**Subject – Math AI(Higher Level)**
**Topic - Number and Algebra**
**Year - May 2021 – Nov 2024**
**Paper -1**
**Answers**

## Question 1

$\log A = x\log B + y\log C + \log k$ ***(M1)***

$\log 5.74 = x\log 2.1 + y\log 3.4 + \log k$
$\log 2.88 = x\log 1.5 + y\log 2.4 + \log k$
$\log 0.980 = x\log 0.8 + y\log 1.9 + \log k$ ***M1A1***

Allow any consistent base, allow numerical equivalents.

attempting to solve their system of equations ***(M1)***

$x = 1.53,\ y = 0.505$ ***A1***

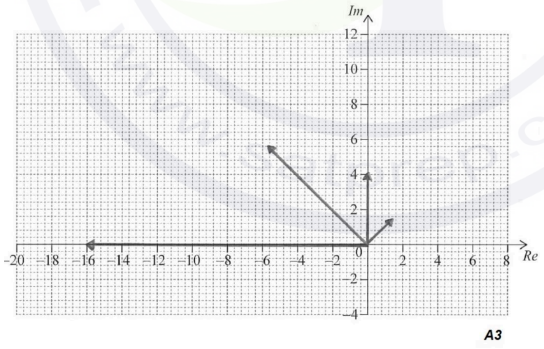
$k = 0.997$ ***A1***

***Total [6 marks]***

## Question 2

(a) (i) $4e^{\frac{\pi}{2}i},\ 8e^{\frac{3\pi}{4}i},\ 16e^{\pi i}\ \left(= 4i,\ -4\sqrt{2} + 4\sqrt{2}i,\ -16\right)$ ***(M1)A1***

(ii)




***A3***

***[5 marks]***

(b) $2^2 + 1^2 = a^2$ ***M1***

$a = \sqrt{5}\ \ (= 2.24)$ ***A1***

***[2 marks]***

***Total [7 marks]***

**Question 3**

*(Model A)*

$R = 3pe^{-0.5p}$ **M1**

predicted values

| $p$ | $R$ |
|---|---|
| 1 | 1.8196 |
| 2 | 2.2073 |
| 3 | 2.0082 |

**(A1)**

$SS_{res} = (1.8196 - 1.5)^2 + (2.2073 - 1.8)^2 + (2.0082 - 1.5)^2$ **(M1)**

$= 0.5263\ldots$ **A1**

*(Model B)*

$R = 2.5pe^{-0.6p}$

predicted values

| $p$ | $R$ |
|---|---|
| 1 | 1.372 |
| 2 | 1.506 |
| 3 | 1.2397 |

**(A1)**

$SS_{res} = 0.170576\ldots$ **A1**

chose model B **A1**

**Total [7 marks]**

**Question 4**

(a) (i) $i^2 = -1$ ***(M1)***

$w = -2 + 1 = -1$ ***A1***

(ii) $w = -1 + i + 1 = i$ ***A1***

***[3 marks]***

(b) **EITHER**

rotation of $90^\circ$ (anticlockwise, centre at the origin) ***A1A1***

**Note:** Award ***A1*** for "rotation" and ***A1*** for "$90^\circ$".

followed by a translation of $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ***A1***

**OR**

translation of $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$ ***A1***

followed by rotation of $90^\circ$ (anticlockwise, centre at the origin) ***A1A1***

**Note:** Award ***A1*** for "rotation" and ***A1*** for "$90^\circ$".

***[3 marks]***

(c) **EITHER**

move $1$ to left to $1 - i$ ***(M1)***

then rotate by $-90^\circ$ to

$-1 - i$ ***A1***

**OR**

$iz + 1 = 2 - i$

$iz = 1 - i$

$z = \dfrac{1 - i}{i}$ ***(M1)***

$-1 - i$ ***A1***

***[2 marks]***

***Total [8 marks]***

## Question 5

(a) (i) attempt to find $u_{20}$ using an arithmetic sequence ***(M1)***

e.g. $u_1 = 500$ and $d = 100$ **OR** $u_{20} = 500 + 1900$ **OR** $500, 600, 700, \ldots$

(Charlie ran) $2400 \text{ m}$ ***A1***

(ii) $(r =)\ 1.02$ ***(A1)***

attempt to find $u_{20}$ using a geometric sequence ***(M1)***

e.g. identifying $u_1 = 500$ and a value for $r$ **OR** $500 \times r^{19}$ **OR** $500,\ 510,\ 520.2, \ldots$

