Subject – Math AI(Higher Level)
Topic - Geometry and Trigonometry
Year - May 2021 – Nov 2022
Paper -1
Answers

Question 1

(a) $15 \times 0 + 2d + 4 = 0$ **(M1)**

$d = -2$ **A1**

[2 marks]

(b) $a\begin{pmatrix}-15\\2\\4\end{pmatrix} \times \begin{pmatrix}0\\-2\\1\end{pmatrix}$ **(M1)**

$= a\begin{pmatrix}10\\15\\30\end{pmatrix} \left(= 5a\begin{pmatrix}2\\3\\6\end{pmatrix}\right)$ **A1**

magnitude is $5a\sqrt{2^2 + 3^2 + 6^2} = 14$ **M1**

$a = \frac{14}{35}$ $(= 0.4)$ **A1**

[4 marks]

Total [6 marks]

Question 2

(a) $\dfrac{\sin C\hat{A}B}{6} = \dfrac{\sin 15^\circ}{4.5}$ ***(M1)(A1)***

$C\hat{A}B = 20.2^\circ \ (20.187415\ldots)$ ***A1***

> **Note:** Award ***(M1)*** for substituted sine rule formula and award ***(A1)*** for correct substitutions.

[3 marks]

(b) $C\hat{B}D = 20.2 + 15 = 35.2^\circ$ ***A1***
(let X be the point on BD where Ollie activates the sensor)

$\tan 35.18741\ldots^\circ = \dfrac{1.8}{BX}$ ***(M1)***

> **Note:** Award ***A1*** for their correct angle $C\hat{B}D$. Award ***M1*** for correctly substituted trigonometric formula.

$BX = 2.55285\ldots$ ***A1***

$5 - 2.55285\ldots$ ***(M1)***

$= 2.45\,(\text{m})\ (2.44714\ldots)$ ***A1***

[5 marks]

Total [8 marks]

Question 3

(a) $\dfrac{50 \times \pi}{180} = 0.873\ (0.872664\ldots)$ ***A1***

[1 mark]

(b) volume $= 240\left(\pi \times 8.4^2 - \dfrac{1}{2} \times 8.4^2 \times 0.872664\ldots\right)$ ***M1M1M1***

> **Note:** Award ***M1*** $240 \times \text{area}$, award ***M1*** for correctly substituting area sector formula, award ***M1*** for subtraction of the angles or their areas.

$= 45800\ (= 45811.96071)$ ***A1***

[4 marks]

Total [5 marks]

Question 4

(a) $\dfrac{3-1}{7-3}$ *(M1)*

$= 0.5$ ***A1***

[2 marks]

(b) $y - 2 = -2(x - 5)$ ***(A1)(M1)***

Note: Award ***(A1)*** for their -2 seen, award ***(M1)*** for the correct substitution of $(5, 2)$ and their normal gradient in equation of a line.

$2x + y - 12 = 0$ ***A1***

[3 marks]

(c) every point in the cell is closer to E than any other snow shelter ***A1***

[1 mark]

Total [6 marks]

Question 5

(a) $\boldsymbol{M} = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$ ***A1A1A1***

> **Note:** Award ***A1*** for each two correct rows.

[3 marks]

(b) calculating $\boldsymbol{M}^6$ ***(M1)***

143 ***A1***

[2 marks]

Total [6 marks]

Question 6

(a) $\boldsymbol{r} = \begin{pmatrix} 0.8 \\ 1.3 \\ -0.3 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix}$ ***A1A1***

> **Note:** Award ***A1*** for each correct vector. Award ***A0A1*** if their "$\boldsymbol{r} =$" is omitted.

[2 marks]

(b) (i) $-0.3 + \lambda = 0$ ***(M1)***

$\Rightarrow \lambda = 0.3$

$\boldsymbol{r} = \begin{pmatrix} 0.8 \\ 1.3 \\ -0.3 \end{pmatrix} + 0.3 \begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.4 \\ 0 \end{pmatrix}$ ***(M1)***

P has coordinates $(0.2,\ 0.4,\ 0)$ ***A1***

> **Note:** Accept the coordinates of P in vector form.

