Subject – Math AI(Higher Level)
Topic - Geometry and Trigonometry
Year - May 2021 – Nov 2024
Paper -1
Questions

Question 1

[Maximum mark: 6]

A particle P moves with velocity $\boldsymbol{v} = \begin{pmatrix} -15 \\ 2 \\ 4 \end{pmatrix}$ in a magnetic field, $\boldsymbol{B} = \begin{pmatrix} 0 \\ d \\ 1 \end{pmatrix}$, $d \in \mathbb{R}$.

(a) Given that $\boldsymbol{v}$ is perpendicular to $\boldsymbol{B}$, find the value of d. [2]

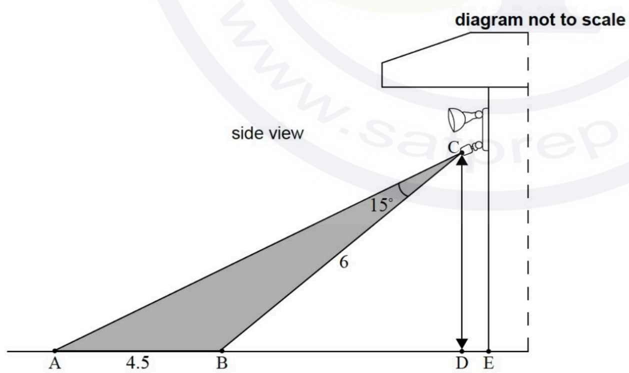
The force, $\boldsymbol{F}$, produced by P moving in the magnetic field is given by the vector equation $\boldsymbol{F} = a\boldsymbol{v} \times \boldsymbol{B}$, $a \in \mathbb{R}^+$.

(b) Given that $|\boldsymbol{F}| = 14$, find the value of a. [4]

Question 2

[Maximum mark: 8]

Ollie has installed security lights on the side of his house that are activated by a sensor. The sensor is located at point C directly above point D. The area covered by the sensor is shown by the shaded region enclosed by triangle ABC. The distance from A to B is 4.5 m and the distance from B to C is 6 m. Angle $\text{A}\hat{\text{C}}\text{B}$ is $15°$.



(a) Find $\text{C}\hat{\text{A}}\text{B}$. [3]

Point B on the ground is 5 m from point E at the entrance to Ollie's house. He is 1.8 m tall and is standing at point D, below the sensor. He walks towards point B.

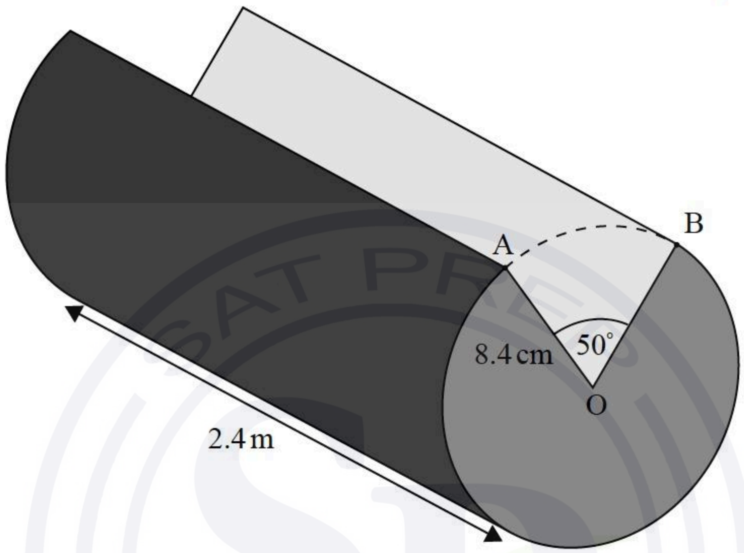
(b) Find the distance Ollie is **from the entrance to his house** when he first activates the sensor. [5]

Question 3

[Maximum mark: 5]

Helen is building a cabin using cylindrical logs of length $2.4\,\text{m}$ and radius $8.4\,\text{cm}$. A wedge is cut from one log and the cross-section of this log is illustrated in the following diagram.

diagram not to scale



(a) Find 50° in radians. [1]

(b) Find the volume of this log. [4]

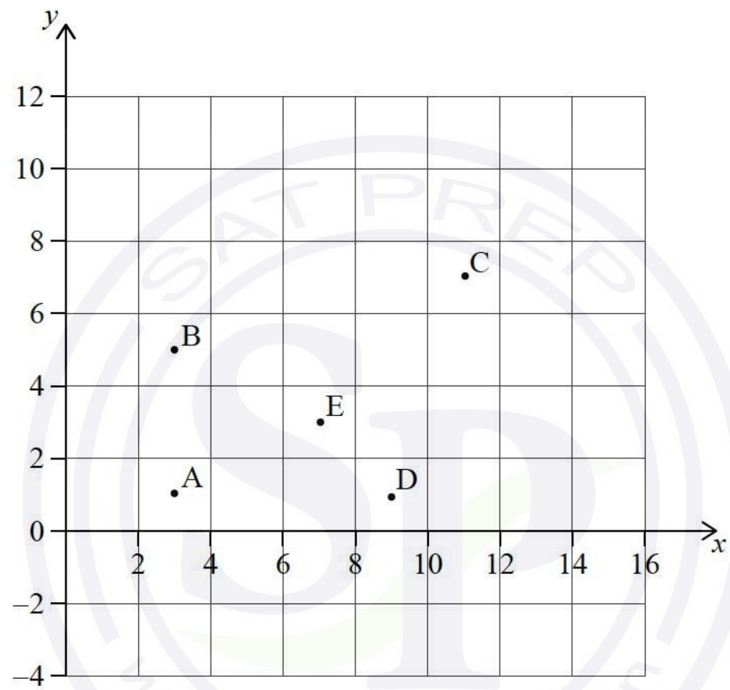
Question 4

[Maximum mark: 6]

Points $A(3, 1)$, $B(3, 5)$, $C(11, 7)$, $D(9, 1)$ and $E(7, 3)$ represent snow shelters in the Blackburn National Forest. These snow shelters are illustrated in the following coordinate axes.

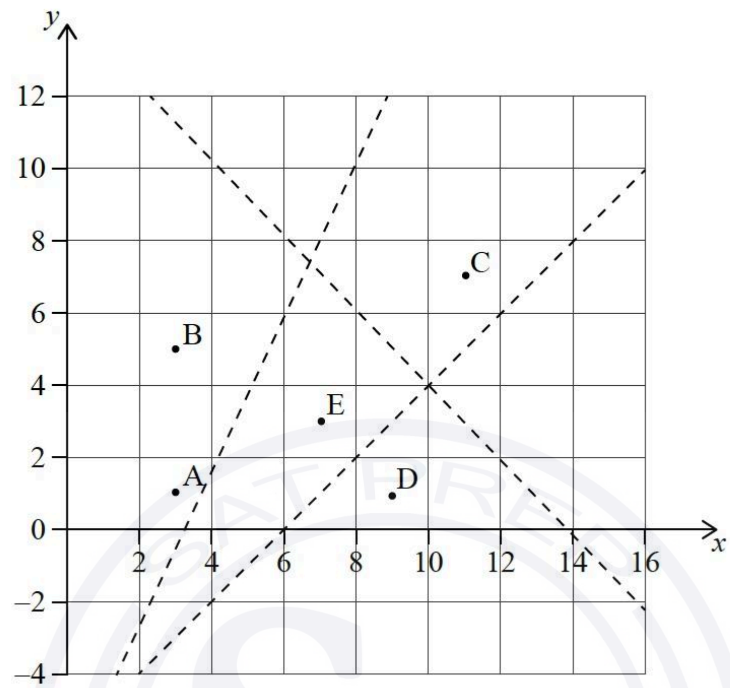
Horizontal scale: 1 unit represents 1 km.

Vertical scale: 1 unit represents 1 km.



(a) Calculate the gradient of the line segment AE. [2]

The Park Ranger draws three straight lines to form an incomplete Voronoi diagram.



