

Subject - Math AI(Higher Level)
Topic - Function
Year - May 2021 - Nov 2022
Paper -1
Answers

Question 1

(a) (i) 1750

A1

(ii) $1350 + 400(1.25)^{-5}$

(M1)

$= 1480$

A1

Note: Accept 1481.

[3 marks]

(b) $1400 = 1350 + 400(1.25)^{-t}$
9.32 (days (9.31885...)) (days))

(M1)

A1

[2 marks]

(c) 1350

A1

Note: Accept 1351 as a valid interpretation of the model as $P=1350$ is an asymptote.

[1 mark]

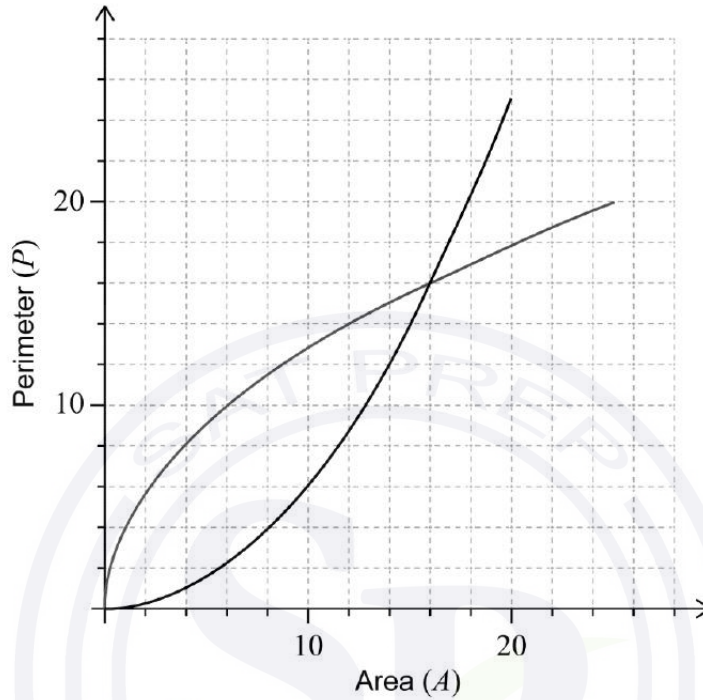
Total [6 marks]

Question 2

(a) 20

A1
[1 mark]

(b)



(M1)A1A1

Note: Award **(M1)** for reflection in the line $P = A$, award **A1** for endpoint at $(20, 25)$, award **A1** for passing through $(16, 16)$.

[3 marks]

(c) when the perimeter is 8, the area is 4

A1
[1 mark]

Total [5 marks]

Question 3

new function is $f(x-a)+b (= \ln(x-a)+b)$

(M1)

$$f(0) = \ln(-a) + b = 1$$

A1

$$f(e^3) = \ln(e^3 - a) + b = 1 + \ln 2$$

A1

$$\ln(-a) = \ln(e^3 - a) - \ln 2$$

(M1)

$$\ln(-a) = \ln\left(\frac{e^3 - a}{2}\right)$$

$$-a = \frac{e^3 - a}{2}$$

$$-2a = e^3 - a$$

$$a = -e^3 \quad (= -20.0855\dots)$$

A1

$$b = 1 - \ln e^3 = 1 - 3 = -2$$

(M1)A1

Total [7 marks]

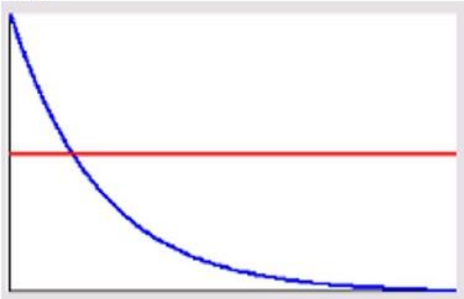


Question 4

(a) $50 = 100e^{-1 \times p}$ OR $0.5 = e^{-1 \times p}$

(M1)

OR



(M1)

0.693 (0.693147..., $\ln 2$)

A1

[2 marks]

(b) $R(1.5) = 100e^{-0.693147... \times 1.5}$

(M1)

35.4(%) (35.3553...)