(ii) $\sqrt{0.2^2 + 0.4^2}$ ***(M1)***

$= 0.447\text{ km}\ \ (=447\text{ m})$ ***A1***

[5 marks]

Total [7 marks]

Question 7

(a)	(i)	use of Prim's algorithm	***M1***
		BC 46	***A1***
		BD 58	***A1***
		DE 23	
		EF 47	
		Total 174	***A1***

> **Note:** Award ***M0A0A0A1*** for 174 without correct working e.g. use of Kruskal's, or with no working.
> Award ***M1A0A0A1*** for 174 by using Prim's from an incorrect starting point.

(ii) $AB + AC = 55 + 63 = 118$ ***(M1)***

$174 + 118 = 292$ minutes ***A1***

[6 marks]

(b) delete a different vertex ***A1***

[1 mark]

Total [7 marks]

Question 8

attempt to find any relevant maximum value ***(M1)***

largest sides are 56.5 and 82.5 ***(A1)***

smallest possible angle is 102.5 ***(A1)***

attempt to substitute into area of a triangle formula ***(M1)***

$$\frac{1}{2} \times 56.5 \times 82.5 \times \sin(102.5^\circ)$$

$= 2280 \text{ m}^2 \quad (2275.37...)$ ***A1***

Total [5 marks]

Question 9

(a) (i) $\overrightarrow{CA} = \begin{pmatrix} -3 \\ -4 \\ -1 \end{pmatrix}$ ***A1***

(ii) $\overrightarrow{CB} = \begin{pmatrix} 3 \\ -4 \\ -1 \end{pmatrix}$ ***A1***

[2 marks]

(b) $\overrightarrow{CA} \times \overrightarrow{CB} = \begin{pmatrix} 0 \\ -6 \\ 24 \end{pmatrix}$ ***(M1)A1***

Note: Do not award ***(M1)*** if less than 2 entries are correct.

[2 marks]

(c) area is $\frac{1}{2}\sqrt{6^2 + 24^2} = 12.4 \text{ m}^2 \quad (12.3693\ldots, 3\sqrt{17})$ ***(M1)A1***

[2 marks]

Total [6 marks]

Question 10

(a) every point in the shaded region is closer to tower T4 ***R1***

Note: Specific reference must be made to the closeness of tower T4.

[1 mark]

(b) $(-9, 1)$ ***A1A1***

Note: Award ***A1*** for each correct coordinate. Accept $x = -9$ and $y = 1$.
Award at most ***A0A1*** if parentheses are missing.

[2 marks]

(c) correct use of gradient formula ***(M1)***

e.g. $(m =) \frac{5-3}{-9--13} \quad \left(= \frac{1}{2}\right)$

taking negative reciprocal of **their** m (at any point) ***(M1)***

edge gradient $= -2$ ***A1***

[3 marks]

Total [6 marks]

Question 11

(a) transition matrix is

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array}\begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{pmatrix} 0 & \frac{1}{3} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{5} \\ 0 & \frac{2}{3} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{5} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & \frac{2}{5} \\ \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{5} \\ \frac{1}{3} & 0 & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix} \end{array}$$

M1A1A1

Note: Allow the transposed matrix.

Award ***M1*** for a 6x6 matrix with all values between 0 and 1, and all columns (or rows if transposed) adding up to 1, award ***A1*** for one correct row (or column if transposed) and ***A1*** for all rows (or columns if transposed) correct.

[3 marks]

(b) attempting to raise the transition matrix to a large power ***(M1)***

steady state vector is $\begin{pmatrix} (0.157) \\ (0.0868) \\ (0.256) \\ (0.241) \\ (0.0868) \\ 0.173 \end{pmatrix}$ ***(A1)***

so percentage of time spent at vertex F is 17.3% ***A1***

Note: Accept 17.2%.

[3 marks]

(c) the model assumes instantaneous travel from junction to junction, ***R1***
and hence the answer obtained would be an overestimate ***R1***
OR
the mouse may eat the sugar over time ***R1***
and hence the probabilities would change ***R1***

Note: Accept any other sensible answer.

[2 marks]

Total [7 marks]

Question 12

(a) Odd vertices are A, B, D, H **A1**

Consider pairings: **M1**

Note: Award ***(M1)*** if there are four vertices not necessarily all correct.

