a) (i) 1750 **A1**

(ii) $1350 + 400\,(1.25)^{-5}$ **(M1)**

$= 1480$ **A1**

Note: Accept 1481.

[3 marks]

(b) $1400 = 1350 + 400\,(1.25)^{-t}$ **(M1)**

9.32 (days $(9.31885\ldots)$ (days)) **A1**

[2 marks]

(c) 1350 **A1**

Note: Accept 1351 as a valid interpretation of the model as $P = 1350$ is an asymptote.

[1 mark]

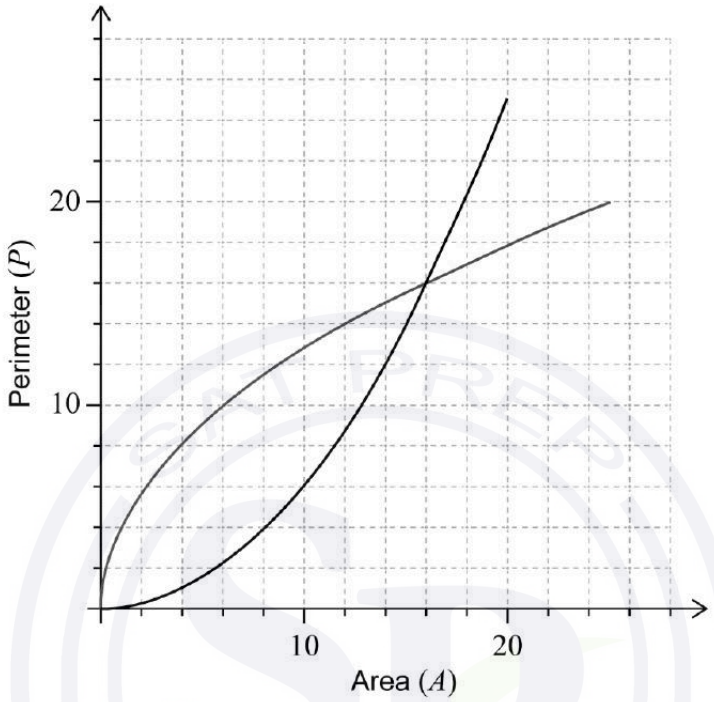
Total [6 marks]

Question 2

(a) 20 ***A1***

[1 mark]

(b)



(M1)A1A1

Note: Award ***(M1)*** for reflection in the line $P = A$, award ***A1*** for endpoint at $(20, 25)$, award ***A1*** for passing through $(16, 16)$.

[3 marks]

(c) when the perimeter is 8, the area is 4 ***A1***

[1 mark]

Total [5 marks]

Question 3

new function is $f(x-a)+b\left(=\ln(x-a)+b\right)$ ***(M1)***

$f(0)=\ln(-a)+b=1$ ***A1***

$f(\mathrm{e}^3)=\ln(\mathrm{e}^3-a)+b=1+\ln 2$ ***A1***

$\ln(-a)=\ln(\mathrm{e}^3-a)-\ln 2$ ***(M1)***

$$\ln(-a)=\ln\left(\frac{\mathrm{e}^3-a}{2}\right)$$

$$-a=\frac{\mathrm{e}^3-a}{2}$$

$$-2a=\mathrm{e}^3-a$$

$a=-\mathrm{e}^3\ \ (=-20.0855...)$ ***A1***

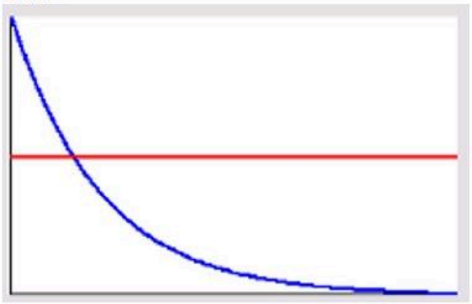
$b=1-\ln\mathrm{e}^3=1-3=-2$ ***(M1)A1***

Total [7 marks]

Question 4

(a) $50 = 100\,e^{-1\times p}$ **OR** $0.5 = e^{-1\times p}$ ***(M1)***

OR



(M1)

