Subject – Math AI(Higher Level)
Topic - Geometry and Trigonometry
Year - May 2021 – Nov 2024
Paper -1
Answers

Question 1

(a) $15 \times 0 + 2d + 4 = 0$ **(M1)**

$d = -2$ **A1**

[2 marks]

(b) $a\begin{pmatrix}-15\\2\\4\end{pmatrix} \times \begin{pmatrix}0\\-2\\1\end{pmatrix}$ **(M1)**

$= a\begin{pmatrix}10\\15\\30\end{pmatrix} \left(= 5a\begin{pmatrix}2\\3\\6\end{pmatrix}\right)$ **A1**

magnitude is $5a\sqrt{2^2 + 3^2 + 6^2} = 14$ **M1**

$a = \frac{14}{35}$ $(= 0.4)$ **A1**

[4 marks]

Total [6 marks]

Question 2

(a) $\frac{\sin C\hat{A}B}{6} = \frac{\sin 15^\circ}{4.5}$ *(M1)(A1)*

$C\hat{A}B = 20.2^\circ$ $(20.187415\ldots)$ ***A1***

Note:	Award ***(M1)*** for substituted sine rule formula and award ***(A1)*** for correct substitutions.

[3 marks]

(b) $C\hat{B}D = 20.2 + 15 = 35.2^\circ$ ***A1***

(let X be the point on BD where Ollie activates the sensor)

$\tan 35.18741\ldots^\circ = \frac{1.8}{BX}$ ***(M1)***

Note:	Award ***A1*** for their correct angle $C\hat{B}D$. Award ***M1*** for correctly substituted trigonometric formula.

$BX = 2.55285\ldots$ ***A1***

$5 - 2.55285\ldots$ ***(M1)***

$= 2.45\,(\text{m})$ $(2.44714\ldots)$ ***A1***

[5 marks]

Total [8 marks]

Question 3

(a) $\frac{50 \times \pi}{180} = 0.873$ $(0.872664\ldots)$ ***A1***

[1 mark]

(b) volume $= 240\left(\pi \times 8.4^2 - \frac{1}{2} \times 8.4^2 \times 0.872664\ldots\right)$ ***M1M1M1***

Note:	Award ***M1*** $240 \times$ area, award ***M1*** for correctly substituting area sector formula, award ***M1*** for subtraction of the angles or their areas.

$= 45800$ $(= 45811.96071)$ ***A1***

[4 marks]

Total [5 marks]

Question 4

(a) $\frac{3-1}{7-3}$ *(M1)*

$= 0.5$ ***A1***

[2 marks]

(b) $y-2=-2(x-5)$ ***(A1)(M1)***

Note: Award ***(A1)*** for their -2 seen, award ***(M1)*** for the correct substitution of $(5, 2)$ and their normal gradient in equation of a line.

$2x + y - 12 = 0$ ***A1***

[3 marks]

(c) every point in the cell is closer to E than any other snow shelter ***A1***

[1 mark]

Total [6 marks]

Question 5

(a) $M = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$ ***A1A1A1***

Note: Award ***A1*** for each two correct rows.

[3 marks]

(b) calculating M^6 ***(M1)***

143 ***A1***

[2 marks]

Total [6 marks]

Question 6

(a) $r = \begin{pmatrix} 0.8 \\ 1.3 \\ -0.3 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix}$ ***A1A1***

Note: Award ***A1*** for each correct vector. Award ***A0A1*** if their "$r =$" is omitted.

[2 marks]

(b) (i) $-0.3 + \lambda = 0$ ***(M1)***

$\Rightarrow \lambda = 0.3$

$r = \begin{pmatrix} 0.8 \\ 1.3 \\ -0.3 \end{pmatrix} + 0.3 \begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.4 \\ 0 \end{pmatrix}$ ***(M1)***

P has coordinates $(0.2, 0.4, 0)$ ***A1***

Note: Accept the coordinates of P in vector form.

(ii) $\sqrt{0.2^2 + 0.4^2}$ ***(M1)***

$= 0.447$ km $(= 447$ m$)$ ***A1***

[5 marks]

Total [7 marks]

Question 7

(a) (i) use of Prim's algorithm **M1**

BC 46 **A1**

BD 58 **A1**

DE 23

EF 47

Total 174 **A1**

Note: Award ***M0A0A0A1*** for 174 without correct working e.g. use of Kruskal's, or with no working.
Award ***M1A0A0A1*** for 174 by using Prim's from an incorrect starting point.

(ii) $AB + AC = 55 + 63 = 118$ ***(M1)***

$174 + 118 = 292$ minutes ***A1***

[6 marks]

(b) delete a different vertex ***A1***

[1 mark]

Total [7 marks]

Question 8

attempt to find any relevant maximum value ***(M1)***

largest sides are 56.5 and 82.5 ***(A1)***

smallest possible angle is 102.5 ***(A1)***

attempt to substitute into area of a triangle formula ***(M1)***

$$\frac{1}{2}\times 56.5\times 82.5\times \sin(102.5^{\circ})$$

$= 2280\ \text{m}^2 \quad (2275.37\ldots)$ ***A1***

Total [5 marks]

Question 9

(a) (i) $\overrightarrow{CA} = \begin{pmatrix} -3 \\ -4 \\ -1 \end{pmatrix}$ ***A1***

(ii) $\overrightarrow{CB} = \begin{pmatrix} 3 \\ -4 \\ -1 \end{pmatrix}$ ***A1***

[2 marks]

(b) $\overrightarrow{CA} \times \overrightarrow{CB} = \begin{pmatrix} 0 \\ -6 \\ 24 \end{pmatrix}$ ***(M1)A1***

Note: Do not award ***(M1)*** if less than 2 entries are correct.

[2 marks]

(c) area is $\frac{1}{2}\sqrt{6^2 + 24^2} = 12.4 \text{ m}^2 \quad (12.3693..., 3\sqrt{17})$ ***(M1)A1***

[2 marks]

Total [6 marks]

Question 10

(a) every point in the shaded region is closer to tower T4 ***R1***

Note: Specific reference must be made to the closeness of tower T4.

[1 mark]

(b) $(-9, 1)$ ***A1A1***

Note: Award ***A1*** for each correct coordinate. Accept $x = -9$ and $y = 1$.
Award at most ***A0A1*** if parentheses are missing.

[2 marks]

(c) correct use of gradient formula ***(M1)***

e.g. $(m =) \frac{5-3}{-9--13} \left(= \frac{1}{2}\right)$

taking negative reciprocal of **their** m (at any point) ***(M1)***

edge gradient $= -2$ ***A1***

[3 marks]

Total [6 marks]

Question 11

(a) transition matrix is

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{pmatrix} 0 & \frac{1}{3} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{5} \\ 0 & \frac{2}{3} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{5} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & \frac{2}{5} \\ \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{5} \\ \frac{1}{3} & 0 & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix} \end{array}$$

M1A1A1

Note: Allow the transposed matrix.

Award ***M1*** for a 6x6 matrix with all values between 0 and 1, and all columns (or rows if transposed) adding up to 1, award ***A1*** for one correct row (or column if transposed) and ***A1*** for all rows (or columns if transposed) correct.

[3 marks]

(b) attempting to raise the transition matrix to a large power ***(M1)***

steady state vector is $\begin{pmatrix} (0.157) \\ (0.0868) \\ (0.256) \\ (0.241) \\ (0.0868) \\ 0.173 \end{pmatrix}$ ***(A1)***

so percentage of time spent at vertex F is 17.3% ***A1***

Note: Accept 17.2%.