AB DH has shortest route AD, DE, EB and DE, EH,
so repeated edges $(19+16+19)+(16+27)=97$

Note: Condone AB in place of AD, DE, EB giving $56+(16+27)=99$.

AD BH has shortest route AD and BE, EH,
so repeated edges $19+(19+27)=65$

AH BD has shortest route AD, DE, EH and BE, ED,
so repeated edges $(19+16+27)+(19+16)=97$ **A2**

Note: Award ***A1*** if only one or two pairings are correctly considered.

so best pairing is AD, BH
weight of route is therefore $582+65=647$ **A1**

[5 marks]

(b) least value of the pairings is 19 therefore repeat AD **R1**

B and H **A1**

Note: Do not award ***R0A1***.

[2 marks]

Total [7 marks]

Question 13

(a) setting a dot product of the direction vectors equal to zero *(M1)*

$$\begin{pmatrix} p \\ 2p \\ 4 \end{pmatrix} \cdot \begin{pmatrix} p+4 \\ 4 \\ -7 \end{pmatrix} = 0$$

$p(p+4)+8p-28=0$ *(A1)*

$p^2+12p-28=0$

$(p+14)(p-2)=0$

$p=-14,\ p=2$ ***A1***

[3 marks]

(b) $p=-14 \Rightarrow$

$$L_1 : r = \begin{pmatrix} 2 \\ -5 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} -14 \\ -28 \\ 4 \end{pmatrix}$$

$$L_2 : r = \begin{pmatrix} 14 \\ 7 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} -10 \\ 4 \\ -7 \end{pmatrix}$$

a common point would satisfy the equations

$2-14\lambda = 14-10\mu$

$-5-28\lambda = 7+4\mu$ *(M1)*

$-3+4\lambda = -2-7\mu$

METHOD 1

solving the first two equations simultaneously

$\lambda = -\frac{1}{2}, \mu = \frac{1}{2}$ ***A1***

substitute into the third equation: ***M1***

$-3+4\left(-\frac{1}{2}\right) \neq -2+\frac{1}{2}(-7)$

so lines do not intersect. ***R1***

Note: Accept equivalent methods based on the order in which the equations are considered.

METHOD 2

attempting to solve the equations using a GDC ***M1***

GDC indicates no solution ***A1***

so lines do not intersect ***R1***

[4 marks]

Total [7 marks]

Question 14

(a) $AC=\dfrac{380}{\tan 25^\circ}$ **OR** $AC=\sqrt{\left(\dfrac{380}{\sin 25^\circ}\right)^2-380^2}$ **OR** $\dfrac{380}{\sin 25^\circ}=\dfrac{AC}{\sin 65^\circ}$ ***(M1)***

$AC=815\text{ m}\ \ (814.912\ldots)$ ***A1***

[2 marks]

(b) **METHOD 1**

attempt to find AB ***(M1)***

$AB=\dfrac{380}{\tan 40^\circ}$

$=453\text{ m}\ \ (452.866\ldots)$ ***(A1)***

$BC=814.912\ldots-452.866\ldots$

$=362\text{ m}\ \ (362.046\ldots)$ ***A1***

METHOD 2

attempt to find HB ***(M1)***

$HB=\dfrac{380}{\sin 40^\circ}$

$591\text{ m}\ \ (=591.175\ldots)$ ***(A1)***

$BC=\dfrac{591.175\ldots\times\sin 15^\circ}{\sin 25^\circ}$

$=362\text{ m}\ \ (362.046\ldots)$ ***A1***

[3 marks]

(c) $362.046\ldots\times 4$

$=1450\text{ m h}^{-1}\ \ (1448.18\ldots)$ ***A1***

[1 mark]

Total [6 marks]

Question 15

(a) gradient $AB = \frac{4}{12} \left(\frac{1}{3}\right)$ *(A1)*

midpoint $AB: (8, 22)$ *(A1)*

gradient of bisector $= -\frac{1}{\text{gradient AB}} = -3$ *(M1)*

perpendicular bisector: $22 = -3 \times 8 + b$ **OR** $(y-22) = -3(x-8)$ *(M1)*

perpendicular bisector: $y = -3x + 46$ *A1*

[5 marks]