0.693 $(0.693147..., \ln 2)$ ***A1***

[2 marks]

(b) $R(1.5) = 100\,e^{-0.693147...\times 1.5}$ ***(M1)***

$35.4\,(\%)$ $(35.3553...)$ ***A1***

[2 marks]

(c) $R(t) > 0$ **OR** $R(t)$ has a horizontal asymptote ***R1***

[1 mark]

(d) *Award **A1** for **one** reasonable limitation of the domain:* ***A1***

small values of t produce unrealistic results
$R(0) = 100\%$
large values of t are not possible
people do not live forever
model is not valid at small or large values of t

The reason should focus on the domain $t \geq 0$. Do not accept answers such as:

recollection varies for different people
memories are discrete not continuous
the nature of the information will change how easily it is recalled
emotional/physical stress can affect recollection/concentration

Note: Do not accept $t \geq 0$ as this is a limitation that has been given in the question.

[1 mark]

Total [6 marks]

Question 5

(a) $\begin{pmatrix} 7 & -10 \\ 2 & -3 \end{pmatrix}\begin{pmatrix} 6 \\ -2 \end{pmatrix}+\begin{pmatrix} -5 \\ 4 \end{pmatrix}$ ***(M1)***

$=\begin{pmatrix} 57 \\ 22 \end{pmatrix}$ **OR** $(57,\ 22)$ ***A1***

[2 marks]

(b) $\begin{pmatrix} 2p \\ 2q \end{pmatrix}=\begin{pmatrix} 7 & -10 \\ 2 & -3 \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix}+\begin{pmatrix} -5 \\ 4 \end{pmatrix}$ ***(M1)***

$7p-10q-5=2p$

$2p-3q+4=2q$ ***(A1)***

solve simultaneously:

$p=13,\ q=6$ ***A1***

Note: Award ***A0*** if 13 and 6 are not labelled or are labelled the other way around.

[3 marks]

(c) $\det\begin{pmatrix} 7 & -10 \\ 2 & -3 \end{pmatrix}=-1$ $\left(\textbf{OR}\ \left|\det\begin{pmatrix} 7 & -10 \\ 2 & -3 \end{pmatrix}\right|=1\right)$ ***A1***

scale factor of image area is therefore $\left(\left|-1\right|=\right)1$ (and the translation does not affect the area)

A1

[2 marks]

Total [7 marks]

Question 6

(a) 3 ***A1***

Note: Accept $(3, 0)$ seen.

[1 mark]

(b) **METHOD 1**

$0 = 4a - 2b + c$, $0 = 9a + 3b + c$, $-\frac{25}{2} = \frac{1}{4}a + \frac{1}{2}b + c$ ***(M1)(A1)***

(i) 2 ***A1***

(ii) -2 ***A1***

(iii) -12 ***A1***

Note: Award the ***(M1)(A1)*** if at least one correct value is seen. Do not apply ***FT*** form part (a) if workings are not shown.

METHOD 2

$-12.5 = a(0.5 + 2)(0.5 - 3)$ ***(M1)***

(i) $a = 2$ ***A1***

$0 = 2x(3)^2 + 3b + c$

$0 = 2x(-2)^2 + (-2)b + c$ ***(M1)***

(ii) $b = -2$ ***A1***

(iii) $c = -12$ ***A1***

[5 marks]

(c) $x = 0.5$ ***A1***

Note: Do not ***FT*** from their part (b), this is a contradiction with the diagram.

[1 mark]

Total [7 marks]

Question 7

(a) $(f(-7)=)\ 8$ and $(f(7)=)\ 1$ ***(A1)***

range is $f(x)\leq 1,\ f(x)\geq 8$ ***A1A1***

Note: Award at most ***A1A1A0*** if strict inequalities are used.