[3 marks]

(c) the model assumes instantaneous travel from junction to junction, ***R1***
and hence the answer obtained would be an overestimate ***R1***
OR
the mouse may eat the sugar over time ***R1***
and hence the probabilities would change ***R1***

Note: Accept any other sensible answer.

[2 marks]

Total [7 marks]

Question 12

(a) Odd vertices are A, B, D, H **_A1_**
Consider pairings: **_M1_**

Note: Award ***(M1)*** if there are four vertices not necessarily all correct.

AB DH has shortest route AD, DE, EB and DE, EH,
so repeated edges $(19+16+19)+(16+27)=97$

Note: Condone AB in place of AD, DE, EB giving $56+(16+27)=99$.

AD BH has shortest route AD and BE, EH,
so repeated edges $19+(19+27)=65$

AH BD has shortest route AD, DE, EH and BE, ED,
so repeated edges $(19+16+27)+(19+16)=97$ **_A2_**

Note: Award ***A1*** if only one or two pairings are correctly considered.

so best pairing is AD, BH
weight of route is therefore $582+65=647$ **_A1_**

[5 marks]

(b) least value of the pairings is 19 therefore repeat AD **_R1_**

B and H **_A1_**

Note: Do not award ***R0A1***.

[2 marks]

Total [7 marks]

Question 13

(a) setting a dot product of the direction vectors equal to zero *(M1)*

$$\begin{pmatrix} p \\ 2p \\ 4 \end{pmatrix} \cdot \begin{pmatrix} p+4 \\ 4 \\ -7 \end{pmatrix} = 0$$

$p(p+4)+8p-28=0$ *(A1)*

$p^2+12p-28=0$

$(p+14)(p-2)=0$

$p=-14,\ p=2$ ***A1***

[3 marks]

(b) $p=-14 \Rightarrow$

$$L_1 : r = \begin{pmatrix} 2 \\ -5 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} -14 \\ -28 \\ 4 \end{pmatrix}$$

$$L_2 : r = \begin{pmatrix} 14 \\ 7 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} -10 \\ 4 \\ -7 \end{pmatrix}$$

a common point would satisfy the equations

$2-14\lambda = 14-10\mu$

$-5-28\lambda = 7+4\mu$ *(M1)*

$-3+4\lambda = -2-7\mu$

METHOD 1

solving the first two equations simultaneously

$\lambda = -\frac{1}{2}, \mu = \frac{1}{2}$ ***A1***

substitute into the third equation: ***M1***

$-3+4\left(-\frac{1}{2}\right) \neq -2+\frac{1}{2}(-7)$

so lines do not intersect. ***R1***

Note: Accept equivalent methods based on the order in which the equations are considered.

METHOD 2

attempting to solve the equations using a GDC ***M1***

GDC indicates no solution ***A1***

so lines do not intersect ***R1***

[4 marks]

Total [7 marks]

Question 14

(a) $AC = \dfrac{380}{\tan 25^\circ}$ **OR** $AC = \sqrt{\left(\dfrac{380}{\sin 25^\circ}\right)^2 - 380^2}$ **OR** $\dfrac{380}{\sin 25^\circ} = \dfrac{AC}{\sin 65^\circ}$ ***(M1)***

$AC = 815 \text{ m } (814.912\ldots)$ ***A1***

[2 marks]

(b) **METHOD 1**

attempt to find AB ***(M1)***

$AB = \dfrac{380}{\tan 40^\circ}$

$= 453 \text{ m } (452.866\ldots)$ ***(A1)***

$BC = 814.912\ldots - 452.866\ldots$

$= 362 \text{ m } (362.046\ldots)$ ***A1***

METHOD 2

attempt to find HB ***(M1)***

$HB = \dfrac{380}{\sin 40^\circ}$

$591 \text{ m } (= 591.175\ldots)$ ***(A1)***

$BC = \dfrac{591.175\ldots \times \sin 15^\circ}{\sin 25^\circ}$

$= 362 \text{ m } (362.046\ldots)$ ***A1***

[3 marks]

(c) $362.046\ldots \times 4$

$= 1450 \text{ m h}^{-1} \; (1448.18\ldots)$ ***A1***

[1 mark]

Total [6 marks]

Question 15

(a) gradient $AB = \frac{4}{12}\left(\frac{1}{3}\right)$ *(A1)*

midpoint $AB: (8, 22)$ *(A1)*

gradient of bisector $= -\frac{1}{\text{gradient AB}} = -3$ *(M1)*

perpendicular bisector: $22 = -3 \times 8 + b$ **OR** $(y-22) = -3(x-8)$ *(M1)*

perpendicular bisector: $y = -3x + 46$ ***A1***

[5 marks]

(b) attempt to solve simultaneous equations *(M1)*

$x + 4 = -3x + 46$

$(10.5, 14.5)$ ***A1***

[2 marks]

Total [7 marks]

Question 16

(a) $\vec{OS} = \begin{pmatrix} 300 \\ 100 \end{pmatrix} + t\begin{pmatrix} -12 \\ 15 \end{pmatrix}$ ***A1***

[1 mark]

(b) attempt to find the vector from L to S *(M1)*

$\vec{LS} = \begin{pmatrix} 171 \\ -183 \end{pmatrix} + t\begin{pmatrix} -12 \\ 15 \end{pmatrix}$ ***A1***

EITHER

$\left|\vec{LS}\right| = \sqrt{(171-12t)^2 + (15t-183)^2}$ *(M1)(A1)*

minimize to find *t* on GDC *(M1)*

OR

S closest when $\vec{LS} \bullet \begin{pmatrix} -12 \\ 15 \end{pmatrix} = 0$ *(M1)*

$\left(\begin{pmatrix} 171 \\ -183 \end{pmatrix} + t\begin{pmatrix} -12 \\ 15 \end{pmatrix}\right) \bullet \begin{pmatrix} -12 \\ 15 \end{pmatrix} = 0$

$-2052 + 144t - 2745 + 225t = 0$ *(M1)(A1)*

(c) the alarm will sound ***A1***

$|\vec{LS}| = 19.2.... < 20$ ***R1***

Note: Do not award ***A1R0***.

[2 marks]

Total: [9 marks]

Question 17

METHOD 1

attempt to find AC using cosine rule **M1**

$7^2 = 10^2 + AC^2 - 2 \times 10 \times AC \times \cos 40°$ **(A1)**

attempt to solve a quadratic equation **(M1)**

$AC = 4.888...$ **AND** $10.432...$ **(A1)**

minimum area = $\frac{1}{2} \times 10 \times 4.888... \times \sin(40°)$ **M1**

$= 15.7\ \text{m}^2$ **A1**

METHOD 2

attempt to find $A\hat{C}B$ using the sine Rule **M1**

$\frac{\sin C}{10} = \frac{\sin 40}{7}$ **(A1)**

$C = 66.674...°$ **OR** $113.325....°$ **(A1)**

EITHER

$B = 180 - 40 - 113.325...$

$B = 26.675...°$ **(A1)**

area $= \frac{1}{2} \times 10 \times 7 \times \sin(26.675...°)$ **M1**

OR

sine rule or cosine rule to find $AC = 4.888...$ **(A1)**

minimum area = $\frac{1}{2} \times 10 \times 4.888... \times \sin(40°)$ **M1**

THEN

$= 15.7\ \text{m}^2$ **A1**

Question 18

(a) (the best placement is either point P or point Q)