(b) attempt to solve simultaneous equations *(M1)*

$x + 4 = -3x + 46$

$(10.5, 14.5)$ *A1*

[2 marks]

Total [7 marks]

Question 16

(a) $\vec{OS} = \begin{pmatrix} 300 \\ 100 \end{pmatrix} + t\begin{pmatrix} -12 \\ 15 \end{pmatrix}$ *A1*

[1 mark]

(b) attempt to find the vector from L to S *(M1)*

$\vec{LS} = \begin{pmatrix} 171 \\ -183 \end{pmatrix} + t\begin{pmatrix} -12 \\ 15 \end{pmatrix}$ *A1*

EITHER

$\left|\vec{LS}\right| = \sqrt{(171-12t)^2 + (15t-183)^2}$ *(M1)(A1)*

minimize to find *t* on GDC *(M1)*

OR

S closest when $\vec{LS} \bullet \begin{pmatrix} -12 \\ 15 \end{pmatrix} = 0$ *(M1)*

$\left(\begin{pmatrix} 171 \\ -183 \end{pmatrix} + t\begin{pmatrix} -12 \\ 15 \end{pmatrix}\right) \bullet \begin{pmatrix} -12 \\ 15 \end{pmatrix} = 0$

$-2052 + 144t - 2745 + 225t = 0$ *(M1)(A1)*

(c) the alarm will sound *A1*

$|\vec{LS}| = 19.2.... < 20$ *R1*

Note: Do not award ***A1R0***.

[2 marks]

Total: [9 marks]

Question 17

METHOD 1

attempt to find AC using cosine rule **M1**

$7^2 = 10^2 + AC^2 - 2 \times 10 \times AC \times \cos 40°$ ***(A1)***

attempt to solve a quadratic equation ***(M1)***

$AC = 4.888...$ **AND** $10.432...$ ***(A1)***

minimum area = $\frac{1}{2} \times 10 \times 4.888... \times \sin(40°)$ ***M1***

$= 15.7 \text{ m}^2$ ***A1***

METHOD 2

attempt to find $A\hat{C}B$ using the sine Rule ***M1***

$\frac{\sin C}{10} = \frac{\sin 40}{7}$ ***(A1)***

$C = 66.674...°$ **OR** $113.325....°$ ***(A1)***

EITHER

$B = 180 - 40 - 113.325...$

$B = 26.675...°$ ***(A1)***

area $= \frac{1}{2} \times 10 \times 7 \times \sin(26.675...°)$ ***M1***

OR

sine rule or cosine rule to find $AC = 4.888...$ ***(A1)***

minimum area = $\frac{1}{2} \times 10 \times 4.888... \times \sin(40°)$ ***M1***

THEN

$= 15.7 \text{ m}^2$ ***A1***

Question 18

(a) (the best placement is either point P or point Q)