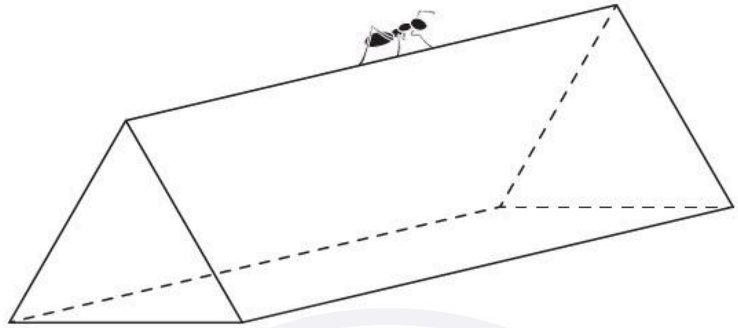
(b) Find the equation of the line which would complete the Voronoi cell containing site E. Give your answer in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$. [3]

(c) In the context of the question, explain the significance of the Voronoi cell containing site E. [1]

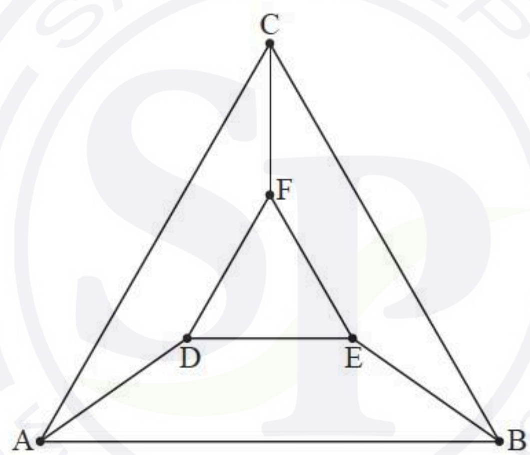
Question 5

[Maximum mark: 5]

An ant is walking along the edges of a wire frame in the shape of a triangular prism.



The vertices and edges of this frame can be represented by the graph below.



(a) Write down the adjacency matrix, M, for this graph. [3]

(b) Find the number of ways that the ant can start at the vertex A, and walk along exactly 6 edges to return to A. [2]

Question 6

[Maximum mark: 7]

A submarine is located in a sea at coordinates $(0.8, 1.3, -0.3)$ relative to a ship positioned at the origin O. The x direction is due east, the y direction is due north and the z direction is vertically upwards.

All distances are measured in kilometres.

The submarine travels with direction vector $\begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix}$.

(a) Assuming the submarine travels in a straight line, write down an equation for the line along which it travels. [2]

The submarine reaches the surface of the sea at the point P.

(b) (i) Find the coordinates of P.

(ii) Find OP. [5]

Question 7

[Maximum mark: 7]

An engineer plans to visit six oil rigs (A–F) in the Gulf of Mexico, starting and finishing at A. The travelling time, in minutes, between each of the rigs is shown in the table.

	A	B	C	D	E	F
A		55	63	79	87	93
B	55		46	58	88	92
C	63	46		87	77	66
D	79	58	87		23	70
E	87	88	77	23		47
F	93	92	66	70	47	

The data above can be represented by a graph G.

(a) (i) Use Prim's algorithm to find the weight of the minimum spanning tree of the subgraph of G obtained by deleting A and starting at B. List the order in which the edges are selected.

(ii) Hence find a lower bound for the travelling time needed to visit all the oil rigs. [6]

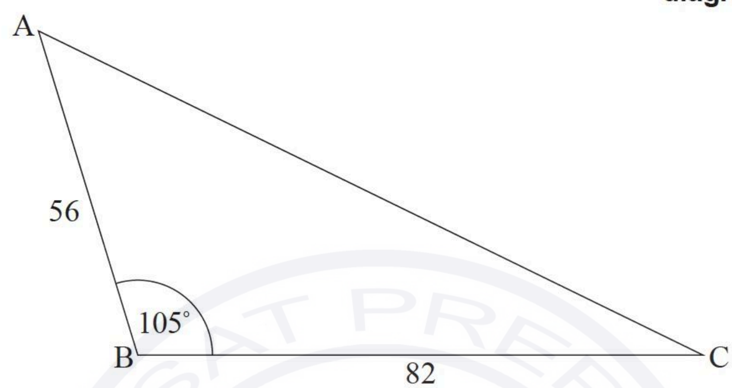
(b) Describe how an improved lower bound might be found. [1]

Question 8

[Maximum mark: 5]

A triangular field ABC is such that $\mathrm{AB} = 56\,\mathrm{m}$ and $\mathrm{BC} = 82\,\mathrm{m}$, each measured correct to the nearest metre, and the angle at B is equal to 105°, measured correct to the nearest 5°.

diagram not to scale



Calculate the maximum possible area of the field.

Question 9

[Maximum mark: 6]

A garden has a triangular sunshade suspended from three points $\mathrm{A}(2, 0, 2)$, $\mathrm{B}(8, 0, 2)$ and $\mathrm{C}(5, 4, 3)$, relative to an origin in the corner of the garden. All distances are measured in metres.

(a) (i) Find $\overrightarrow{\mathrm{CA}}$.

(ii) Find $\overrightarrow{\mathrm{CB}}$. [2]

(b) Find $\overrightarrow{\mathrm{CA}} \times \overrightarrow{\mathrm{CB}}$. [2]

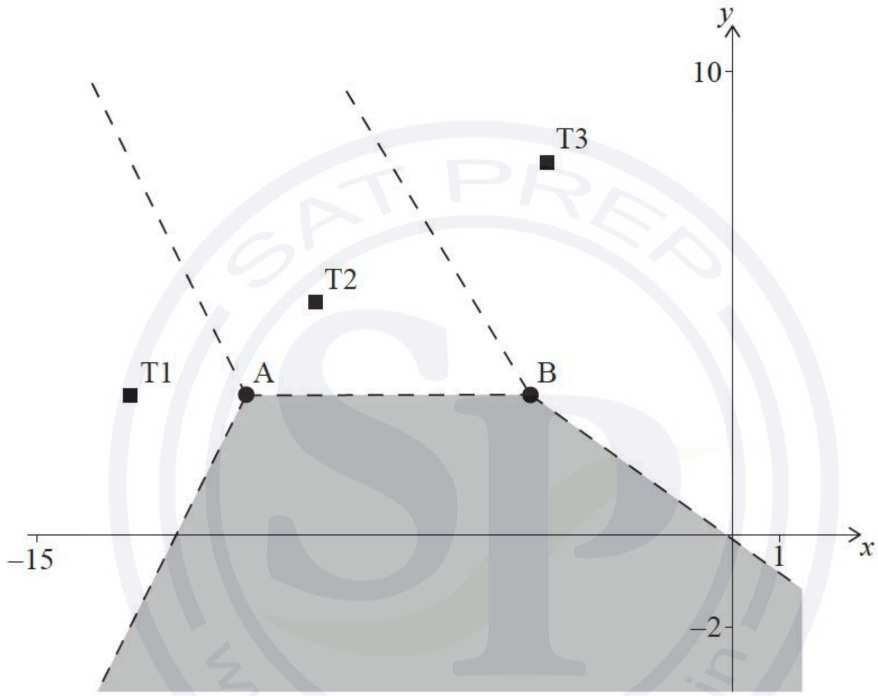
(c) Hence find the area of the triangle ABC. [2]

Question 10

[Maximum mark: 6]

The Voronoi diagram below shows three identical cellular phone towers, T1, T2 and T3. A fourth cellular phone tower, T4 is located in the shaded region. The dashed lines in the diagram below represent the edges in the Voronoi diagram.

Horizontal scale: 1 unit represents 1 km.
Vertical scale: 1 unit represents 1 km.



Tim stands inside the shaded region.

(a) Explain why Tim will receive the strongest signal from tower T4. [1]

Tower T2 has coordinates $(-9, 5)$ and the edge connecting vertices A and B has equation $y = 3$.

(b) Write down the coordinates of tower T4. [2]

Tower T1 has coordinates $(-13, 3)$.

(c) Find the gradient of the edge of the Voronoi diagram between towers T1 and T2. [3]

Question 11

[Maximum mark: 8]

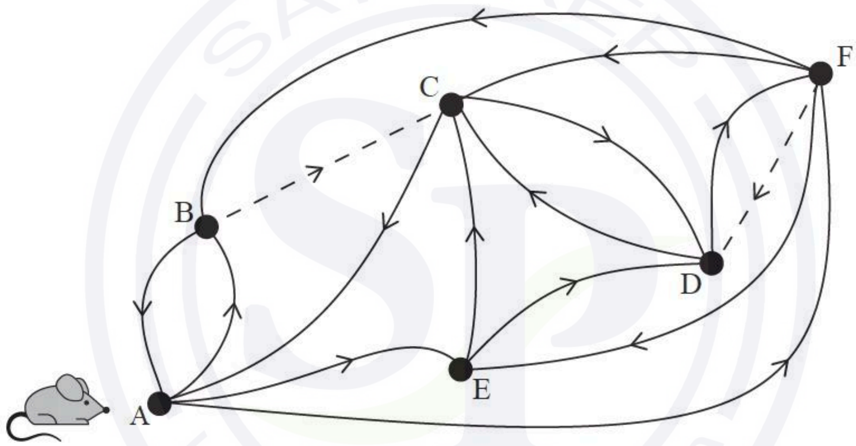
The graph below shows a small maze, in the form of a network of directed routes. The vertices A to F show junctions in the maze and the edges show the possible paths available from one vertex to another.