attempt at using the distance formula **(M1)**

$AP = \sqrt{(10-6)^2 + (6-2)^2}$ **OR**

$BP = \sqrt{(10-14)^2 + (6-2)^2}$ **OR**

$DP = \sqrt{(10-10.8)^2 + (6-11.6)^2}$ **OR**

$BQ = \sqrt{(13-14)^2 + (7-2)^2}$ **OR**

$CQ = \sqrt{(13-18)^2 + (7-6)^2}$ **OR**

$DQ = \sqrt{(13-10.8)^2 + (7-11.6)^2}$

(AP or BP or DP $=$) $\sqrt{32} = 5.66$ $(5.65685\ldots)$ **AND**

(BQ or CQ or DQ $=$) $\sqrt{26} = 5.10$ $(5.09901\ldots)$ ***A1***

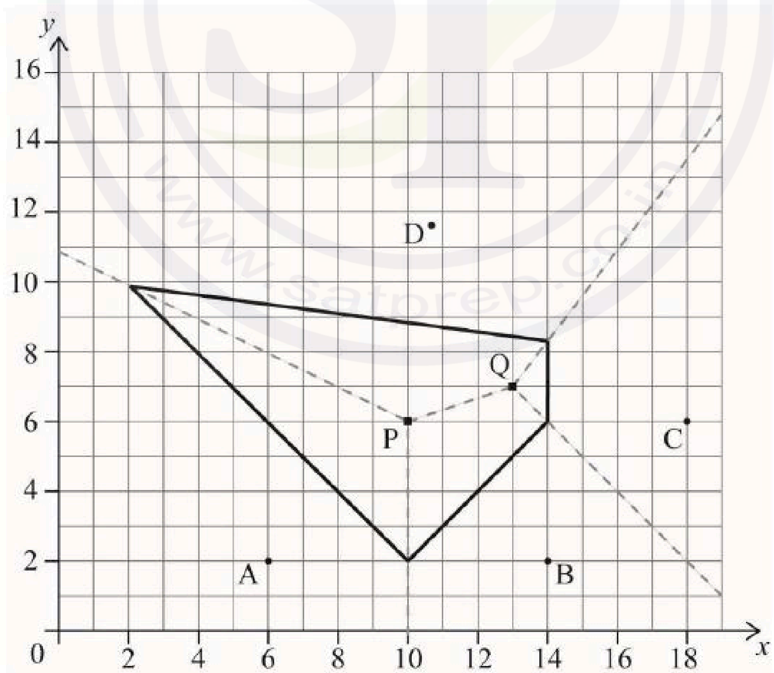
$\sqrt{32} > \sqrt{26}$ **OR** AP (or BP or DP) is greater than BQ (or CQ or DQ) ***A1***

point P is the furthest away ***AG***

[3 marks]

(b) (i) $x = 14$ ***A1***

(ii)



A1A1

[3 marks]

Total: [6 marks]

Question 19

(a) $m = \frac{6-0}{4-2} = 3$ **(M1)A1**

[2 marks]

(b) $(m =)\ -\frac{1}{3}\ (-0.333,\ -0.333333...)$ **A1**

[1 mark]

(c) an equation of line with a correct intercept and either of their gradients from (a) or (b) ***(M1)***

e.g. $y = -\frac{1}{3}x + 4$ **OR** $y - 4 = -\frac{1}{3}(x - 0)$

Note: Award ***(M1)*** for substituting either of their gradients from parts (a) or (b) and point B or $(3, 3)$ into equation of a line.

$x + 3y - 12 = 0$ or any integer multiple **A1**

[2 marks]

(d) $(x =)\ 12$ **A1**

[1mark]
Total: [6 marks]

Question 20

(a) $(\boldsymbol{r} =) \begin{pmatrix} 1 \\ 4 \end{pmatrix} + t\begin{pmatrix} 1.2 \\ -0.6 \end{pmatrix}$ **A1**

Note: Do not condone the use of λ or any other variable apart from t.

[1 mark]

(b) when the bearing from the port is 045°, the distance east from the port is equal to the distance north from the port ***(M1)***

$1 + 1.2t = 4 - 0.6t$ ***(A1)***
$1.8t = 3$

$t = \frac{5}{3}$ (1.6666..., 1 hour 40 minutes) ***(A1)***

time is 2:40 pm (14:40) **A1**

[4 marks]
[Total 5 marks]

Question 21

(a) $\begin{pmatrix}-3.2\\-4.5\\6.1\end{pmatrix}$ ***A1***

[1 mark]

(b) $\sqrt{(-3.2)^2+(-4.5)^2+6.1^2}$ ***(M1)***

$8.22800\ldots \approx 8.23\text{ m}$ ***A1***

[2 marks]

(c) **EITHER**

$\vec{AO}=\begin{pmatrix}-3.2\\-4.5\\0.3\end{pmatrix}$ ***A1***

$$\cos\theta=\frac{\vec{AO}\bullet\vec{AF}}{\left|\vec{AO}\right|\left|\vec{AF}\right|}$$

$\vec{AO}\bullet\vec{AF}=(-3.2)^2+(-4.5)^2+(0.3\times 6.1)\ (=32.32)$ ***(A1)***

$\cos\theta=\dfrac{32.32}{\sqrt{3.2^2+4.5^2+0.3^2}\times 8.22800\ldots}$ ***(M1)***

$=0.710326\ldots$ ***(A1)***

Note: If $\vec{OA}$ is used in place of $\vec{AO}$ then $\cos\theta$ will be negative.
Award ***A1(A1)(M1)(A1)*** as above. In order to award the final ***A1***, some justification for changing the resulting obtuse angle to its supplementary angle **must** be seen.

OR

$AO=\sqrt{3.2^2+4.5^2+0.3^2}\ \ (=5.52991\ldots)$ ***(A1)***

$\cos\theta=\dfrac{8.22800\ldots^2+5.52991\ldots^2-5.8^2}{2\times 8.22800\ldots\times 5.52991\ldots}$ ***(M1)(A1)***

$=0.710326\ldots$ ***(A1)***

THEN

$\theta=0.780833\ldots\approx 0.781$ **OR** $44.7384\ldots^\circ\approx 44.7^\circ$ ***A1***

[5 marks]

[Total 8 marks]

Question 22

(a)

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

A2

Note: Award ***A2*** for the transposed matrix. Presentation in markscheme assumes columns/rows ordered A-E; accept a matrix with rows and/or columns in a different order only if appropriately communicated. Do not ***FT*** from part (a) into part (b).

[2 marks]

(b) raising their matrix to a power of 5 ***(M1)***

$$\boldsymbol{M}^5 = \begin{pmatrix} 17 & 9 & 2 & 3 & 5 \\ 17 & 10 & 3 & 4 & 4 \\ 13 & 6 & 2 & 2 & 4 \\ 8 & 5 & 1 & 2 & 2 \\ 18 & 11 & 2 & 4 & 5 \end{pmatrix}$$

(A1)

Note: The numbers along the diagonal are sufficient to award ***M1A1***.

(the required number is $17+10+2+2+5=$) 36 ***A1***

[3 marks]
Total [5 marks]

Question 23

(a) $\sin\theta = \frac{2.1}{2.8}$ **OR** $\tan\theta = \frac{2.1}{1.85202\ldots}$ ***(M1)***

$(\theta =)\ 48.6°\ \ (48.5903\ldots°)$ ***A1***

[2 marks]

(b) **METHOD 1**

$\sqrt{2.8^2 - 2.1^2}$ **OR** $2.8\cos(48.5903...)$ **OR** $\frac{2.1}{\tan(48.5903\ldots)}$ ***(M1)***

> **Note:** Award ***M1*** for attempt to use Pythagorean Theorem with 2.1 seen or for attempt to use cosine or tangent ratio.

1.85 m $(1.85202\ldots)$ ***(A1)***

> **Note:** Award the ***M1A1*** if 1.85 is seen in part (a).