attempt at using the distance formula **(M1)**

$AP = \sqrt{(10-6)^2 + (6-2)^2}$ **OR**

$BP = \sqrt{(10-14)^2 + (6-2)^2}$ **OR**

$DP = \sqrt{(10-10.8)^2 + (6-11.6)^2}$ **OR**

$BQ = \sqrt{(13-14)^2 + (7-2)^2}$ **OR**

$CQ = \sqrt{(13-18)^2 + (7-6)^2}$ **OR**

$DQ = \sqrt{(13-10.8)^2 + (7-11.6)^2}$

(AP or BP or DP $=$) $\sqrt{32} = 5.66$ $(5.65685\ldots)$ **AND**

(BQ or CQ or DQ $=$) $\sqrt{26} = 5.10$ $(5.09901\ldots)$ ***A1***

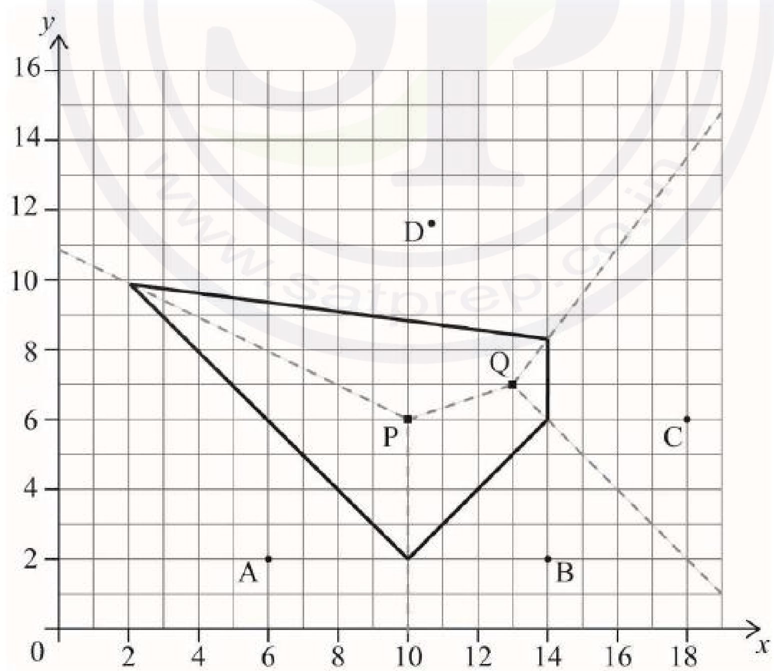
$\sqrt{32} > \sqrt{26}$ **OR** AP (or BP or DP) is greater than BQ (or CQ or DQ) ***A1***

point P is the furthest away ***AG***

[3 marks]

(b) (i) $x = 14$ ***A1***

(ii)



A1A1

[3 marks]

Total: [6 marks]

Question 19

(a) $m = \frac{6-0}{4-2} = 3$ **(M1)A1**

[2 marks]

(b) $(m =)\ -\frac{1}{3}\ (-0.333,\ -0.333333...)$ **A1**

[1 mark]

(c) an equation of line with a correct intercept and either of their gradients from (a) or (b) **(M1)**

e.g. $y = -\frac{1}{3}x + 4$ **OR** $y - 4 = -\frac{1}{3}(x - 0)$

Note: Award ***(M1)*** for substituting either of their gradients from parts (a) or (b) and point B or $(3,\ 3)$ into equation of a line.

$x + 3y - 12 = 0$ or any integer multiple **A1**

[2 marks]

(d) $(x =)\ 12$ **A1**

[1mark]

Total: [6 marks]

Question 20

(a) $(\boldsymbol{r} =) \begin{pmatrix} 1 \\ 4 \end{pmatrix} + t \begin{pmatrix} 1.2 \\ -0.6 \end{pmatrix}$ **A1**

Note: Do not condone the use of λ or any other variable apart from t.

[1 mark]

(b) when the bearing from the port is 045°, the distance east from the port is equal to the distance north from the port **(M1)**

$1 + 1.2t = 4 - 0.6t$ **(A1)**

$1.8t = 3$

$t = \frac{5}{3}$ $(1.6666...,\ 1$ hour 40 minutes$)$ **(A1)**

time is 2:40 pm $(14{:}40)$ **A1**

[4 marks]

[Total 5 marks]

Question 21

(a) $\begin{pmatrix} -3.2 \\ -4.5 \\ 6.1 \end{pmatrix}$ ***A1***

[1 mark]

(b) $\sqrt{(-3.2)^2 + (-4.5)^2 + 6.1^2}$ ***(M1)***

$8.22800\ldots \approx 8.23 \text{ m}$ ***A1***

[2 marks]

(c) **EITHER**

$\vec{AO} = \begin{pmatrix} -3.2 \\ -4.5 \\ 0.3 \end{pmatrix}$ ***A1***

$\cos\theta = \dfrac{\vec{AO} \bullet \vec{AF}}{\left|\vec{AO}\right|\left|\vec{AF}\right|}$

$\vec{AO} \bullet \vec{AF} = (-3.2)^2 + (-4.5)^2 + (0.3 \times 6.1) \ (= 32.32)$ ***(A1)***

$\cos\theta = \dfrac{32.32}{\sqrt{3.2^2 + 4.5^2 + 0.3^2} \times 8.22800\ldots}$ ***(M1)***

$= 0.710326\ldots$ ***(A1)***

Note: If $\vec{OA}$ is used in place of $\vec{AO}$ then $\cos\theta$ will be negative.
Award ***A1(A1)(M1)(A1)*** as above. In order to award the final ***A1***, some justification for changing the resulting obtuse angle to its supplementary angle **must** be seen.