A mouse is placed at vertex A and left to wander the maze freely. The routes shown by dashed lines indicate paths sprinkled with sugar.

When the mouse reaches any junction, she rests for a constant time before continuing.

At any junction, it may also be assumed that

- the mouse chooses any available normal path with equal probability
- if the junction includes a path sprinkled with sugar, the probability of choosing this path is twice that of a normal path.



(a) Determine the transition matrix for this graph. [3]

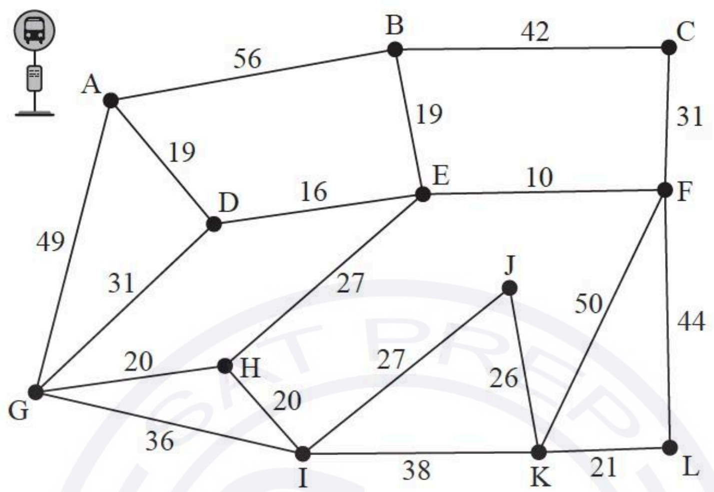
(b) If the mouse was left to wander indefinitely, use your graphic display calculator to estimate the percentage of time that the mouse would spend at point F. [3]

(c) Comment on your answer to part (b), referring to at least one limitation of the model. [2]

Question 12

[Maximum mark: 7]

The diagram below shows a network of roads in a small village with the weights indicating the distance of each road, in metres, and junctions indicated with letters.



Musab is required to deliver leaflets to every house on each road. He wishes to minimize his total distance.

(a) Musab starts and finishes from the village bus-stop at A. Determine the total distance Musab will need to walk. [5]

Instead of having to catch the bus to the village, Musab's sister offers to drop him off at any junction and pick him up at any other junction of his choice.

(b) Explain which junctions Musab should choose as his starting and finishing points. [2]

Question 13

[Maximum mark: 7]

Two lines L_1 and L_2 are given by the following equations, where $p \in \mathbb{R}$.

$$L_1 : r = \begin{pmatrix} 2 \\ p+9 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} p \\ 2p \\ 4 \end{pmatrix}$$

$$L_2 : r = \begin{pmatrix} 14 \\ 7 \\ p+12 \end{pmatrix} + \mu \begin{pmatrix} p+4 \\ 4 \\ -7 \end{pmatrix}$$

It is known that L_1 and L_2 are perpendicular.

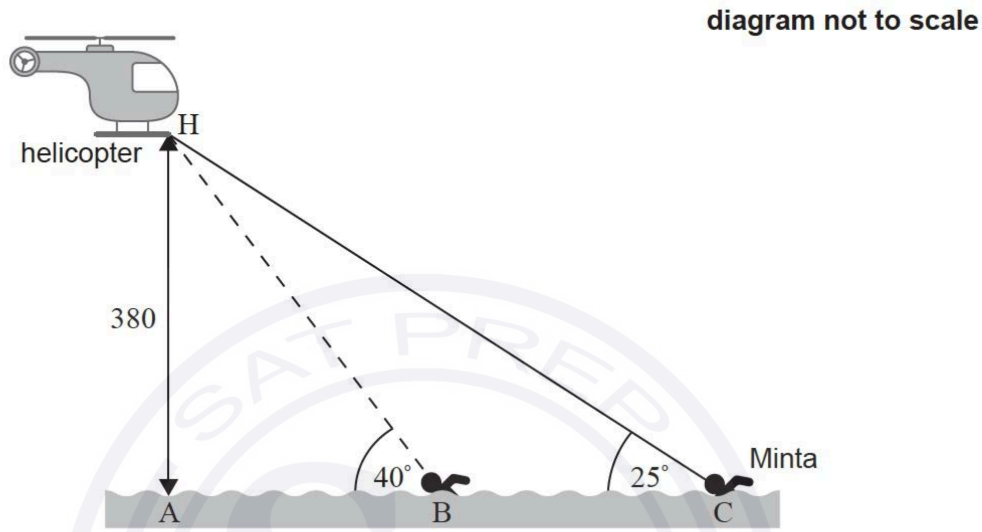
(a) Find the possible value(s) for p. [3]

(b) In the case that $p < 0$, determine whether the lines intersect. [4]

Question 14

[Maximum mark: 6]

The diagram below shows a helicopter hovering at point H, 380 m vertically above a lake. Point A is the point on the surface of the lake, directly below the helicopter.



Minta is swimming at a constant speed in the direction of point A. Minta observes the helicopter from point C as she looks upward at an angle of 25°. After 15 minutes, Minta is at point B and she observes the same helicopter at an angle of 40°.

(a) Find the distance from A to C. [2]

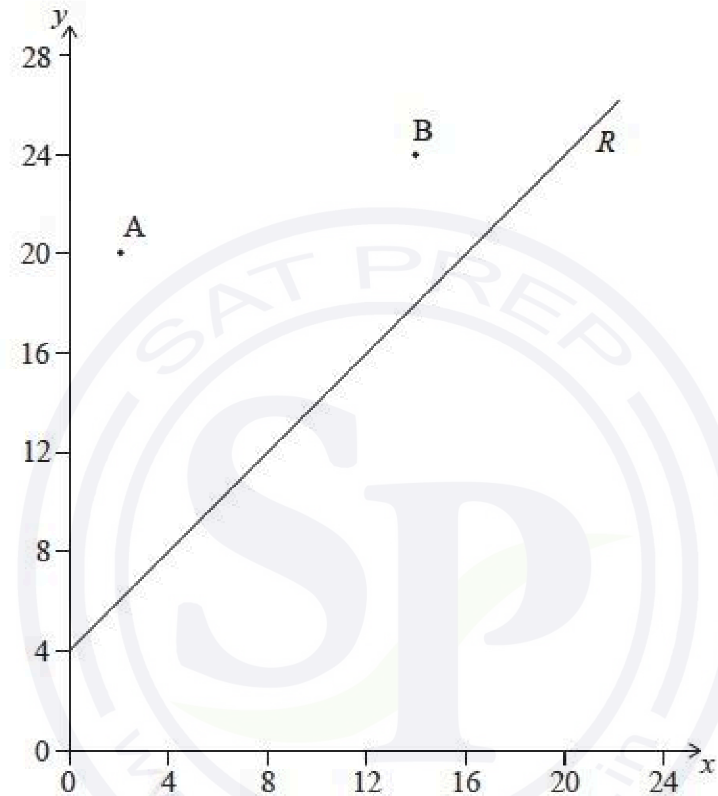
(b) Find the distance from B to C. [3]

(c) Find Minta's speed, in metres per hour. [1]

Question 15

[Maximum mark: 7]

Two schools are represented by points $\text{A}(2, 20)$ and $\text{B}(14, 24)$ on the graph below. A road, represented by the line R with equation $-x+y=4$, passes near the schools. An architect is asked to determine the location of a new bus stop on the road such that it is the same distance from the two schools.



(a) Find the equation of the perpendicular bisector of $[\text{AB}]$. Give your equation in the form $y = mx + c$. [5]

(b) Determine the coordinates of the point on R where the bus stop should be located. [2]

Question 16

[Maximum mark: 9]

A ship S is travelling with a constant velocity, $\boldsymbol{v}$, measured in kilometres per hour, where

$$\boldsymbol{v} = \begin{pmatrix} -12 \\ 15 \end{pmatrix}.$$

At time $t = 0$ the ship is at a point $\mathrm{A}(300,\ 100)$ relative to an origin O, where distances are measured in kilometres.

(a) Find the position vector $\overrightarrow{\mathrm{OS}}$ of the ship at time t hours. [1]

A lighthouse is located at a point $(129,\ 283)$.

(b) Find the value of t when the ship will be closest to the lighthouse. [6]

An alarm will sound if the ship travels within 20 kilometres of the lighthouse.