A1

[2 marks]

(c) $R(t) > 0$ OR $R(t)$ has a horizontal asymptote

R1

[1 mark]

(d) Award **A1** for **one** reasonable limitation of the domain:

A1

small values of t produce unrealistic results

$R(0) = 100\%$

large values of t are not possible

people do not live forever

model is not valid at small or large values of t

The reason should focus on the domain $t \geq 0$. Do not accept answers such as:

recollection varies for different people

memories are discrete not continuous

the nature of the information will change how easily it is recalled

emotional/physical stress can affect recollection/concentration

Note: Do not accept $t \geq 0$ as this is a limitation that has been given in the question.

[1 mark]

Total [6 marks]

Question 5

$$(a) \begin{pmatrix} 7 & -10 \\ 2 & -3 \end{pmatrix} \begin{pmatrix} 6 \\ -2 \end{pmatrix} + \begin{pmatrix} -5 \\ 4 \end{pmatrix} \\ = \begin{pmatrix} 57 \\ 22 \end{pmatrix} \text{ OR } (57, 22)$$

(M1)

A1

[2 marks]

$$(b) \begin{pmatrix} 2p \\ 2q \end{pmatrix} = \begin{pmatrix} 7 & -10 \\ 2 & -3 \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} + \begin{pmatrix} -5 \\ 4 \end{pmatrix} \\ 7p - 10q - 5 = 2p \\ 2p - 3q + 4 = 2q \\ \text{solve simultaneously:} \\ p = 13, q = 6$$

(M1)

(A1)

A1

Note: Award A0 if 13 and 6 are not labelled or are labelled the other way around.

[3 marks]

$$(c) \det \begin{pmatrix} 7 & -10 \\ 2 & -3 \end{pmatrix} = -1 \left(\text{OR } \left| \det \begin{pmatrix} 7 & -10 \\ 2 & -3 \end{pmatrix} \right| = 1 \right)$$

A1

scale factor of image area is therefore $(|-1| =) 1$ (and the translation does not affect the area)

A1

[2 marks]

Total [7 marks]

Question 6

(a) 3

A1

Note: Accept (3, 0) seen.

[1 mark]

(b) **METHOD 1**

$$0 = 4a - 2b + c, \quad 0 = 9a + 3b + c, \quad -\frac{25}{2} = \frac{1}{4}a + \frac{1}{2}b + c$$

(M1)(A1)

(i) 2

A1

(ii) -2

A1

(iii) -12

A1

Note: Award the (M1)(A1) if at least one correct value is seen.
Do not apply FT from part (a) if workings are not shown.

METHOD 2

$$-12.5 = a(0.5 + 2)(0.5 - 3)$$

(M1)

(i) $a = 2$

A1

$$0 = 2x(3)^2 + 3b + c$$

$$0 = 2x(-2)^2 + (-2)b + c$$

(M1)

(ii) $b = -2$

A1

(iii) $c = -12$

A1

[5 marks]

(c) $x = 0.5$

A1

Note: Do not FT from their part (b), this is a contradiction with the diagram.

[1 mark]

Total [7 marks]

Question 7

- (a) $(f(-7)=) 8$ and $(f(7)=) 1$
range is $f(x) \leq 1, f(x) \geq 8$

(A1)

A1A1

Note: Award at most **A1A1A0** if strict inequalities are used.