[3 marks]

(b) interchanging x, y at any stage ***(A1)***

$$y=2-\frac{12}{x+5}$$

$$\frac{12}{x+5}=2-y$$

$$\frac{12}{2-y}=x+5$$ ***(A1)***

$$\frac{12}{2-y}-5=x$$

$$\left(f^{-1}(x)=\right)\ \frac{12}{2-x}-5\ \left(=\frac{2+5x}{2-x}\right)$$ ***A1***

[3 marks]

(c) range is $-7\leq f^{-1}(x)\leq 7,\ f^{-1}(x)\neq -5$ ***A1***

[1 mark]

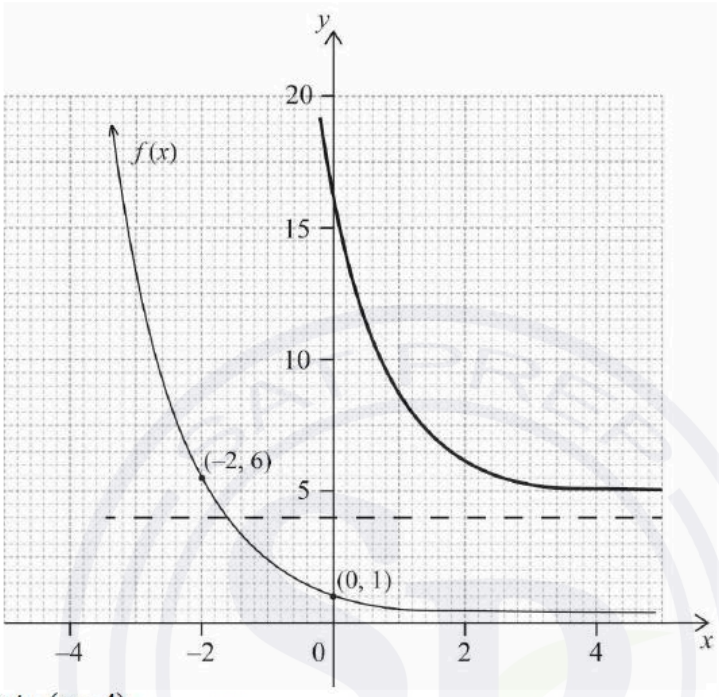
Total [7 marks]

Question 8

(a) $g(0) = 16$ **M1A1**

[2 marks]

(b)



y-asymptote $(y = 4)$ **A1**

concave up decreasing curve and passing through $(0, 16)$ **A1**

[2 marks]

Total: [4 marks]

Question 9

(a) $h(4)=\frac{640}{4^2}+0.5$ **OR** $h(14)=\frac{640}{14^2}+0.5$ ***(M1)***

Note: Award ***(M1)*** for substituting 4 or 14 into h. This can be implicit from seeing 3.77 $(3.76530\ldots)$ or 40.5.

$3.77 \le h(x) \le 40.5$ $(3.76530... \le h(x) \le 40.5)$ ***A1A1***

Note: Award ***A1*** for both correct endpoints seen, ***A1*** for the endpoints in a correct interval.

[3 marks]

(b) (i) $h(x)=10$ **OR** $h^{-1}(x)=\sqrt{\frac{640}{x-0.5}}$ **OR** $h^{-1}(10)=\sqrt{\frac{640}{10-0.5}}$ ***(M1)***

$(x=)$ 8.21 cm $(8.20782...)$ ***A1***

(ii) a tin that is 10 cm high will have a diameter of 8.21 cm $(8.20782...)$ ***A1***

Note: Condone a correct answer expressed as the converse.

(iii) $4 \le h^{-1} \le 14$ ***A1***

Note: Accept $4 \le y \le 14$. Do not ***FT*** in this part.

[4 marks]
Total: [7 marks]

Question 10

(a) 1.2 metres ***A1***

[1 mark]

(b) $-4.8t^2+21t+1.2=0$ ***(M1)***

$(t=)$ 4.43 s $(4.431415...\ s)$ ***A1***

Note: If both values for t are seen do not award the ***A1*** mark unless the negative is explicitly excluded.

[2 marks]

(c) $0 \le t \le 4.43$ **OR** $[0,\ 4.43]$ ***A1A1***

Note: Award ***A1*** for correct endpoints and ***A1*** for expressing answer with correct notation. Award at most ***A1A0*** for use of x instead of t.