(c) State whether the alarm will sound. Give a reason for your answer. [2]

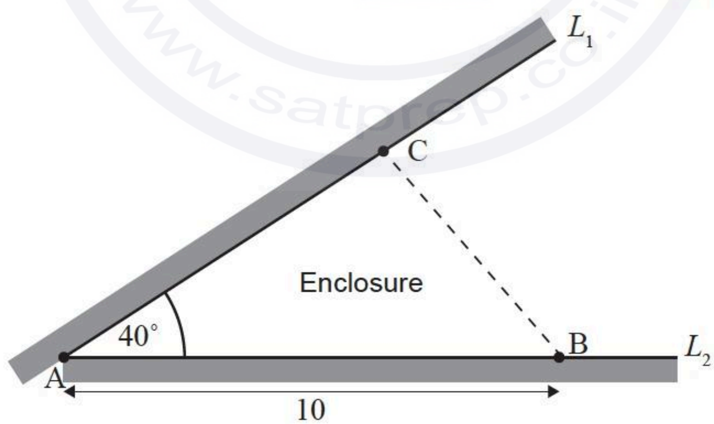
Question 17

[Maximum mark: 6]

The following diagram shows a corner of a field bounded by two walls defined by lines L_1 and L_2. The walls meet at a point A, making an angle of 40°.

Farmer Nate has $7\,\mathrm{m}$ of fencing to make a triangular enclosure for his sheep. One end of the fence is positioned at a point B on L_2, $10\,\mathrm{m}$ from A. The other end of the fence will be positioned at some point C on L_1, as shown on the diagram.

diagram not to scale



He wants the enclosure to take up as little of the current field as possible.

Find the minimum possible area of the triangular enclosure ABC. [6]

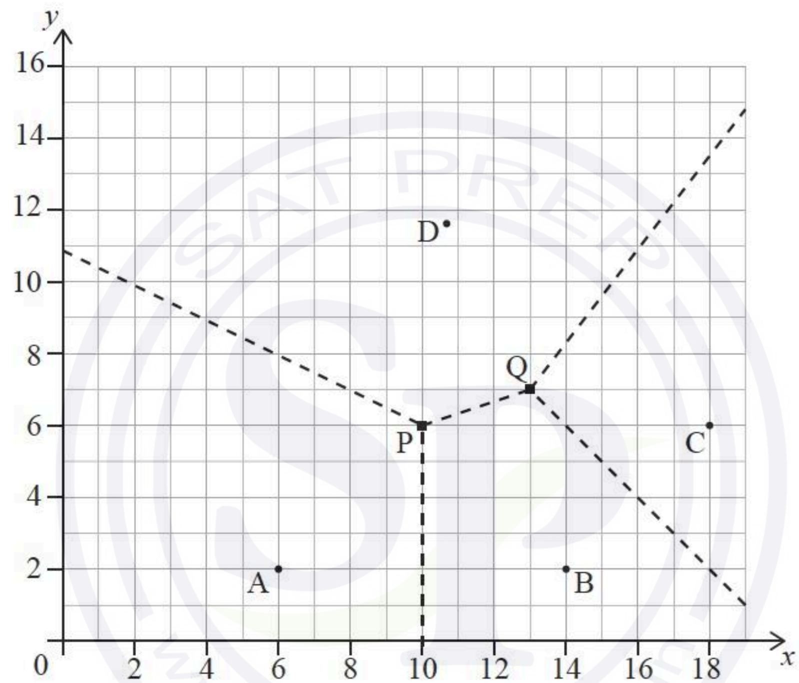
Question 18

[Maximum mark: 6]

There are four stations used by the fire wardens in a national forest.

On the following Voronoi diagram, the coordinates of the stations are $\mathrm{A}(6, 2)$, $\mathrm{B}(14, 2)$, $\mathrm{C}(18, 6)$ and $\mathrm{D}(10.8, 11.6)$ where distances are measured in kilometres.

The dotted lines represent the boundaries of the regions patrolled by the fire warden at each station. The boundaries meet at $\mathrm{P}(10, 6)$ and $\mathrm{Q}(13, 7)$.



To reduce the areas of the regions that the fire wardens patrol, a new station is to be built within the quadrilateral ABCD. The new station will be located so that it is as far as possible from the nearest existing station.

(a) Show that the new station should be built at P. [3]

The Voronoi diagram is to be updated to include the region around the new station at P. The edges defined by the perpendicular bisectors of [AP] and [BP] have been added to the following diagram.

(b) (i) Write down the equation of the perpendicular bisector of [PC].

(ii) Hence draw the missing boundaries of the region around P on the following diagram. [3]

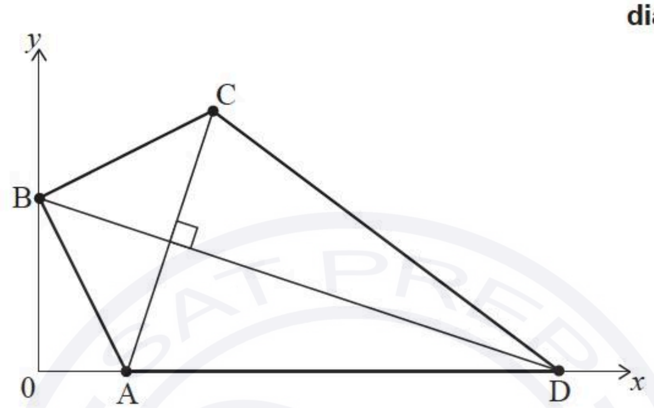
Question 19

[Maximum mark: 6]

Dilara is designing a kite ABCD on a set of coordinate axes in which one unit represents $10\,\text{cm}$.

The coordinates of A, B and C are $(2,\ 0)$, $(0,\ 4)$ and $(4,\ 6)$ respectively. Point D lies on the x-axis. [AC] is perpendicular to [BD]. This information is shown in the following diagram.

diagram not to scale



(a) Find the gradient of the line through A and C. [2]

(b) Write down the gradient of the line through B and D. [1]

(c) Find the equation of the line through B and D. Give your answer in the form $ax + by + d = 0$, where a, b and d are integers. [2]

(d) Write down the x-coordinate of point D. [1]

Question 20

[Maximum mark: 5]

At $1{:}00\ \text{pm}$ a ship is $1\,\text{km}$ east and $4\,\text{km}$ north of a harbour. A coordinate system is defined with the harbour at the origin. The position vector of the ship at $1{:}00\ \text{pm}$ is given by $\begin{pmatrix}1\\4\end{pmatrix}$.

The ship has a constant velocity of $\begin{pmatrix}1.2\\-0.6\end{pmatrix}$ kilometres per hour (km h^{-1}).

(a) Write down an expression for the position vector $\boldsymbol{r}$ of the ship, t hours after $1{:}00\ \text{pm}$. [1]

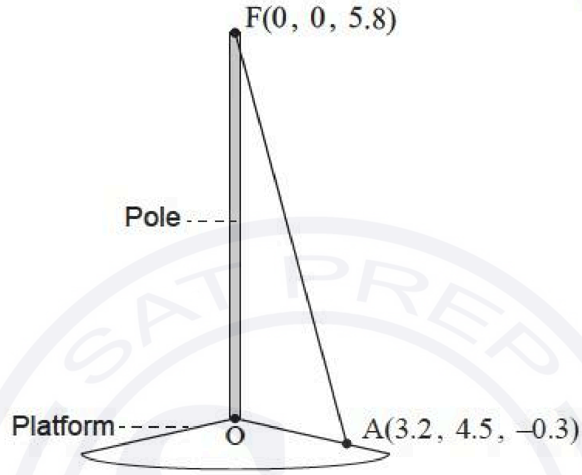
(b) Find the time at which the bearing of the ship from the harbour is 045°. [4]

Question 21

[Maximum mark: 8]

A vertical pole stands on a sloped platform. The bottom of the pole is used as the origin, O, of a coordinate system in which the top, F, of the pole has coordinates $(0, 0, 5.8)$. All units are in metres.

diagram not to scale



The pole is held in place by ropes attached at F.

One of these ropes is attached to the platform at point $A(3.2, 4.5, -0.3)$. The rope forms a straight line from A to F.

(a) Find $\overrightarrow{AF}$. [1]

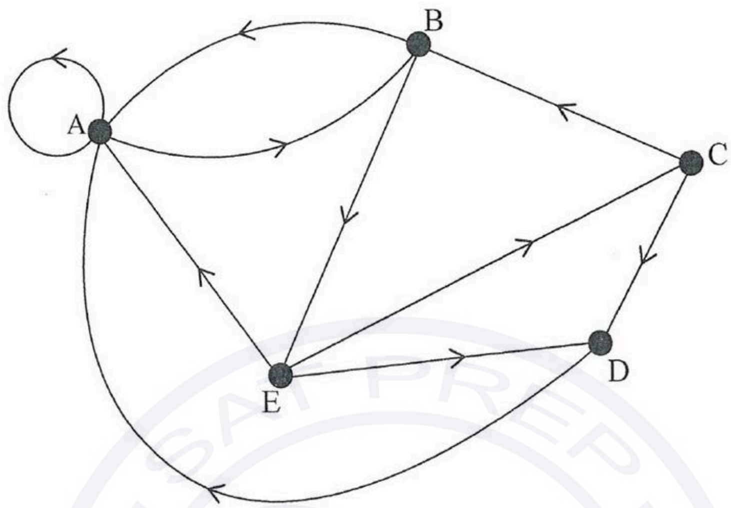
(b) Find the length of the rope. [2]

(c) Find $F\hat{A}O$, the angle the rope makes with the platform. [5]

Question 22

[Maximum mark: 5]

Consider the following directed network.