- (b) interchanging x, y at any stage

[3 marks]

(A1)

$$y = 2 - \frac{12}{x+5}$$

$$\frac{12}{x+5} = 2 - y$$

$$\frac{12}{2-y} = x+5$$

$$\frac{12}{2-y} - 5 = x$$

$$(f^{-1}(x)) = \frac{12}{2-x} - 5 \left(= \frac{2+5x}{2-x} \right)$$

(A1)

A1

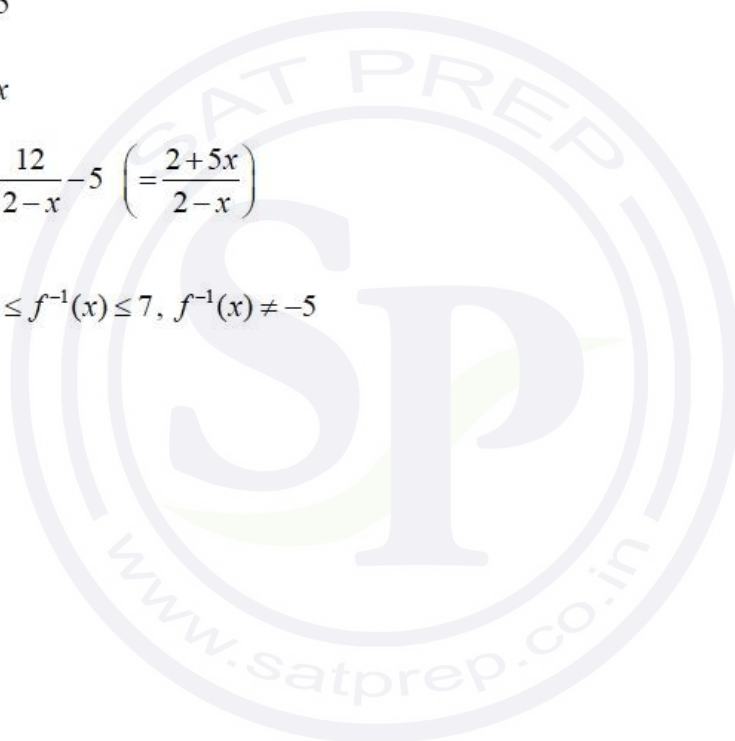
[3 marks]

- (c) range is $-7 \leq f^{-1}(x) \leq 7, f^{-1}(x) \neq -5$

A1

[1 mark]

Total [7 marks]



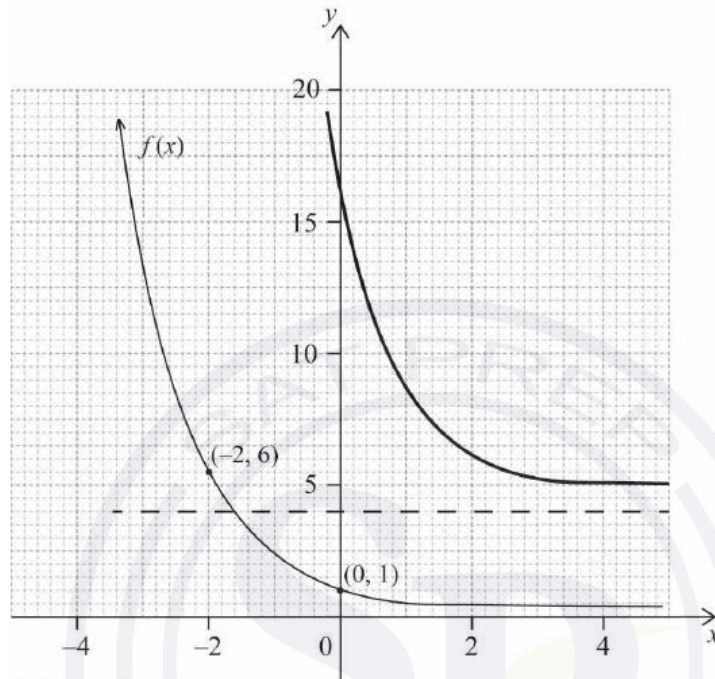
Question 8

(a) $g(0) = 16$

M1A1

[2 marks]

(b)



y-asymptote ($y = 4$)

concave up decreasing curve and passing through $(0, 16)$

A1

A1

[2 marks]

Total: [4 marks]

Question 9

(a) $h(4) = \frac{640}{4^2} + 0.5$ OR $h(14) = \frac{640}{14^2} + 0.5$ (M1)

Note: Award (M1) for substituting 4 or 14 into h . This can be implicit from seeing 3.77 (3.76530...) or 40.5.