$(6.4 - 1.85202\ldots)$
4.55 m $(4.54797\ldots)$ ***(A1)***

> **Note:** Award ***A1*** for 4.55 or equivalent seen, either as a separate calculation or in Pythagorean Theorem.

$\sqrt{(4.54797\ldots)^2 + 2.1^2}$
5.01 m $(5.00939\ldots\text{m})$ ***A1***

METHOD 2
attempt to use cosine rule ***(M1)***
$(c^2 =)\ 2.8^2 + 6.4^2 - 2(2.8)(6.4)\cos(48.5903\ldots)$ ***(A1)(A1)***

> **Note:** Award ***A1*** for 48.5903...° substituted into cosine rule formula, ***A1*** for correct substitution.

$(c =)\ 5.01$ m $(5.00939\ldots\text{m})$ ***A1***

[4 marks]

(c) camera 1 is closer to the cash register than camera 2 (and both cameras are at the same height on the wall) ***R1***
the larger angle of depression is from camera 1 ***A1***

> **Note:** Do not award ***R0A1***. Award ***R0A0*** if additional calculations are completed and used in their justification, as per the question. Accept "1.85<4.55" or "2.8<5.01" as evidence for the ***R1***.

[2 marks]
Total [8 marks]

Question 24

(a) setting up at least two simultaneous equations *(M1)*

$p = -0.8$ **OR** $q = 3.6$ *(A1)*

M has coordinates $(1.6, -2.6, 8.2)$ ***A1***

[3 marks]

(b) using vectors $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ *(M1)*

$$\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = 3$$ *(A1)*

$$\cos\theta = \frac{\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}}{\sqrt{3^2+2^2+1^2}\sqrt{1^2+(-1)^2+2^2}} \quad \left(\cos\theta = \frac{3}{\sqrt{14}\sqrt{6}}\right)$$ *(M1)*

Note: Accept correct use of vector product.

$(\theta =)$ 1.24 radians $(1.23732\ldots)$ $(70.9^\circ \ (70.8933...))$ ***A1***

[4 marks]

Total [7 marks]

Question 25

(a) 78 — *A1*

[1 mark]

(b) (i) 65 — *A1*

(ii) **EITHER**

(period $=$) 16 (could be seen on sketch) — *(M1)*

$b = \frac{2\pi}{16}$ **OR** $b = \frac{360^\circ}{16}$

$(b =)\ 0.393 \left(0.392699\ldots, \frac{\pi}{8}\right)$ **OR** $(b =)\ 22.5^\circ$ — *A1*

OR

$143 = 65\sin(4b) + 78$ — *(M1)*

$(\sin(4b) = 1)$

$(4b = \frac{\pi}{2}$ **OR** $4b = 90^\circ)$

$(b =)\ 0.393 \left(0.392699\ldots, \frac{\pi}{8}\right)$ **OR** $(b =)\ 22.5^\circ$ — *A1*

[3 marks]

(c) 13 — *A1*

Note: Apply follow through marking only if their final answer is positive.

[1 mark]

(d) $(b =)\ 0.196 \left(0.196349\ldots, \frac{\pi}{16}\right)$ **OR** $(b =)\ 11.3^\circ\ (11.25^\circ)$ — *A1*

[1 mark]
Total [6 marks]

Question 26

(a)

$$\begin{array}{c} \\ S \\ A \\ B \\ C \\ D \\ T \end{array} \begin{array}{c} \begin{array}{cccccc} S & A & B & C & D & T \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 1 & \boxed{2} & 0 \\ 1 & 0 & 1 & 1 & \boxed{0} & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ \boxed{2} & \boxed{0} & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix} \end{array}$$

$SD = DS = 2$ ***A1***

$AD = DA = 0$ ***A1***

[2 marks]

(b) attempt to calculate at least one of $\boldsymbol{M}^2$, $\boldsymbol{M}^3$ and $\boldsymbol{M}^4$ ***(M1)***

attempt to calculate all of $\boldsymbol{M}^2$, $\boldsymbol{M}^3$ and $\boldsymbol{M}^4$ ***(M1)***

finding at least one of the top right entries, $4, 10, 64$ ***(A1)***

78 walks ***A1***

Note: If $SD = DS = 1$ is their answer in part (a), their ***FT*** answer is $(3 + 8 + 41 =)$ 52 walks.

[4 marks]

(c) because some of the walks will pass through T, before returning to T ***R1***

[1 mark]

Total [7 marks]

Question 27

(a) $\sin(21^\circ) = \dfrac{17}{\text{BF}}$ ***(M1)***

$\text{BF} = 47.4 \text{ m} \ \ (47.4372...)$ ***A1***

[2 marks]

(b) **EITHER**

$\text{BE} = \sqrt{47.4372...^2 + 44^2} = 64.7015...$ ***(A1)***

$\sin^{-1}\left(\dfrac{17}{\text{BE}}\right)$ ***(M1)***

$= 15.2^\circ \ \ (15.2329...^\circ) \quad \left(\text{or } 0.266 \text{ radians } \ (0.265866...)\right)$ ***A1***

OR

$\text{AD} = \sqrt{47.4372...^2 - 17^2} = 44.2865...$

$\text{DB} = \sqrt{64.7015...^2 + 44^2} = 62.4832...$ ***(A1)***

$\tan^{-1}\left(\dfrac{17}{62.4832...}\right)$ ***(M1)***

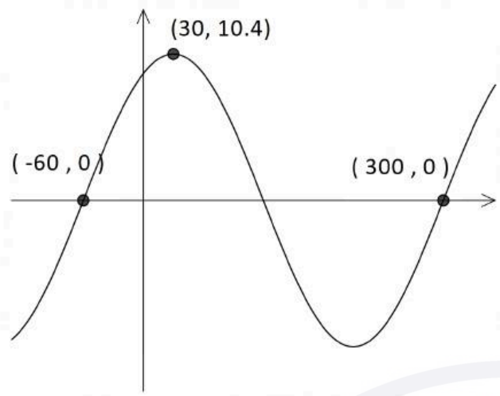
$= 15.2^\circ \ \ (15.2329...^\circ) \quad \left(\text{or } 0.266 \text{ radians } \ (0.265866...)\right)$ ***A1***

[3 marks]

Total [5 marks]

Question 28

let $at = x$ and plot $V_1 + V_2$ curves on GDC **(M1)**



attempt to find maximum **(M1)**

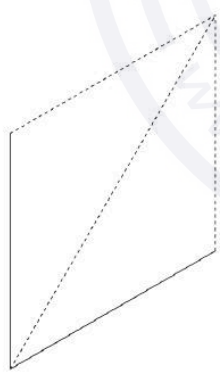
$V = 10.4$ **A1**

attempt to find any x-axis intercept (either -60 or 300) **(M1)**

$\theta = 60$ (degrees) $(\theta = -300$ (degrees)$)$ **A1**

OR

considering the rhombus



(M1)

$V = \sqrt{6^2 + 6^2 - 2 \times 6 \times 6 \cos 120^\circ}$ **(M1)**

$\left(= \sqrt{108} = 6\sqrt{3}\right) = 10.4$ $(10.3923\ldots)$ **A1**

$\theta = 60$ (degrees) **A2**

[Total: 5 marks]

Question 29

(a) attempt to find the vector product (e.g. one term correct) ***(M1)***

$$\begin{pmatrix}0\\6\\1\end{pmatrix}\times\begin{pmatrix}7\\3\\0\end{pmatrix}=\begin{pmatrix}-3\\7\\-42\end{pmatrix}$$ ***A1***

[2 marks]