[2 marks]
[Total 5 marks]

Question 11

(a) $y = \ln\left(\frac{1}{x-2}\right)$

an attempt to isolate x (or y if switched) ***(M1)***

$e^y = \frac{1}{x-2}$

$x - 2 = e^{-y}$

$x = e^{-y} + 2$

switching x and y (seen anywhere) ***M1***

$f^{-1}(x) = e^{-x} + 2$ ***A1***

[3 marks]

(b) sketch of $f(x)$ and $f^{-1}(x)$ ***(M1)***

$x = 2.12 \ \ (2.12002\ldots)$ ***A1***

[2 marks]

Total [5 marks]

Question 12

(a) $y = -0.00855x^3 - 0.234x^2 - 0.225x + 3.20$ ***A2***

$(y = -0.00854819...x^3 - 0.234002...x^2 - 0.224884...x + 3.20056...)$

Note: Award ***A0A1*** for at least two terms correct.

[2 marks]

(b) $y(2x)$ (for horizontal stretch) ***(A1)***

attempt to stretch vertically by factor $\frac{1}{2}$ ***(M1)***

$y = 0.0332x^3 - 0.15x^2 - 0.58x \ \ (+1.1)$ ***A1***

Note: Award ***A0M1A0*** for a vertical stretch, factor 2. Although a d value of 1.1 is preferred, technically this value can be wrong/omitted and the question is still answered (hence it is presented in brackets).

[3 marks]

Total [5 marks]

Question 13

(a) $71e^{-0.0514(16)} + 23$ ***(M1)***

$54.2\ ^{\circ}\text{C}\ (54.1956...)$ ***A1***

[2 marks]

(b) $23\ ^{\circ}\text{C}$ ***A1***

[1 mark]

(c) $50 = 71e^{-0.0514(k)} + 23$ ***(M1)***

$k = 18.8\ \left(\frac{-5000}{257}\ln\left(\frac{27}{71}\right), 18.8101...\right)$ ***A1***

Note: Award ***M1*** for a sketch showing a point of intersection between the exponential function and $y = 50$.

[2 marks]

Total [5 marks]

Question 14

(a) (i) **METHOD 1**

attempt to find change in height of the ball using gradient ***(M1)***

$\frac{a}{0.43} = (-)0.045$

$a = (-)0.045 \times 0.43$

$a = (-)0.0194\,(\text{m})\ \ (0.01935\ (\text{m}))$ ***A1***

METHOD 2

attempt to find height at back of home plate ***(M1)***

horizontal distance to the front of the home plate $= 16.6666\ldots\ (\text{m})$

height at the back of the home plate $= -0.045(16.6666\ldots + 0.43) + 2$

$(= 1.23065\ (\text{m}))$

Note: The ***M1*** can be awarded for $16.6666\ldots + 0.43$ seen at some point.

$(a = 1.25 - 1.23065\ldots)$

$(a =)\ (-)0.0194\ (\text{m})\ \ (0.01935\ (\text{m}))$ ***A1***

(ii) $1.25 - 0.01935 = 1.23065$ (may be seen in part (a)(i)) ***A1***

$0.53 < 1.23065 < 1.24$ ***R1***

therefore a strike ***AG***

Note: Do not award ***A0R1***.

[4 marks]

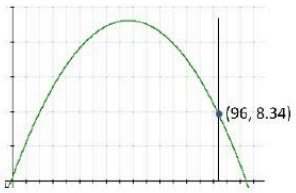
(b) **METHOD 1**

indication of $d = 96$ in the function $h(d)$ or its graph ***(M1)***

EITHER

$(h(96) =) -0.01(96)^2 + 1.04(96) + 0.66$

OR



THEN

$(h(96) =)\ 8.34\ (\text{m})$ ***A1***

$8.34 > 5$ so the ball will go over the wall. ***A1***

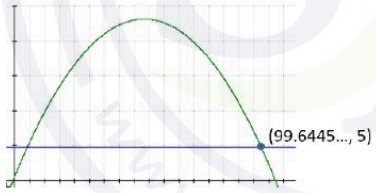
METHOD 2

indication of $h = 5$ in the function $h(d)$ or its graph ***(M1)***

EITHER

$5 = -0.01d^2 + 1.04d + 0.66$

OR



THEN

$d = 99.6\ (\text{m})\ (99.6445\ldots\ (\text{m}))\ (d = 4.35548\ldots\ (\text{m})$ may also be seen) ***A1***