$3.77 \leq h(x) \leq 40.5$ (3.76530... $\leq h(x) \leq 40.5$) A1A1

Note: Award A1 for both correct endpoints seen, A1 for the endpoints in a correct interval.

[3 marks]

(b) (i) $h(x) = 10$ OR $h^{-1}(x) = \sqrt{\frac{640}{x-0.5}}$ OR $h^{-1}(10) = \sqrt{\frac{640}{10-0.5}}$ (M1)
(x =) 8.21 cm (8.20782...) A1

(ii) a tin that is 10 cm high will have a diameter of 8.21 cm (8.20782...) A1

Note: Condone a correct answer expressed as the converse.

(iii) $4 \leq h^{-1} \leq 14$ A1

Note: Accept $4 \leq y \leq 14$. Do not FT in this part.

[4 marks]
Total: [7 marks]

Question 10

(a) 1.2 metres A1

[1 mark]

(b) $-4.8t^2 + 21t + 1.2 = 0$ (M1)
(t =) 4.43 s (4.431415... s) A1

Note: If both values for t are seen do not award the A1 mark unless the negative is explicitly excluded.

[2 marks]

(c) $0 \leq t \leq 4.43$ OR $[0, 4.43]$ A1A1

Note: Award A1 for correct endpoints and A1 for expressing answer with correct notation. Award at most A1A0 for use of x instead of t .

[2 marks]
[Total 5 marks]

Question 11

(a) $y = \ln\left(\frac{1}{x-2}\right)$

an attempt to isolate x (or y if switched)

$$e^y = \frac{1}{x-2}$$

$$x-2 = e^{-y}$$

$$x = e^{-y} + 2$$

switching x and y (seen anywhere)

$$f^{-1}(x) = e^{-x} + 2$$

(M1)

M1

A1

[3 marks]

(b) sketch of $f(x)$ and $f^{-1}(x)$

$$x = 2.12 \text{ (2.12002...)}$$

(M1)

A1

[2 marks]

Total [5 marks]

Question 12

(a) $y = -0.00855x^3 - 0.234x^2 - 0.225x + 3.20$

$$(y = -0.00854819...x^3 - 0.234002...x^2 - 0.224884...x + 3.20056...)$$

A2

Note: Award **A0A1** for at least two terms correct.

[2 marks]

(b) $y(2x)$ (for horizontal stretch)

(A1)

attempt to stretch vertically by factor $\frac{1}{2}$

(M1)

$$y = 0.0332x^3 - 0.15x^2 - 0.58x \text{ (+1.1)}$$

A1

Note: Award **A0M1A0** for a vertical stretch, factor 2. Although a d value of 1.1 is preferred, technically this value can be wrong/omitted and the question is still answered (hence it is presented in brackets).

[3 marks]

Total [5 marks]

Question 13

(a) $71e^{-0.0514(16)} + 23$

$54.2 \text{ } ^\circ\text{C}$ (54.1956...)

(M1)

A1

[2 marks]

(b) $23 \text{ } ^\circ\text{C}$

A1

[1 mark]

(c) $50 = 71e^{-0.0514(k)} + 23$

$k = 18.8 \left(\frac{-5000}{257} \ln\left(\frac{27}{71}\right), 18.8101\dots \right)$

(M1)

A1

Note: Award **M1** for a sketch showing a point of intersection between the exponential function and $y = 50$.

[2 marks]

Total [5 marks]