(b) **METHOD 1**

attempt to use the vector product formula for the area of triangle

(condone incorrect signs and missing $\frac{1}{2}$) ***(M1)***

area $=\frac{1}{2}\sqrt{3^2+7^2+42^2}$

$=21.3\ (\text{m}^2)\ \left(21.3424\ldots,\ \frac{1}{2}\sqrt{1822}\right)$ ***A1***

METHOD 2

find θ using $\vec{\text{AB}}\times\vec{\text{AC}}=\left|\vec{\text{AB}}\right|\left|\vec{\text{AC}}\right|\sin\theta$ ***(M1)***

$\theta=67.1^\circ\ \ (67.1350^\circ\ldots,\ 1.171728\ldots.$ radians)

then area $=\frac{1}{2}\left|\vec{\text{AB}}\right|\left|\vec{\text{AC}}\right|\sin\theta$

$=21.3\ (\text{m}^2)\ \left(21.3424\ldots,\ \frac{1}{2}\sqrt{1822}\right)$ ***A1***

[2 marks]

(c) $\text{AC} = 7.61577... \left(\sqrt{58}\right)$ ***(A1)***

setting the area formula $\frac{1}{2} \times \text{base} \times \text{height}$ equal to their part (b) ***(M1)***

$$\text{BX} = \frac{2 \times 21.3424...}{\sqrt{58}}$$

$= 5.60 \ (5.60480...)$ ***A1***

Note: Award ***A1*** for 5.6.
Award ***A1*** for 5.59 (5.5936…) from the use of 21.3 to 3 sf.

[3 marks]

(d) attempting to set up a trig ratio ***(M1)***

angle is $\arcsin\left(\frac{1}{\text{BX}}\right)$

$10.3^\circ \ (10.2776...^\circ,\ 0.179378\ldots$ radians) ***A1***

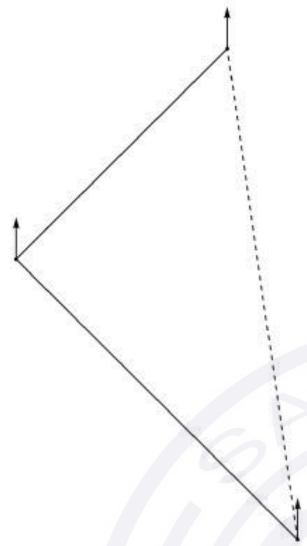
[2 marks]

[Total: 9 marks]

Question 30

METHOD 1

diagram showing (approximately) correct directions (and order) for the 315° and 045° ***(A1)***



recognizing right angle triangle ***(M1)***

correct expression to find second angle in triangle ***(A1)***

e.g. $\arctan\left(\frac{6}{8}\right)$ **OR** $\arctan\left(\frac{8}{6}\right)$

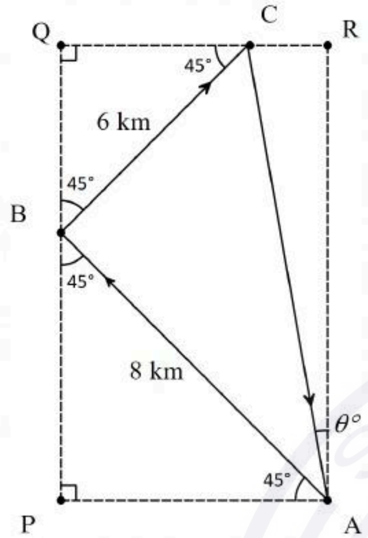
correct expression to find bearing ***(A1)***

e.g. $\arctan\left(\frac{6}{8}\right)+135^\circ$ **OR** $360^\circ-\left(\arctan\left(\frac{8}{6}\right)+135^\circ\right)$

$=172^\circ$ $(171.869...^\circ)$ ***A1***

METHOD 2

diagram showing (approximately) correct directions (and order) for the $315°$ and $045°$ (these may be shown in reverse as the return journey) ***(A1)***



finding the lengths marked AP, BP, CQ and BQ in the diagram ***(M1)***

$$\mathrm{AP} = \mathrm{BP} = 8\frac{\sqrt{2}}{2} = 5.6568...$$

$$\mathrm{CQ} = \mathrm{BQ} = 6\frac{\sqrt{2}}{2} = 4.2426...$$

Note: This may be done using a vector approach.

using $\tan\theta° = \dfrac{\mathrm{AP} - \mathrm{CQ}}{\mathrm{PB} + \mathrm{BQ}}$ or equivalent to find the direction of AC ***(A1)***

correct expression to find bearing ***(A1)***

$$180° - \arctan\left(\frac{8\frac{\sqrt{2}}{2} + 6\frac{\sqrt{2}}{2}}{8\frac{\sqrt{2}}{2} - 6\frac{\sqrt{2}}{2}}\right)$$

$= 172° \quad (171.869...°)$ ***A1***

[Total: 5 marks]

Question 31

(a) attempt to use distance formula for points D and A *(M1)*

$DA = \sqrt{11^2 + 7^2}$

$= 13.0$ (miles) $(13.0384\ldots,\ \sqrt{170})$ ***A1***

> **Note:** Accept 13 miles. Award ***M0A0*** for finding the equation of the line DA.
> DA may be seen in part (b) but this should not be accepted as answer for part (a).

[2 marks]

(b) $\left(DB = \sqrt{13^2 + 5^2} =\right)\ 13.9\ \left(13.9283\ldots,\ \sqrt{194}\right)$ **AND**

$\left(DC = \sqrt{4^2 + 12^2} =\right)\ 12.6\ \left(12.6491\ldots,\ \sqrt{160}\right)$ ***A1***

recognizing closest town is best estimate *(M1)*

(town C is closest)

$30\ ^{\circ}C$ ***A1***

> **Note:** If their DA from part (a) is the shortest length, then allow ***FT*** in (b).

[3 marks]

[Total: 5 marks]

Question 32

(a) $v_B = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ ***(A1)***

attempt to find any relevant angle ***(M1)***

$\tan^{-1}\left(\frac{3}{2}\right) \quad (= 56.3099...^\circ)$

$(90^\circ + 56.3099...^\circ =)\ 146^\circ \quad (146.3099...^\circ)$ ***A1***

[3 marks]

(b) setting $1 + 2t = -2 + 4t$ ***(M1)***

$t = 1.5$ (hrs.) ***A1***

[2 marks]

(c) $\boldsymbol{r}_B - \boldsymbol{r}_A = (-3 + 2t)\boldsymbol{i} + (-7 + 4t)\boldsymbol{j}$ ***(M1)***

$-3 + 2t = -(-7 + 4t)$ ***(M1)***

$t = 1.67\ (\text{hrs.})\ \left(1.66666..., \frac{5}{3}\right)$ ***A1***

[3 marks]

Total [8 marks]

Question 33

(a) $30\sin(t + 60^\circ) + 60\sin(t + 10^\circ)$ ***(M1)***

finding maximum graphically

$82.5\ (\text{V})\ \ (82.5471...)$ ***A1***

Note: Award ***M1A0*** for 83.

[2 marks]

(b) recognizing that a is still 1 ***A1***

$V_0 = 82.5$ ***A1***

attempt to find an x-intercept of combined voltage ***(M1)***

$b = 26.2^\circ\ \ (26.1643...^\circ)$ **OR** any other correct x-intercept ***A1***

Note: May be seen in the final answer. Award ***M1A0*** for $b = 26$ with no working.

$\left(V_{\text{TOT}} = 82.5\sin(t + 26.2^\circ)\ \ (82.5471...\sin(t + 26.1643...^\circ))\right)$

[4 marks]

Total [6 marks]

Question 34

(a) attempt to create a 5x5 adjacency matrix **(M1)**

$$M = \begin{pmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$ **A2**

> **Note:** Allow the transposed matrix. Award ***A2*** for all entries correct, ***A1*** if one or two entries are incorrect, ***A0*** otherwise.
> Answer presented in markscheme assumes ABCDE ordering of rows and columns; accept other orders provided they are clearly communicated.
> Award ***A1*** if the zeroes are replaced by blank cells.