$96 < 99.6445\ldots$ so the ball will go over the wall. ***A1***

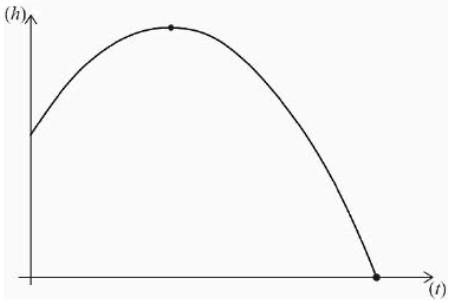
[3 marks]

[Total: 7 marks]

Question 15

(a) **METHOD 1**

correct sketch with some indication of maximum point ***(M1)***



0.921 (seconds) $\left(0.921052..., \frac{35}{38}\right)$ ***A1***

METHOD 2

correct substitution into equation for line of symmetry ***(M1)***

$(t =) \ -\frac{8.75}{2\times -4.75}$

0.921 (seconds) $\left(0.921052..., \frac{35}{38}\right)$ ***A1***

METHOD 3

equating the correct derivative to 0 ***(M1)***

$-9.5t + 8.75 = 0$

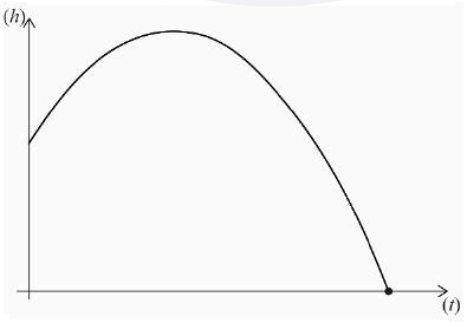
0.921 (seconds) $\left(0.921052..., \frac{35}{38}\right)$ ***A1***

Note: Award ***M1A0*** for a final answer of 0.92 seen with no working.

[2 marks]

(b) **METHOD 1**

correct sketch with some indication of x-intercept



(M1)

Note: May be seen in part (a).

2 (seconds) ***A1***

METHOD 2
setting the equation to zero *(M1)*

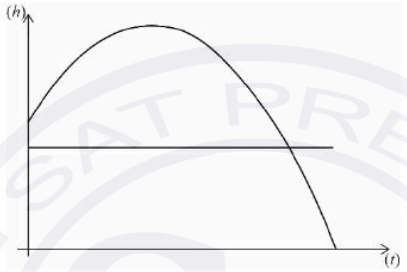
$-4.75t^2 + 8.75t + 1.5 = 0$

2 (seconds) *A1*

Note: If both roots are given, with or without working, award ***(M1)A0***.

[2 marks]

(c) **METHOD 1**
correct sketch of quadratic function and a straight line in approximate correct position *(M1)*



1.88 (seconds) (1.87577…(seconds)) *A1*

METHOD 2
setting the equation equal to 1.2 *(M1)*
$-4.75t^2 + 8.75t + 1.5 = 1.2$
1.88 (seconds) (1.87577…(seconds)) *A1*

Note: Award ***(M1)A0*** if -0.0336702… seen as (part of) a final answer.
Award ***M1A0*** for answer of 1.9 seen without working.

[2 marks]

(d) *Award **R1** for a sensible reason in the context of the question:* ***R1***
e.g.
The model ignores air resistance (or wind)
The model treats the ball as a point
The model assumes gravity is constant
The model assumes that the ball continues to follow the trajectory even after hitting the ground
This model ignores the bouncing back of the ball after hitting the ground

Note: Do not accept generic criticisms of any mathematical model, such as:
There are assumptions being made
Models are never accurate / It is only a model

[1 mark]

Total [7 marks]

Question 16

(a) $2.36 = a(3)^2 + b(3) + c$ **OR** $2.36 = 9a + 3b + c$ ***A1***

[1 mark]