(a) Write down the adjacency matrix for this network. [2]

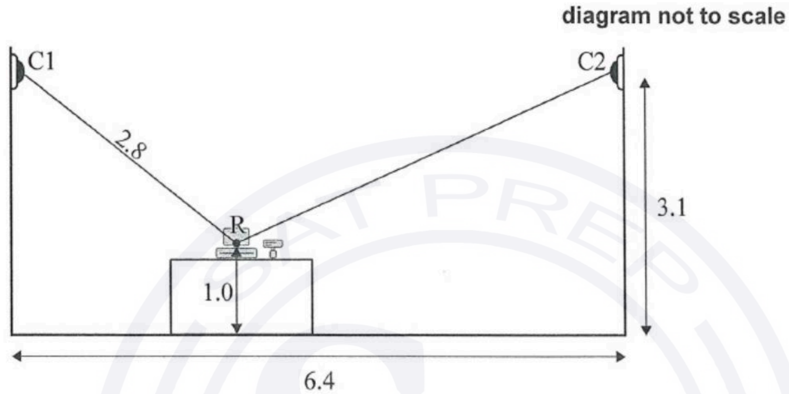
(b) Determine the number of different walks of length 5 that start and end at the same vertex. [3]

Question 23

[Maximum mark: 8]

The owner of a convenience store installs two security cameras, represented by points C1 and C2. Both cameras point towards the centre of the store's cash register, represented by the point R.

The following diagram shows this information on a cross-section of the store.



The cameras are positioned at a height of $3.1\,\text{m}$, and the horizontal distance between the cameras is $6.4\,\text{m}$. The cash register is sitting on a counter so that its centre, R, is $1.0\,\text{m}$ above the floor.

The distance from Camera 1 to the centre of the cash register is $2.8\,\text{m}$.

(a) Determine the angle of depression from Camera 1 to the centre of the cash register. Give your answer in degrees. [2]

(b) Calculate the distance from Camera 2 to the centre of the cash register. [4]

(c) Without further calculation, determine which camera has the largest angle of depression to the centre of the cash register. Justify your response. [2]

Question 24

[Maximum mark: 7]

Line L_1 has a vector equation $\boldsymbol{r} = \begin{pmatrix} 3p+4 \\ 2p-1 \\ p+9 \end{pmatrix}$, where $p \in \mathbb{R}$.

Line L_2 has a vector equation $\boldsymbol{r} = \begin{pmatrix} q-2 \\ 1-q \\ 2q+1 \end{pmatrix}$, where $q \in \mathbb{R}$.

The two lines intersect at point M.

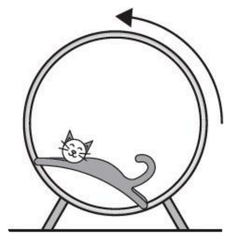
(a) Find the coordinates of M. [3]

(b) Find the acute angle between the two lines. [4]

Question 25

[Maximum mark: 6]

A cat runs inside a circular exercise wheel, making the wheel spin at a constant rate in an anticlockwise direction. The height, h cm, of a fixed point, P, on the wheel can be modelled by $h(t) = a\sin(bt) + c$ where t is the time in seconds and $a,\ b,\ c \in \mathbb{R}^+$.



When $t = 0$, point P is at a height of 78 cm.

(a) Write down the value of c. [1]

When $t = 4$, point P first reaches its maximum height of 143 cm.

(b) Find the value of

(i) a.

(ii) b. [3]

(c) Write down the minimum height of point P. [1]

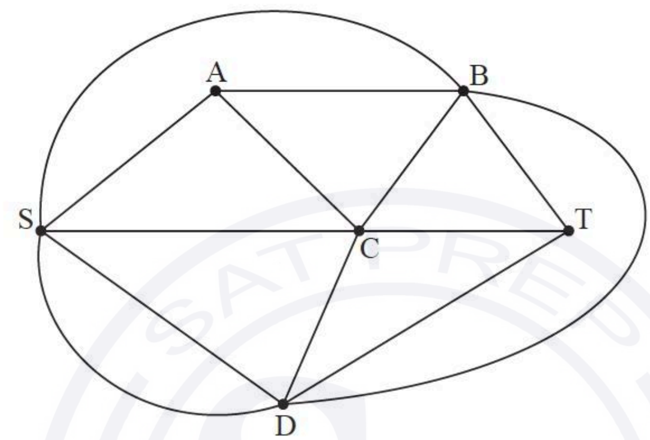
Later, the cat is tired, and it takes twice as long for point P to complete one revolution at a new constant rate.

(d) Write down the new value of b. [1]

Question 26

[Maximum mark: 7].

In a competition, a contestant has to move through a maze to find treasure. A graph of the maze is shown below, where each edge represents a corridor in the maze. The contestant starts at S and the treasure is located at T.



(a) Complete the adjacency matrix, $\boldsymbol{M}$, for the graph. [2]

	S	A	B	C	D	T
S	0	1	1	1		0
A	1	0	1	1		0
B	1	1	0	1	1	1
C	1	1	1	0	1	1
D			1	1	0	1
T	0	0	1	1	1	0

The competition rules state that the contestant can walk along a maximum of four corridors.

(b) Find the number of walks from S to T with a maximum of 4 edges. [4]

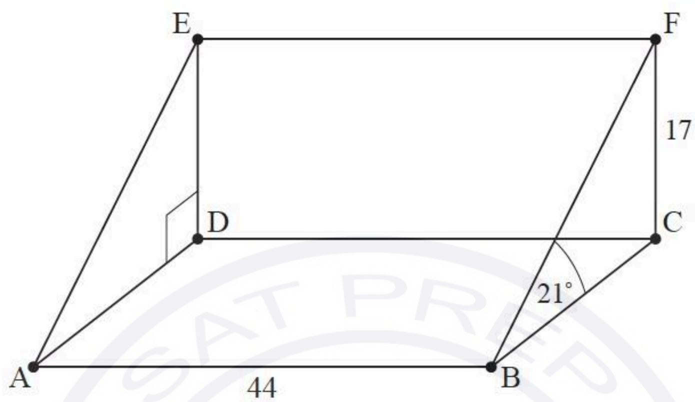
(c) Explain why the number of ways the contestant can reach the treasure is less than the answer to part (b). [1]

Question 27

[Maximum mark: 5]

An artificial ski slope can be modelled as a triangular prism, as shown in the diagram. Rectangle ABCD is horizontal, and rectangle CDEF is vertical.

diagram not to scale



The maximum height of the ski slope, CF, is 17 metres and the steepest angle of the ski slope, $F\hat{B}C$, is 21°.

(a) Calculate the length of [BF]. [2]

The width of the base of the ski slope, AB, is 44 metres. Mayumi skis in a straight line, starting from point E and finishing at the base of the ski slope.

(b) Find the value of the least steep angle that Mayumi can ski. [3]

Question 28

[Maximum mark: 5]

Two AC (alternating current) electrical sources with the same frequencies are combined. The voltages from these sources can be expressed as $V_1 = 6\sin(at + 30^\circ)$ and $V_2 = 6\sin(at + 90^\circ)$.

The combined total voltage can be expressed in the form $V_1 + V_2 = V\sin(at + \theta^\circ)$.

Determine the value of V and the value of θ.

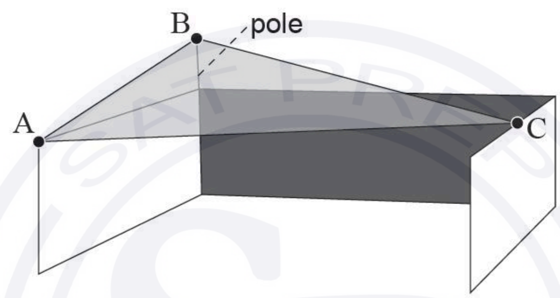
Question 29

[Maximum mark: 9]

A triangular cover is positioned over a walled garden to provide shade. It is anchored at points A and C, located at the top of a $2\,\mathrm{m}$ wall, and at a point B, located at the top of a $1\,\mathrm{m}$ vertical pole fixed to a top corner of the wall.

The three edges of the cover can be represented by the vectors

$\overrightarrow{AB} = \begin{pmatrix} 0 \\ 6 \\ 1 \end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix} 7 \\ 3 \\ 0 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} 7 \\ -3 \\ -1 \end{pmatrix}$, where distances are measured in metres.