[3 marks]

(b) (i) recognizing need to find M^7 **(M1)**

$$M^7 = \begin{pmatrix} 8 & 8 & 17 & 8 & 13 \\ 8 & 10 & 19 & 17 & 14 \\ 6 & 11 & 16 & 10 & 17 \\ 11 & 8 & 19 & 14 & 10 \\ 2 & 6 & 8 & 11 & 8 \end{pmatrix}$$

2 (routes) ***A1***

(ii) vertices visited in order are

EITHER

$E \to D \to C \to B \to C \to B \to D \to A$ ***A2***

OR

$E \to D \to C \to B \to C \to E \to D \to A$ ***A2***

[4 marks]

Total [7 marks]

Question 35

(a) **EITHER**

$\frac{4}{3}\pi(3.4)^3$ ***(A1)***

Multiplying their volume by $\frac{4}{5}$ ***(M1)***

OR

$\frac{4}{3}\pi(3.4)^3$ ***(A1)***

Subtracting $\frac{1}{5}$ of their volume ***(M1)***

$\left(\frac{4}{3}\pi(3.4)^3 - \frac{1}{5}\times\frac{4}{3}\pi(3.4)^3\right)$

Note: The ***M1*** can be awarded for a final answer of 32.9272… seen without working.

THEN

$132\ \text{cm}^3\ \ (131.708\ldots\ \text{cm}^3)$ ***A1***

[3 marks]

(b) $\pi\times3\times11$ ***(A1)***

$103.672\ldots\ (\text{cm}^2)$ **OR** $33\pi(\text{cm}^2)$

$104\ (\text{cm}^2)$ ***A1***

[2 marks]

Total [5 marks]

Question 36

(a) vector from Q to any point in L or vice versa

$$=\begin{pmatrix}1+\lambda\\3+\lambda\\2\lambda\end{pmatrix}-\begin{pmatrix}11\\-1\\3\end{pmatrix}=\begin{pmatrix}-10+\lambda\\4+\lambda\\2\lambda-3\end{pmatrix}$$ ***(M1)***

EITHER (scalar product)

attempt to use scalar product ***(M1)***

$$\begin{pmatrix}-10+\lambda\\4+\lambda\\2\lambda-3\end{pmatrix}\bullet\begin{pmatrix}1\\1\\2\end{pmatrix}=0$$

$$-10+\lambda+4+\lambda+4\lambda-6=0$$

OR (distance formula)

attempt to use distance formula ***(M1)***

minimizing $(-10+\lambda)^2+(4+\lambda)^2+(-3+2\lambda)^2$

THEN

$\lambda=2$ ***(A1)***

point $\mathrm{P}(3, 5, 4)$ ***A1***

Note: Do not award final ***A1*** for P given as a vector.

[4 marks]

(b) $\overrightarrow{\mathrm{PQ}}=\begin{pmatrix}8\\-6\\-1\end{pmatrix}$ ***(A1)***

attempt to use vector product ***(M1)***

(perpendicular vector =) $\begin{pmatrix}8\\-6\\-1\end{pmatrix}\times\begin{pmatrix}1\\1\\2\end{pmatrix}$

$\begin{pmatrix}-11\\-17\\14\end{pmatrix}$ ***A1***

Note: Award final ***A1*** for any multiple (positive or negative) of the answer given here.

[3 marks]

[Total 7 marks]

Question 37

(a) $(4, 8)$ **A1**

[1 mark]

(b) attempt to find the gradient of AC **(M1)**

$\frac{13-3}{8-0}, \frac{10}{8}, \left(\frac{5}{4}\right), (1.25)$

attempt to substitute **their** coordinates and the negative reciprocal of **their** gradient into the equation of a straight line **(M1)**

$y-8=-\frac{4}{5}(x-4)$ **OR** $8=-\frac{4}{5}(4)+c$ **OR** $c=11.2$

$y-8=-\frac{4}{5}(x-4) \quad (y=-0.8x+11.2,\ 4x+5y-56=0)$ **A1**

[3 marks]

(c) (i) attempt to find one distance from a farm to any closest vertex **M1**

finding a correct distance from at least two distinct vertices **A1**

$7.58968\ldots, 4.472135\ldots \left(\sqrt{20}\right), 5.830951\ldots \left(\sqrt{34}\right)$

$\left(\frac{9}{11}, \frac{116}{11}\right)$ (is furthest) **A1**

(ii) $7.59\ \text{(km)}\ (=7.58968\ldots)$ **A1**

[4 marks]
[Total 8 marks]

Question 38

(a) recognizing supplementary angles or acute angles in right-triangles **(M1)**

$(\hat{ABC}=)\ 41^\circ+(180^\circ-112^\circ),\ 41^\circ+(90^\circ-22^\circ)$

Note: Values may be seen on diagram.

$\hat{ABC}=109^\circ$ **A1**

[2 marks]

(b) $\hat{ACB}=49^\circ$ (may be seen in part (a)) **(A1)**

attempt to substitute into the sine rule (or equivalent) **(M1)**

$\frac{AC}{\sin 109^\circ}=\frac{100}{\sin 49^\circ}$ **(A1)**

$AC=125\ \text{(km)}\ (=125.282\ldots)$ **A1**

[4 marks]
[Total 6 marks]

Question 39

(a) (upper bound =) 0.525 (m) **A1**
(lower bound =) 0.515 (m) **A1**

Note: Accept an answer in interval notation or written as an inequality.

[2 marks]

(b) **METHOD 1 Convert REC to linear metres**
attempt to convert REC to metres using their lower bound ***(M1)***
$440 \times 0.515 \ (= 226.6)$ **OR** $280 \times 0.515 \ (= 144.2)$ seen

attempt to use the formula for the volume of a right pyramid ***(M1)***

$(V =) \ \frac{1}{3}(440 \times 0.515)^2(280 \times 0.515)$ ***(A1)***

$2\,470\,000 \ (\text{m}^3) \quad (2\,468106.051\ldots, \ 2.47 \times 10^6)$ ***A1***

METHOD 2 Convert REC to cubic metres
attempt to use the formula for the volume of a right pyramid ***(M1)***

$(V =) \ \frac{1}{3}(440)^2(280) \ (= 18\,069\,333.33\ldots)$
attempt to convert 1 cubic REC to cubic metres using their lower bound ***(M1)***
(1 cubic REC =) 0.515^3

$(V =) \ \frac{1}{3}(440)^2(280) \times (0.515)^3$ ***(A1)***
$2\,470\,000 \ (\text{m}^3) \quad (2\,468106.051\ldots, \ 2.47 \times 10^6)$ ***A1***

[4 marks]
[Total 6 marks]

Question 40

(a) $\sqrt{6^2+6^2+3^2}$ *(A1)*

$=9\ \ (\mathrm{m\,s^{-1}})$ ***A1***

[2 marks]

(b) $\boldsymbol{h}=\begin{pmatrix}-38\\134\\315\end{pmatrix}+t\begin{pmatrix}15\\-20\\-60\end{pmatrix}$ ***A1***

[1 mark]

(c) equating one component from each *(M1)*

e.g. $7+6t=-38+15t$

$t=5$ *(A1)*

substituting their t-value into either equation *(M1)*

$\begin{pmatrix}-38\\134\\315\end{pmatrix}+5\begin{pmatrix}15\\-20\\-60\end{pmatrix}$

$=\begin{pmatrix}37\\34\\15\end{pmatrix}$ ***A1***

[4 marks]

[Total: 7 marks]

Question 41

(a) $\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ ***A1***

Note: Accept any valid vector notation. Do not accept $(1, 1, 2)$.