OR

$\text{AO} = \sqrt{3.2^2 + 4.5^2 + 0.3^2} \ (= 5.52991\ldots)$ ***(A1)***

$\cos\theta = \dfrac{8.22800\ldots^2 + 5.52991\ldots^2 - 5.8^2}{2 \times 8.22800\ldots \times 5.52991\ldots}$ ***(M1)(A1)***

$= 0.710326\ldots$ ***(A1)***

THEN

$\theta = 0.780833\ldots \approx 0.781$ **OR** $44.7384\ldots^\circ \approx 44.7^\circ$ ***A1***

[5 marks]
[Total 8 marks]

Question 22

(a)

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

A2

Note: Award ***A2*** for the transposed matrix. Presentation in markscheme assumes columns/rows ordered A-E; accept a matrix with rows and/or columns in a different order only if appropriately communicated.
Do not ***FT*** from part (a) into part (b).

[2 marks]

(b) raising their matrix to a power of 5 ***(M1)***

$$\boldsymbol{M}^5 = \begin{pmatrix} 17 & 9 & 2 & 3 & 5 \\ 17 & 10 & 3 & 4 & 4 \\ 13 & 6 & 2 & 2 & 4 \\ 8 & 5 & 1 & 2 & 2 \\ 18 & 11 & 2 & 4 & 5 \end{pmatrix}$$

(A1)

Note: The numbers along the diagonal are sufficient to award ***M1A1***.

(the required number is $17+10+2+2+5=$) 36 ***A1***

[3 marks]
Total [5 marks]

Question 23

(a) $\sin\theta = \dfrac{2.1}{2.8}$ **OR** $\tan\theta = \dfrac{2.1}{1.85202\ldots}$ ***(M1)***

$(\theta =)\ 48.6\,°\ \ (48.5903\ldots°)$ ***A1***

[2 marks]

(b) **METHOD 1**

$\sqrt{2.8^2 - 2.1^2}$ **OR** $2.8\cos(48.5903...)$ **OR** $\dfrac{2.1}{\tan(48.5903\ldots)}$ ***(M1)***

Note: Award ***M1*** for attempt to use Pythagorean Theorem with 2.1 seen or for attempt to use cosine or tangent ratio.

$1.85\text{ m}\ \ (1.85202\ldots)$ ***(A1)***

Note: Award the ***M1A1*** if 1.85 is seen in part (a).

$(6.4 - 1.85202\ldots)$
$4.55\text{ m}\ \ (4.54797\ldots)$ ***(A1)***

Note: Award ***A1*** for 4.55 or equivalent seen, either as a separate calculation or in Pythagorean Theorem.

$\sqrt{(4.54797\ldots)^2 + 2.1^2}$
$5.01\text{ m}\ \ (5.00939\ldots\text{m})$ ***A1***

METHOD 2
attempt to use cosine rule ***(M1)***
$(c^2 =)\ 2.8^2 + 6.4^2 - 2(2.8)(6.4)\cos(48.5903\ldots)$ ***(A1)(A1)***

Note: Award ***A1*** for 48.5903...° substituted into cosine rule formula, ***A1*** for correct substitution.

$(c =)\ 5.01\text{ m}\ \ (5.00939\ldots\text{m})$ ***A1***

[4 marks]

(c) camera 1 is closer to the cash register than camera 2 (and both cameras are at the same height on the wall) ***R1***
the larger angle of depression is from camera 1 ***A1***

Note: Do not award ***R0A1***. Award ***R0A0*** if additional calculations are completed and used in their justification, as per the question. Accept "1.85<4.55" or "2.8<5.01" as evidence for the ***R1***.