(b) finding other equations to solve simultaneously ***(M1)***

$5 = a(10)^2 + b(10) + c$ **AND** $7.16 = a(17)^2 + b(17) + c$
OR $5 = 100a + 10b + c$ **AND** $7.16 = 289a + 17b + c$

any one coefficient in equation correct ***(A1)***
$f(x) = -0.00490x^2 + 0.441x + 1.08$ ***A1***

$\left(f(x) = -0.00489795\ldots x^2 + 0.440816\ldots x + 1.08163\ldots\right)$
$\left(f(x) = -\frac{6}{1225}x^2 + \frac{108}{245}x + \frac{53}{49}\right)$

Note: Award at most ***(M1)(A1)A0*** if answer is not expressed as an equation.

[3 marks]

(c) attempt to substitute 80 into their equation ***(M1)***

$\left(f(80) =\right) 5$ ***A1***

$5 > 4$ **OR** therefore the ball will go over the fence ***R1***

Note: Do not award ***A0R1***; their value must be seen to credit a correct conclusion.

[3 marks]

(d) setting their equation equal to zero, graph ***(M1)***

$0 = -0.00489795\ldots x^2 + 0.440816\ldots x + 1.08163\ldots$ **OR** $f(x) = 0$

92.4 (92.3902…) (m) ***A1***

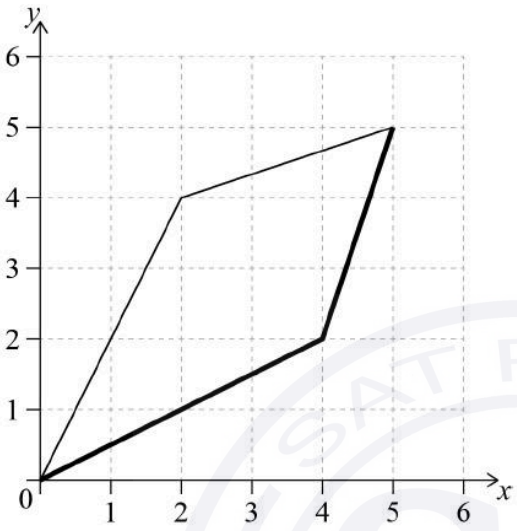
[2 marks]
[Total 9 marks]

Question 17

(a) 4 ***A1***

[1 mark]

(b)



A1A1

Note: Award ***A1*** for passing through $(0, 0)$ and $(4, 2)$,
A1 for passing through $(4, 2)$ and $(5, 5)$.

[2 marks]

(c) attempt to solve $y = 3x - 1$ for x **OR** changing variables ***(M1)***

$\left(g^{-1}(x)\right) = \frac{x+1}{3}$ ***A1***

[2 marks]

(d) sketch of $g(x)$ or $g^{-1}(x)$, algebraic approach ***(M1)***

$\frac{1}{2}x = \frac{x+1}{3}$

$(x =)\ 2$ ***A1***

[2 marks]
[Total 7 marks]

Question 18

(a) correct substitution of 0.0003 into the formula **(A1)**

$\text{pH} = -\log_{10}(0.0003)$

$= 3.52 \quad (3.52287\ldots)$ **A1**

[2 marks]

(b) **EITHER**

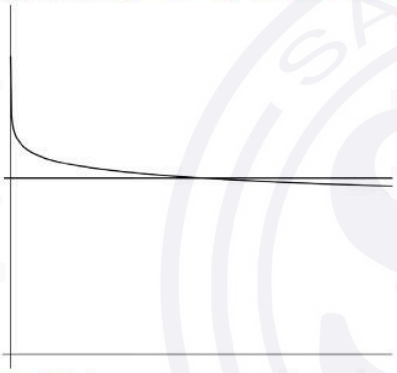
attempt to change to exponential form **(M1)**

$\left[\text{H}^+\right] = 10^{-6.6}$

OR

attempt to solve $6.6 = -\log_{10}\left[\text{H}^+\right]$

(graphically or using numerical solver)



THEN

$\left[\text{H}^+\right]$ = 0.000000251 (moles per litre) $\quad (0.000000251188..., 2.51\times10^{-7})$ **A1**

Note: Award ***M1A0*** for an answer of 2.51 $(2.51188...)$ seen.
Award ***M0A0*** if a substitution of 6.6 into the formula is seen without an answer or some indication of using numerical solver.