[1 mark]

(b) (i) **EITHER**

use of scalar product formula to find angle $\text{B}\hat{\text{A}}\text{D}$ ***(M1)***

$$\cos \text{B}\hat{\text{A}}\text{D} = \frac{\begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}}{5\sqrt{6}}$$ ***(A1)***

OR

use of cosine rule to find $\text{B}\hat{\text{A}}\text{D}$ ***(M1)***

$$\cos \text{B}\hat{\text{A}}\text{D} = \frac{6+25-17}{2 \times \sqrt{6} \times 5}$$ ***(A1)***

THEN

$\text{B}\hat{\text{A}}\text{D} = 55.1^\circ$ $(55.1417...^\circ,\ 0.962405...)$ ***A1***

Note: If the direction of one of the vectors is reversed, leading to an obtuse angle $(124.858...^\circ)$ between the vectors, then award ***M1A1A0***.

(ii) **EITHER**

an attempt at using vector product ***(M1)***

$\frac{1}{2}\left|\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \times \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}\right|$ or equivalent

attempt to apply formula (e.g. one correct component) ***(M1)***

$$= \frac{1}{2}\left((1\times 0 - 4\times 2)\boldsymbol{i} - (1\times 0 - 2\times 3)\boldsymbol{j} - (1\times 4 - 1\times 3)\boldsymbol{k}\right)$$

$\frac{1}{2}\left|\begin{pmatrix} -8 \\ 6 \\ 1 \end{pmatrix}\right|$ ***(A1)***

$= 5.02\ \left(5.02493,\ \frac{\sqrt{101}}{2}\right)$ ***A1***

OR

use of formula for the area of a triangle *(M1)*

area = $\frac{1}{2}\left|\begin{pmatrix}3\\4\\0\end{pmatrix}\right|\left|\begin{pmatrix}1\\1\\2\end{pmatrix}\right|\sin 55.1417\ldots$ *(A1)(A1)*

$= 5.02$ ***A1***

Note: Award ***A1*** for the lengths AB and AD and ***A1*** for the angle.

[7 marks]

(c) **EITHER**

$\vec{AD} = \vec{BC} = \begin{pmatrix}1\\1\\2\end{pmatrix}$ **OR** $\vec{AB} = \vec{DC} = \begin{pmatrix}3\\4\\0\end{pmatrix}$ ***A1***

One pair of opposite sides have equal length AND are parallel ***R1***

OR

$\vec{AD} = \begin{pmatrix}1\\1\\2\end{pmatrix}, \vec{BC} = \begin{pmatrix}1\\1\\2\end{pmatrix}, \vec{AB} = \begin{pmatrix}3\\4\\0\end{pmatrix}, \vec{DC} = \begin{pmatrix}3\\4\\0\end{pmatrix}$ ***A1***

$\vec{AD} = \vec{BC} = \begin{pmatrix}1\\1\\2\end{pmatrix}$ and $\vec{AB} = \vec{DC} = \begin{pmatrix}3\\4\\0\end{pmatrix}$

both pairs of opposite sides are parallel (or have equal length) ***R1***

Note: Both pairs of opposite angles are equal is also valid.

THEN

hence ABCD is a parallelogram ***AG***

[2 marks]

[Total: 8 marks]

Question 42

attempt to substitute into area of triangle formula ***(M1)***

(sheep's field area =) $0.5 \times 15 \times 21 \times \sin(78°)$

$= 154.058\ldots\ (\text{m}^2)$ ***A1***

EITHER

(goat's field area =) $\frac{282}{360} \times \pi \times 8^2$ ***(A1)(A1)***

Note: Award ***A1*** for 282, ***A1*** for correct entries in formula (including their 282).

OR

$\pi \times 8^2 - \frac{78}{360} \times \pi \times 8^2$ ***(M1)(A1)***

Note: Award ***A1*** for minor sector area, ***M1*** for subtracting their sector area from circle area.

THEN

$= 157.498\ldots\left(\frac{752\pi}{15}\right)\ (\text{m}^2)$ ***A1***

the goat has most area by $3.44\ (\text{m}^2)\ (3.44026\ldots)$ ***A1***

Note: Accept 154 and 157 for the intermediate ***A1*** marks, but do NOT follow through within the question; a final answer of $3\ \text{m}^2$ is awarded ***A0***.

[Total: 6 marks]

Question 43

attempt to find gradient **(M1)**

EITHER

gradient of tangent $= -\tan 75^\circ \quad (= -3.73205\ldots,\ -2-\sqrt{3})$ **(A1)(A1)**

Note: Award ***A1*** for negative and ***A1*** for $\tan 75^\circ$ (or equivalent).

OR

gradient of tangent $= \tan 105^\circ \quad (= -3.73205\ldots)$ **(A2)**

THEN

$\frac{dy}{dx} = -5.5\sin(1.1x)$ **(A1)**

Note: Award ***(A1)*** for a labelled sketch of the derivative function.

equating derivative to their gradient **(M1)**

$-5.5\sin(1.1x) = -3.73205...$ **OR** line on graph

$x = 0.677993...$ **(A1)**

Note: Award ***(A1)(M1)A0*** for an answer of $x = 38.8$, from calculator being in degrees. Award ***A0M1A0*** if "$\frac{d}{dx}(5\cos(1.1x)) = -3.73205...$" seen, but leading to an incorrect x-value.

height = $= 5\cos(1.1\times 0.677993...)$ **(M1)**

$= 3.67$ (m) (3.67274...) ***A1***

[Total 8 marks]

Question 44

(recognition that OB is a radius)

(radius $=$) $\sqrt{5^2+8^2}$ $(=\sqrt{89})$ ***(A1)***

EITHER (finding angle BOQ)

correct calculation for finding $\text{B}\hat{\text{O}}\text{A}$ ***(A1)***

$\text{B}\hat{\text{O}}\text{A}=\arctan\left(\frac{8}{5}\right)$ **OR** $\tan \text{B}\hat{\text{O}}\text{A}=\frac{8}{5}$

expressing $\text{B}\hat{\text{O}}\text{Q}$ as $90+\text{B}\hat{\text{O}}\text{A}$ ***(M1)***

$\text{B}\hat{\text{O}}\text{Q}=90+\arctan\left(\frac{8}{5}\right)$ **OR** $\text{B}\hat{\text{O}}\text{Q}=\frac{\pi}{2}+\arctan\left(\frac{8}{5}\right)$

$\left(\text{B}\hat{\text{O}}\text{Q}=\right)$ $147.994°\ldots$ **OR** $2.58299\ldots$

substituting *their* radius and angle BOQ correctly into arc length formula ***(M1)***

(arc BQ $=$) $\frac{90+\arctan\left(\frac{8}{5}\right)}{360}\times 2\pi\left(\sqrt{5^2+8^2}\right)$ **OR** $\left(\frac{\pi}{2}+\arctan\left(\frac{8}{5}\right)\right)\times\left(\sqrt{5^2+8^2}\right)$

24.4 (m) $(24.3679\ldots)$ ***A1***

OR (finding angle BOP)

correct calculation for finding angle $\text{B}\hat{\text{O}}\text{P}$ ***(A1)***

$\text{B}\hat{\text{O}}\text{P}=\arctan\left(\frac{5}{8}\right)$ **OR** $\tan \text{B}\hat{\text{O}}\text{P}=\frac{5}{8}$

substituting *their* radius and $\text{B}\hat{\text{O}}\text{P}$ correctly into arc length formula ***(M1)***