(a) Calculate the vector product $\overrightarrow{AB} \times \overrightarrow{AC}$. [2]

(b) Hence find the area of the triangular cover. [2]

The point X on $[\mathrm{AC}]$ is such that $[\mathrm{BX}]$ is perpendicular to $[\mathrm{AC}]$.

(c) Use your answer to part (b) to find the distance BX. [3]

(d) Find the angle the cover makes with the horizontal plane. [2]

Question 30

[Maximum mark: 5]

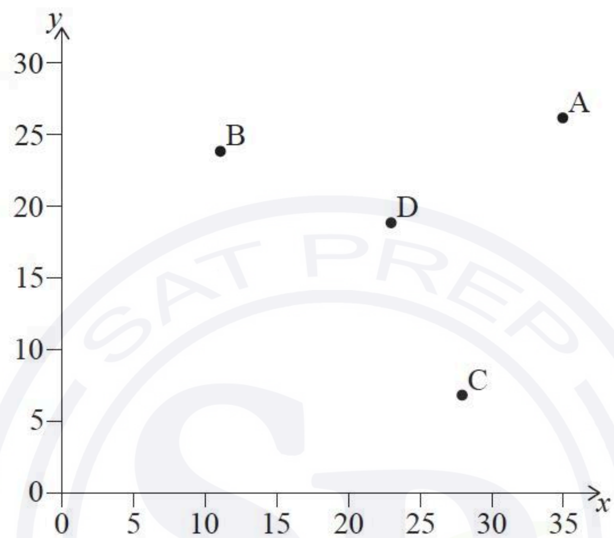
A boat travels $8\,\mathrm{km}$ on a bearing of $315°$ and then a further $6\,\mathrm{km}$ on a bearing of $045°$. Find the bearing on which the boat should travel to return directly to the starting point.

Question 31

[Maximum mark: 5]

Three towns have positions $A(35, 26)$, $B(11, 24)$, and $C(28, 7)$ according to the coordinate system shown where distances are measured in miles.

Dominique's farm is located at the position $D(24, 19)$.



(a) Find AD. [2]

On a particular day, the mean temperatures recorded in each of towns A, B and C are $34°C$, $29°C$ and $30°C$ respectively.

(b) Use nearest neighbour interpolation to estimate the temperature at Dominique's farm on that particular day. [3]

Question 32

[Maximum mark: 6]

Two AC (alternating current) electrical sources of equal frequencies are combined.

The voltage of the first source is modelled by the equation $V = 30\sin(t + 60^\circ)$.

The voltage of the second source is modelled by the equation $V = 60\sin(t + 10^\circ)$.

(a) Determine the maximum voltage of the combined sources. [2]

(b) Using your graphic display calculator, find a suitable equation for the combined voltages, giving your answer in the form $V = V_0\sin(at + b)$, where a, b and V_0 are constants, $a > 0$ and $0^\circ \le b < 180^\circ$. [4]

Question 33

[Maximum mark: 8]

In this question, $\boldsymbol{i}$ denotes a unit vector due east, and $\boldsymbol{j}$ denotes a unit vector due north.

Two ships, A and B, are each moving with constant velocities.

The position vector of ship A, at time t hours, is given as $\boldsymbol{r}_A = (1 + 2t)\boldsymbol{i} + (3 - 3t)\boldsymbol{j}$.

The position vector of ship B, at time t hours, is given as $\boldsymbol{r}_B = (-2 + 4t)\boldsymbol{i} + (-4 + t)\boldsymbol{j}$.

(a) Find the bearing on which ship A is sailing. [3]

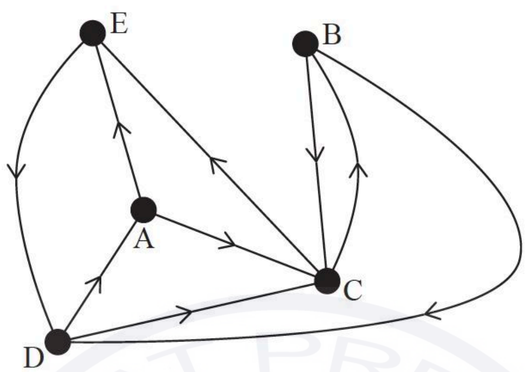
(b) Find the value of t when ship B is directly south of ship A. [2]

(c) Find the value of t when ship B is directly south-east of ship A. [3]

Question 34

[Maximum mark: 7]

The following directed, unweighted, graph shows a simplified road network on an island, connecting five small villages marked A to E.



(a) Construct the adjacency matrix M for this network. [3]

Beatriz the bus driver starts at village E and drives to seven villages, such that the seventh village is A.

(b) (i) Determine how many possible routes Beatriz could have taken, to travel from E to A.

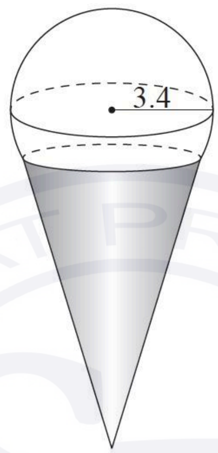
(ii) Describe one possible route taken by Beatriz, by listing the villages visited in order. [4]

Question 35

[Maximum mark: 5]

Ruhi buys a scoop of ice cream in the shape of a sphere with a radius of $3.4\,\text{cm}$. The ice cream is served in a cone, and it may be assumed that $\frac{1}{5}$ of the volume of the ice cream is inside the cone. This is shown in the following diagram.

diagram not to scale



(a) Calculate the volume of ice cream that is not inside the cone. [3]

The cone has a slant height of $11\,\text{cm}$ and a radius of $3\,\text{cm}$.

The outside of the cone is covered with chocolate.

(b) Calculate the surface area of the cone that is covered with chocolate. Give your answer correct to the nearest cm^2. [2]

Question 36

[Maximum mark: 7]

A straight line L has vector equation $\boldsymbol{r} = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ and point Q has coordinates $(11, -1, 3)$.

Point P is the point on L closest to Q.

(a) Find the coordinates of P. [4]

(b) Find a vector that is perpendicular to both L and the line passing through points P and Q. [3]

Question 37

[Maximum mark: 8]

On the following Voronoi diagram, the coordinates of three farmhouses are $A(0, 3)$, $B(8, 3)$ and $C(8, 13)$, where distances are measured in kilometres. Each farmhouse owns the land that is closest to it, and their boundaries are defined by the dotted lines on **Diagram 1**.

Diagram 1

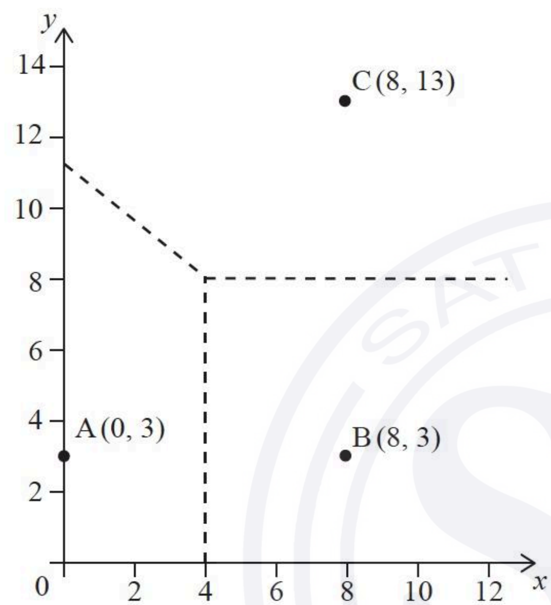
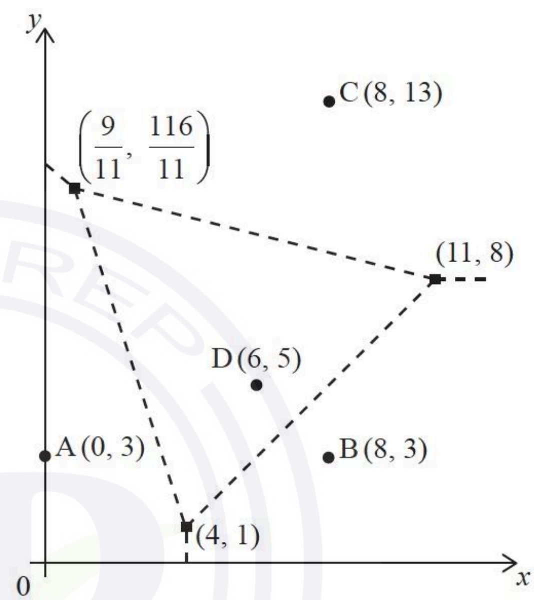


Diagram 2



To provide water to the farms it is decided to construct a well at the point where the boundaries meet on **Diagram 1**.