[2 marks]

(c) $2 = -\log_{10}\left[\text{H}^+\right],\ 4.5 = -\log_{10}\left[\text{H}^+\right]$

$10^{-2} \quad (0.01)$ **OR** $10^{-4.5} \quad (0.0000316227...)$ **(A1)**

substitution of their values into correct ratio **(M1)**

$\dfrac{10^{-2}}{10^{-4.5}}$ **OR** $\dfrac{0.01}{0.0000316227...}$

$= 316.227... = 316$ **A1**

Note: Some candidates may subtract logs and hence look to solve $\log_{10}[\text{H}^+] = 2.5$.

[3 marks]

[Total: 7 marks]

Question 19

(a) $x_B = 4\sqrt{2(t-3)} \quad (t \geq 3)$ ***A1A1***

Note: Award ***A1*** for multiplying by 2 and ***A1*** for $t-3$. Award ***A1A0*** for $4\sqrt{2t-3}$.

[2 marks]

(b) equating their x_B to $4\sqrt{t}$ ***(M1)***

$4\sqrt{2(t-3)} = 4\sqrt{t}$

$t = 6$ (seconds) ***A1***

Note: Do not ***FT*** from part (a) to part (b).

[2 marks]
[Total 4 marks]

Question 20

attempt to substitute $t = 9$ and $P = 433$ (**OR** $t = 12$ and $P = 954$) ***(M1)***

$433 = \frac{L}{1 + Ce^{-0.28(9)}}$ and $954 = \frac{L}{1 + Ce^{-0.28(12)}}$ ***(A1)(A1)***

attempt to rearrange into linear form or substitute for L

$433\left(1 + Ce^{-0.28(9)}\right) = L$ and $954\left(1 + Ce^{-0.28(12)}\right) = L$ **OR** $954 = \frac{433\left(1 + Ce^{-0.28(9)}\right)}{1 + Ce^{-0.28(12)}}$ ***(M1)***

attempt to solve simultaneously (e.g. graph or eliminate L) ***(M1)***

$C = 306.187\ldots$

$L = 11100 \ (11100.2\ldots)$ ***A1***

Note: Accept a correct decimal value.

[6 marks]

Question 21

(a) vertical stretch scale factor 4 ***A1***

Note: The vertical stretch can occur in any order.

EITHER

translation $\begin{pmatrix}3\\0\end{pmatrix}$ followed by horizontal stretch scale factor $\frac{1}{2}$ ***A1A1***

OR

horizontal stretch scale factor $\frac{1}{2}$ followed by translation $\begin{pmatrix}1.5\\0\end{pmatrix}$ ***A1A1***

Note: Award ***A1A0*** in either approach, for correct transformations, but incorrect order.

[3 marks]

(b) $g(2)=5$ ***(A1)***

$f(5)=7.78 \quad (=7.78364\ldots,\ 4\ln 7)$ ***A1***

[2 marks]

(c) $4\ln(2g(x)-3)=2\ln 9$ ***(M1)***

$g(x)=3$ ***(A1)***

$x=1$ ***A1***

[3 marks]

[Total 8 marks]

Question 22

(a) 120 (ng) ***A1***

Note: Only accept an exact answer of 120.

[1 mark]

(b) $(m(20\,000)=)\ 120\,e^{-0.000121\times 20\,000}$ ***(A1)***

$10.7\ (10.6705\ldots..)$ (ng) ***A1***

[2 marks]

(c) attempt to write or solve an inequality (or equality) with a value of 60 seen ***(M1)***

$(m(t)=)120\,e^{-0.000121t}\leq 60$ **OR** sketch **OR** table of values

$(t>)\ 5728.489\ldots.$ ***(A1)***

5729 (years) ***A1***

Note: Do not accept 5728 or 5730 for the final ***A1***.

[3 marks]

[Total: 6 marks]