(arc BP $=$) $\frac{\arctan\left(\frac{5}{8}\right)}{360}\times 2\pi\left(\sqrt{5^2+8^2}\right)$

subtracting *their* arc BP from arc PQ ***(M1)***

(arc BQ $=$) $\pi\sqrt{5^2+8^2}-\frac{\arctan\left(\frac{5}{8}\right)}{360}\times 2\pi\left(\sqrt{5^2+8^2}\right)$

24.4 (m) $(24.3679\ldots)$ ***A1***

[Total: 5 marks]

Question 45

(a) B and C **A1**

[1 mark]

(b) correct intervals seen ($x \le 5$ (or $x < 5$) **AND** $x \ge 5$ (or $x > 5$)) **A1**

Note: The case of $x = 5$ must be included for this ***A1*** to be awarded.

attempt to add edges to $33 + x$ ***(M1)***

(If $x < 5$ (or $x \le 5$) then repeat BC and) length is $33 + 2x$ ***A1***

(If $x > 5$ (or $x \ge 5$) then repeat AB and AC and) length is $(33 + x + 5 =) 38 + x$

A1

Note: If the intervals are not explicit, award at most ***A0(M1)A1A1***.

[4 marks]

[Total 5 marks]

Question 46

(a) $y = 0.5x - 1$ ***A1A1***

Note: Award ***A1*** for $0.5x$ and ***A1*** for -1 (or equivalent equation). Award at most ***A1A0*** if answer is not presented as an equation.

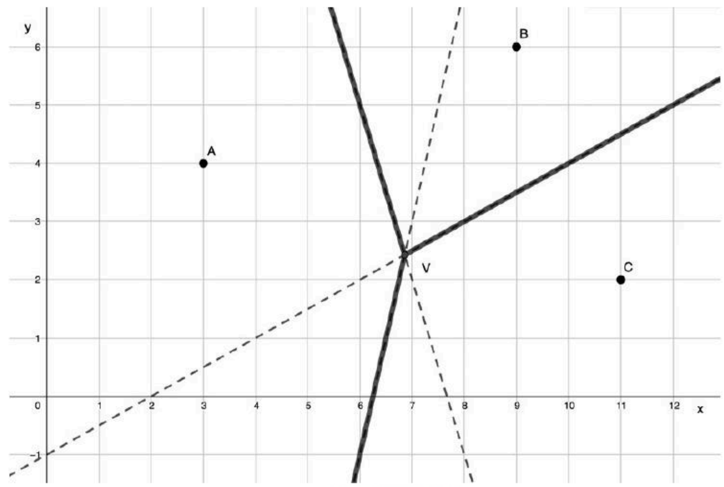
[2 marks]

(b) (6.857, 2.429) ***A1A1***

Note: If both answers are not correct to 4 sig figs, award at most ***A1A0***.
Accept $x = 6.857$, $y = 2.429$.
Award ***A1A0*** for $\left(\frac{48}{7}, \frac{17}{7}\right)$. Award ***A0A1*** for (2.429, 6.857).

[2 marks]

(c)



A2

Note: Award marks as shown in the table below. Condone edges that do not extend to the sides of the graph or beyond the x-axis.

Correct edges	Incorrect edges	Marks
3	0	***A2***
3	1	***A1A0***
3	2 or more	***A0A0***
2	0	***A1A0***
2	1	***A1A0***
2	2 or more	***A0A0***
1	0	***A1A0***
1	1 or more	***A0A0***

[2 marks]
[Total 6 marks]

Question 47

(a) $d = \frac{16.63 + 7.83}{2}$ **(A1)**

$(d =)\ 12.2\ \ (12.23)$ **A1**

[2 marks]

(b) $(a =)\ 4.4$ **A1**

[1 mark]

(c) period $= 2(356 - 173)\ \ (= 366)$ **(A1)**

$(b =)\ \frac{2\pi}{366} = \frac{\pi}{183}\ \ (= 0.0171671...)$ (Accept $b = \frac{360}{366}$ $(= 0.983606\ldots)$) **A1**

[2 marks]

(d) **EITHER**

attempt to find midpoint of x values of max and min **(M1)**

$c = \frac{-10 + 173}{2}$

OR

substitute values and solve **(M1)**

$b(173 - c) = \frac{\pi}{2}$ **OR** $16.63 = 4.4\ \sin\left(b(173 - c)\right) + 12.23$

THEN

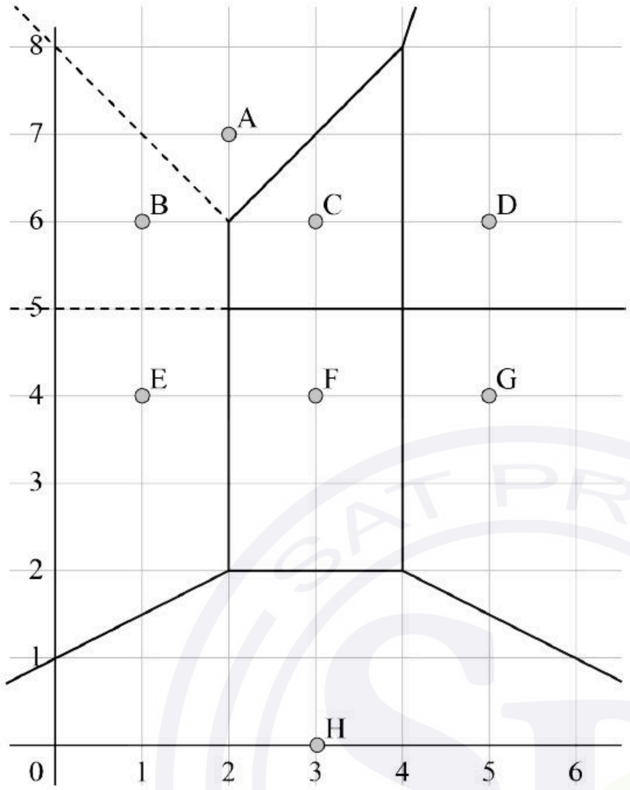
$(c =)\ 81.5$ **A1**

Note: Accept other possible positive values of $c = 81.5 + 366n$.
FT from their "c" consistent with their "a".

[2 marks]
[Total 7 marks]

Question 48

(a)



A1A1

> **Note:** Award at most ***A1A0*** if lines are not straight OR not accurate through the midpoint of AB and $(0, 8)$ and through the midpoint of BE and $(0, 5)$.

[2 marks]

(b) (i) area of $F = 6$ **OR** area of $C = 4$ ***(A1)***

$(6 \times 4 =)\ 24\ (\text{km}^2)$ ***A1***

(ii) $(4 \times 4 =)\ 16\ (\text{km}^2)$ ***A1***

[3 marks]

(c) The population in each cell might be equal ***R1***

[1 mark]

[Total 6 marks]

Question 49

(a) attempt to use the cosine rule to find $\hat{CAB}$ ***(M1)***

$(\hat{CAB} =) \cos^{-1}\left(\frac{42^2 + 35^2 - 36^2}{2 \times 42 \times 35}\right)$ ***(A1)***

$(\hat{CAB} =)$ $54.8°$ $(54.8407\ldots°)$ **OR** 0.957 $(0.957152\ldots)$ ***A1***

[3 marks]

(b) $(\text{Area} =)$ $\frac{1}{2} \times 42 \times 35 \times \sin 54.8407\ldots°$ ***(A1)***

$(\text{Area} =)$ 601 (600.903) cm^2 ***A1***

Note: Correct units must be seen for the final ***A1*** to be awarded.

[2 marks]

[Total 5 marks]