[2 marks]
Total [8 marks]

Question 24

(a) setting up at least two simultaneous equations **(M1)**

$p = -0.8$ **OR** $q = 3.6$ **(A1)**

M has coordinates $(1.6, -2.6, 8.2)$ **A1**

[3 marks]

(b) using vectors $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ **(M1)**

$\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = 3$ **(A1)**

$\cos\theta = \dfrac{\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}}{\sqrt{3^2 + 2^2 + 1^2}\sqrt{1^2 + (-1)^2 + 2^2}} \quad \left(\cos\theta = \dfrac{3}{\sqrt{14}\sqrt{6}}\right)$ **(M1)**

Note: Accept correct use of vector product.

$(\theta =)$ 1.24 radians $(1.23732\ldots)$ $(70.9^\circ$ $(70.8933\ldots))$ **A1**

[4 marks]

Total [7 marks]

Question 25

(a) 78 **A1**

[1 mark]

(b) (i) 65 **A1**

(ii) **EITHER**

(period $=$) 16 (could be seen on sketch) **(M1)**

$b=\frac{2\pi}{16}$ **OR** $b=\frac{360^\circ}{16}$

$(b=)\ 0.393\ \left(0.392699\ldots,\ \frac{\pi}{8}\right)$ **OR** $(b=)\ 22.5^\circ$ **A1**

OR

$143=65\sin(4b)+78$ **(M1)**

$(\sin(4b)=1)$

$(4b=\frac{\pi}{2}$ **OR** $4b=90^\circ)$

$(b=)\ 0.393\ \left(0.392699\ldots,\ \frac{\pi}{8}\right)$ **OR** $(b=)\ 22.5^\circ$ **A1**

[3 marks]

(c) 13 **A1**

Note: Apply follow through marking only if their final answer is positive.

[1 mark]

(d) $(b=)\ 0.196\ \left(0.196349\ldots,\ \frac{\pi}{16}\right)$ **OR** $(b=)\ 11.3^\circ\ (11.25^\circ)$ **A1**

[1 mark]

Total [6 marks]

Question 26

(a)

$$\begin{array}{c c} & \begin{matrix} S & A & B & C & D & T \end{matrix} \\ \begin{matrix} S \\ A \\ B \\ C \\ D \\ T \end{matrix} & \begin{pmatrix} 0 & 1 & 1 & 1 & \boxed{2} & 0 \\ 1 & 0 & 1 & 1 & \boxed{0} & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ \boxed{2} & \boxed{0} & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix} \end{array}$$

$SD = DS = 2$ ***A1***

$AD = DA = 0$ ***A1***

[2 marks]

(b) attempt to calculate at least one of $\boldsymbol{M}^2$, $\boldsymbol{M}^3$ and $\boldsymbol{M}^4$ ***(M1)***

attempt to calculate all of $\boldsymbol{M}^2$, $\boldsymbol{M}^3$ and $\boldsymbol{M}^4$ ***(M1)***

finding at least one of the top right entries, $4, 10, 64$ ***(A1)***

78 walks ***A1***

Note: If $SD = DS = 1$ is their answer in part (a), their ***FT*** answer is $(3+8+41=)$ 52 walks.

[4 marks]

(c) because some of the walks will pass through T, before returning to T ***R1***

[1 mark]

Total [7 marks]

Question 27

(a) $\sin(21^\circ) = \dfrac{17}{\text{BF}}$ *(M1)*

$\text{BF} = 47.4 \text{ m} \ \ (47.4372...)$ ***A1***

[2 marks]

(b) **EITHER**

$\text{BE} = \sqrt{47.4372...^2 + 44^2} = 64.7015...$ *(A1)*

$\sin^{-1}\left(\dfrac{17}{\text{BE}}\right)$ *(M1)*

$= 15.2^\circ \ \ (15.2329...^\circ) \ \ \left(\text{or } 0.266 \text{ radians } \ (0.265866...)\right)$ ***A1***

OR

$\text{AD} = \sqrt{47.4372...^2 - 17^2} = 44.2865...$

$\text{DB} = \sqrt{64.7015...^2 + 44^2} = 62.4832...$ *(A1)*

$\tan^{-1}\left(\dfrac{17}{62.4832...}\right)$ *(M1)*

$= 15.2^\circ \ \ (15.2329...^\circ) \ \ \left(\text{or } 0.266 \text{ radians } \ (0.265866...)\right)$ ***A1***

[3 marks]
Total [5 marks]