(a) Write down the coordinates of this point. [1]

(b) Find the equation of the perpendicular bisector of $[AC]$. [3]

An additional farmhouse $D(6, 5)$ is built on the land. The Voronoi diagram has been redrawn to show the new boundaries. The coordinates of the vertices of these boundaries are indicated on **Diagram 2**.

A wind turbine is to be built at one of the vertices.

(c) The wind turbine should be as far from the nearest farmhouses as possible.

(i) By calculating appropriate distances, find the location of the wind turbine.

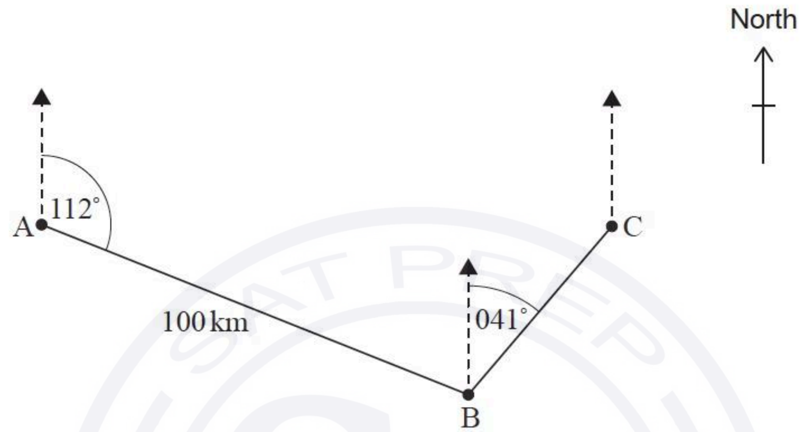
(ii) Hence, write down the distance of the wind turbine to the nearest farmhouse. [4]

Question 38

[Maximum mark: 6]

Jason sails his boat from point A for a distance of 100 km, on a bearing of 112°, to arrive at point B. He then sails on a bearing of 041° to point C. Jason's journey is shown in the diagram.

diagram not to scale



(a) Find $A\hat{B}C$. [2]

Point C is directly east of point A.

(b) Calculate the distance that Jason sails to return directly from point C to point A. [4]

Question 39

[Maximum mark: 6]

The Great Pyramid of Giza is the oldest of the Seven Wonders of the Ancient World. When it was built, 4500 years ago, the measurements of the pyramid were in Royal Egyptian Cubits (REC).

[Source: Nina Aldin Thune. https://en.wikipedia.org/wiki/Great_Pyramid_of_Giza#/media/File:Kheops-Pyramid.jpg. Licensed under CC BY 2.5 https://creativecommons.org/licenses/by/2.5/#. Image adapted.]

Viktor reads online that $1\,\text{REC}$ is equal to 0.52 metres, rounded to two decimal places.

(a) Write down the upper and lower bounds of $1\,\text{REC}$ in metres. [2]

The Great Pyramid of Giza has a square base with side lengths of $440\,\text{REC}$ and a height of $280\,\text{REC}$. Viktor assumes that these two measurements are exact and that the Great Pyramid can be modelled as a square-based pyramid with smooth faces.

(b) Find the minimum possible volume of the pyramid in cubic metres. [4]

Question 40

[Maximum mark: 7]

A duck is sitting in a duck pond at point $A(7, 4, 0)$ relative to an origin O, where lengths are measured in metres and time, t, is measured in seconds. It takes off and flies in a straight line with vector equation

$$\boldsymbol{d} = \begin{pmatrix} 7 \\ 4 \\ 0 \end{pmatrix} + t\begin{pmatrix} 6 \\ 6 \\ 3 \end{pmatrix}.$$

(a) Find the speed of the duck through the air (in $\mathrm{m\,s^{-1}}$). [2]

A hawk hovering at position vector $\begin{pmatrix} -38 \\ 134 \\ 315 \end{pmatrix}$, relative to O, sees the duck take off and immediately dives from its position with constant velocity vector $\begin{pmatrix} 15 \\ -20 \\ -60 \end{pmatrix}$ to intercept the duck.

(b) Write down the vector equation for $\boldsymbol{h}$, that models the flight of the hawk. [1]

(c) Find the position vector at which the hawk intercepts the duck. [4]

Question 41

[Maximum mark: 10]

The quadrilateral ABCD has coordinates $A(1, 3, 5)$, $B(4, 7, 5)$, $C(5, 8, 7)$ and $D(2, 4, 7)$.

(a) Write down $\overrightarrow{AD}$. [1]

(b) Calculate

(i) the size of $B\hat{A}D$.

(ii) the area of triangle BAD. [7]

(c) Show that ABCD is a parallelogram. [2]

Question 42

[Maximum mark: 6]

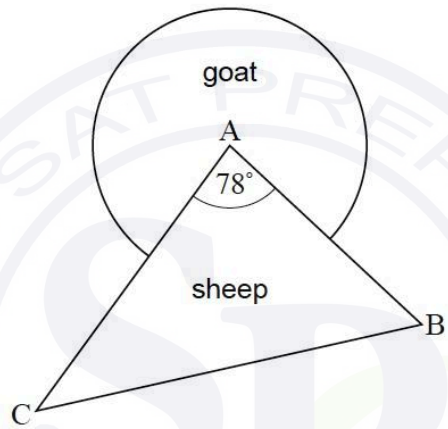
A sheep is in a field in the shape of a triangle, ABC.

$AC = 21$ metres, $AB = 15$ metres and $C\hat{A}B = 78^\circ$.

A goat is in an adjacent field in the shape of a sector of a circle with centre, A, and radius 8 metres.

The fields are shown in the diagram.

diagram not to scale



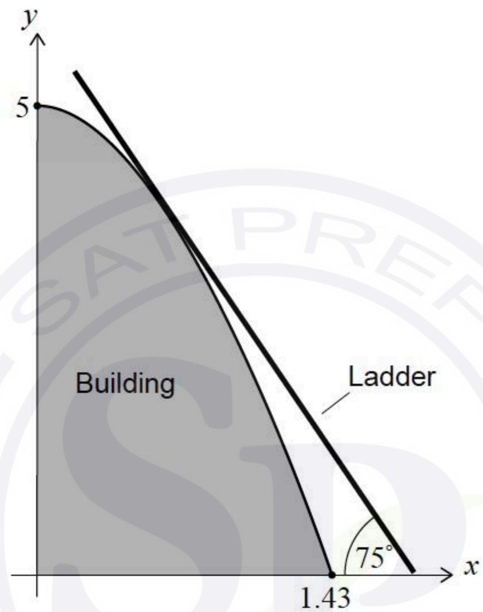
Determine which animal, the sheep or the goat, is in the field with the larger area, and state how many extra square metres are in this larger field.

Question 43

[Maximum mark: 8]

The cross section of the side of a building can be modelled by a curve with equation $y = 5\cos(1.1x), 0 \le x \le 1.43$, as shown in the following diagram. Distances are measured in metres.

diagram not to scale



A builder leans a straight ladder against the building to do repairs. For safety reasons, the angle between the ladder and the horizontal ground must be 75°.

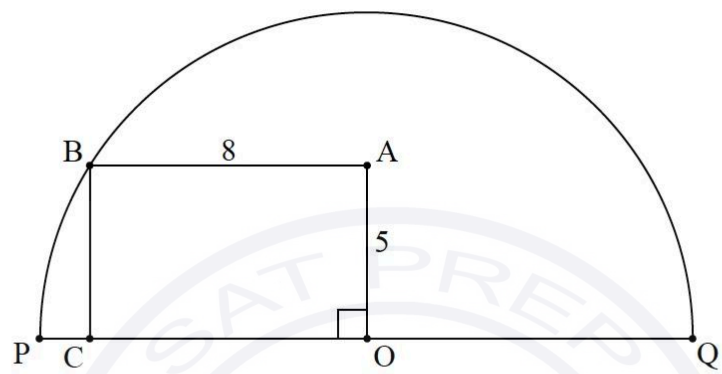
Find the height above the ground at which the ladder touches the building.

Question 44

[Maximum mark: 5]

The following diagram shows a semicircle with centre O and diameter PQ. A rectangle OABC is also shown, such that $\mathrm{AB} = 8$ and $\mathrm{OA} = 5$.

diagram not to scale



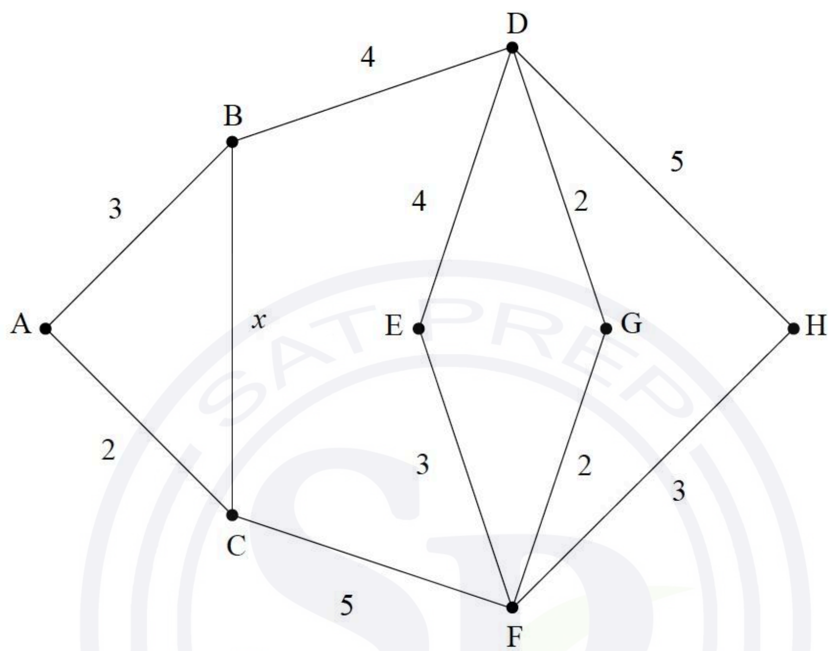
Find the length of the arc BQ.

Question 45

[Maximum mark: 5]

The weights on the following graph represent the lengths of different roads in kilometres.

diagram not to scale



(a) Write down the vertices with odd degree. [1]

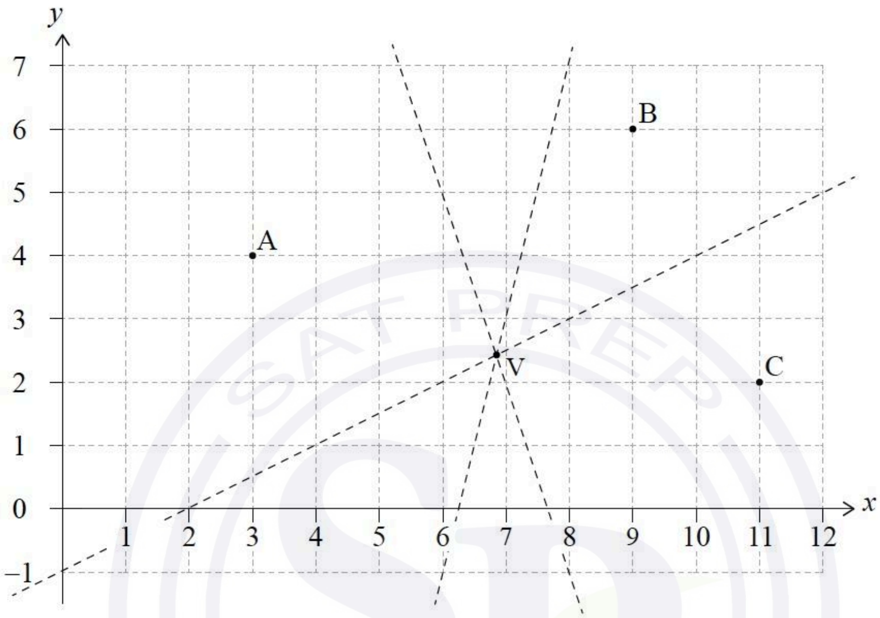
The total length of the roads is $33 + x$ km.

(b) Find **two** expressions, in terms of x, for the shortest distance required to walk along all of the paths, beginning and ending at the same vertex.
Include in your answer the interval of values of x for which each expression is valid. [4]

Question 46

[Maximum mark: 6]

Points $A(3, 4)$, $B(9, 6)$ and $C(11, 2)$ are shown on the following diagram, along with the perpendicular bisectors of [AB], [AC] and [BC].



The perpendicular bisector of [BC] intercepts the axes at coordinates $(0, -1)$ and $(2, 0)$.

(a) Write down the equation of the perpendicular bisector of [BC]. [2]

The equation of the perpendicular bisector of [AB] is $y = -3x + 23$.

(b) Find the coordinates of point V where the perpendicular bisectors meet. Give your answer to four significant figures. [2]

A Voronoi diagram is constructed with points A, B and C as the three sites.

(c) Draw, clearly, the edges of the Voronoi diagram on the given diagram. [2]

Question 47

[Maximum mark: 7]

The amount of daylight, L (in hours), in London in 2024 can be modelled by

$$L = a\sin\left(b\left(t - c\right)\right) + d,$$

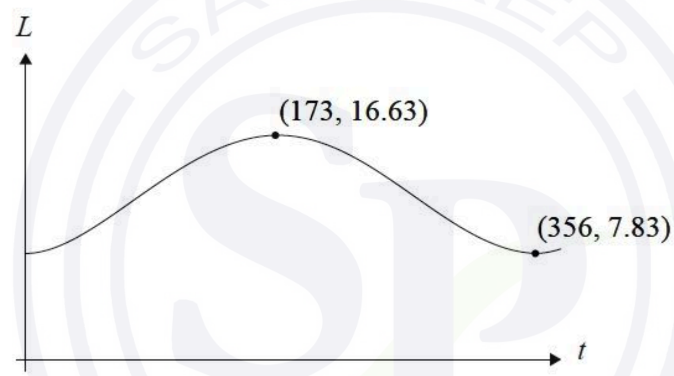
where $a, b, c, d > 0$ and t is the day of the year.

For example, day $1 =$ 1 January, day $2 =$ 2 January, and so on.

The maximum value of L is 16.63 hours on day 173 (21 June 2024).

The minimum value of L is 7.83 hours on day 356 (21 December 2024).

This information is shown in the following diagram.



Find the value of

(a) d [2]

(b) a [1]

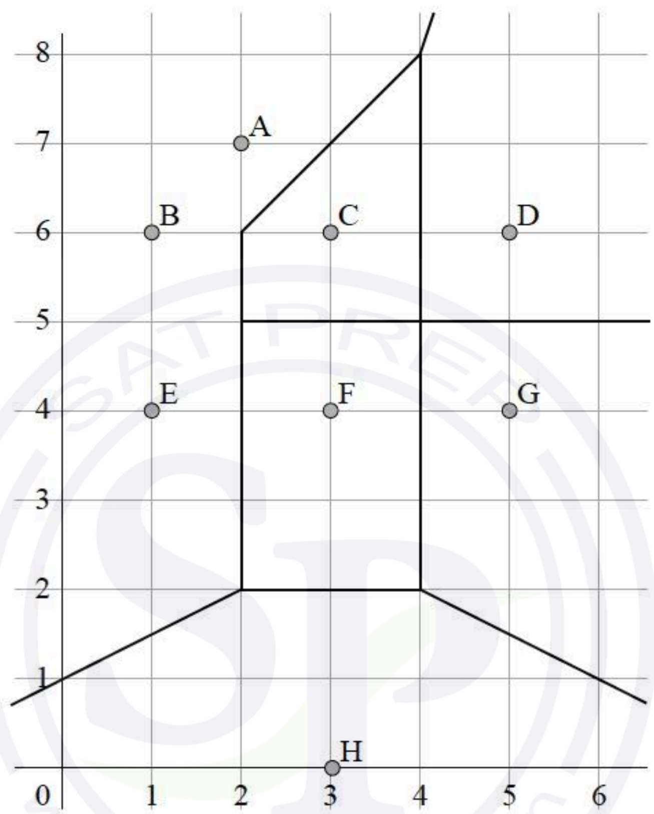
(c) b [2]

(d) c. [2]

Question 48

[Maximum mark: 6]

The sites in the Voronoi diagram represent eight hospitals in a city.



Each site has integer coordinates. Two edges are missing from the diagram.

(a) Draw the missing edges on the diagram. [2]

One square unit on the diagram represents $4\,\text{km}^2$ in the city.

(b) Find the area, in km^2, of the cell containing

(i) site F

(ii) site C. [3]

The hospitals at sites C and F have the same number of patients each year.

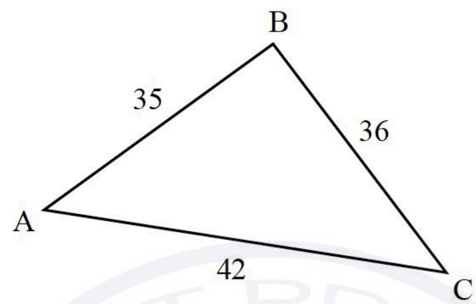
(c) Suggest a reason why the number of patients is not proportional to the area of the cell. [1]

Question 49

[Maximum mark: 5]

Consider the following triangle, ABC, such that $\mathrm{AB} = 35\,\mathrm{cm}$, $\mathrm{BC} = 36\,\mathrm{cm}$, and $\mathrm{CA} = 42\,\mathrm{cm}$.

diagram not to scale



(a) Find the value of $\mathrm{C\hat{A}B}$. [3]

(b) Find the area of the triangle ABC. [2]