(Daniella ran) $728 \text{ m}$ $(728.405...)$ ***A1***

***[5 marks]***

(b) $500 \times 1.02^{n-1} > 500 + (n-1) \times 100$ ***(M1)***

attempt to solve inequality ***(M1)***

$n > 184.215...$

$n = 185$ ***A1***

***[3 marks]***

***Total [8 marks]***

## Question 6

(a) (i) $z_1^3 = 27e^{\frac{i\pi}{4}} (= 27e^{0.785398...i})$ ***A1A1***

**Note:** Award ***A1*** for 27 and ***A1*** for the angle in the correct form.

(ii) $\left(\frac{z_1}{z_2}\right)^4 = \left(\frac{81}{16}\right)e^{\frac{i\pi}{2}} (= 5.0625e^{1.57079...i})$ ***A1A2***

**Note:** Award ***A1*** for $\frac{81}{16}$, ***A2*** for the angle in the correct form and ***A1*** for the angle in incorrect form e.g. $\text{cis}\frac{\pi}{2}$ and/or $\frac{5\pi}{2}$.

Award ***A1*** if i is given in place of $\text{cis}\frac{\pi}{2}$.

***[5 marks]***

(b) $z_1 z_2 = 6\,\text{cis}\left(\frac{3\pi}{4} + \frac{n\pi}{16}\right)$ ***(M1)***

$= 6\,\text{cis}\left(\frac{12\pi + n\pi}{16}\right)$

$12\pi + n\pi = 32\pi$ ***(M1)***

$n = 20$ ***A1***

***[3 marks]***

***Total [8 marks]***

## Question 7

(a) recognition of geometric sequence *eg* $r = 0.82$ **(M1)**

$S_{10} = \dfrac{450\left(1-0.82^{10}\right)}{1-0.82}$ **(A1)**

$= 2160 \text{ m}$ $(2156.37...)$ **A1**

**[3 marks]**

(b) $S_{\infty} = \dfrac{450}{1-0.82}$ **(M1)**

$= 2500 < 2520$ so the balloon will not reach the required height. **A1**

**[2 marks]**

(c) horizontal motion not taken into account,
rate of cooling will not likely be linear,
balloon is considered a point mass / size of balloon not considered,
effects of wind/weather unlikely to be consistent,
a discrete model has been used, whereas a continuous one may offer greater accuracy **R1**

**Note:** Accept any other sensible answer.

**[1 mark]**

**Total [6 marks]**

## Question 8

(a) $£495 \times 0.9^5 = £292$ $(£292.292...)$ **(M1)A1**

**[2 marks]**

(b) $495 \times 0.9^k = 2200 \times 0.85^k$ **(M1)**

$k = 26.1$ $(26.0968...)$ **A1**

**Note:** Award ***M1A0*** for $k-1$ in place of $k$.

**[2 marks]**

(c) depreciation rates unlikely to be constant (especially over a long time period) **R1**

**Note:** Accept reasonable answers based on the magnitude of $k$ or the fact that "value" depends on factors other than time.

**[1 mark]**

**Total [5 marks]**

## Question 9

(a) $m = -0.695\ (-0.695383...)$; $b = 4.63\ (4.62974...)$ **A1A1**

**[2 marks]**

(b) $\ln x = -0.695(\ln 25) + 4.63$ **M1**

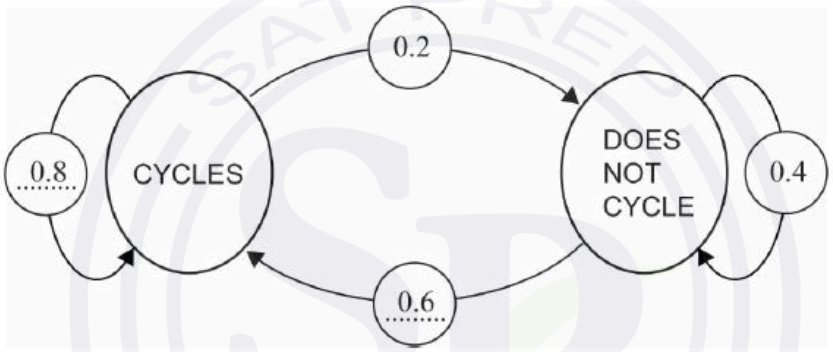
$\ln x = 2.39288...$ **(A1)**

$x = 10.9\%$ **A1**

**[3 marks]**
**Total: [5 marks]**

## Question 10

(a)




**A1A1**

**[2 marks]**

(b) $A = \begin{pmatrix} 0.8 & 0.6 \\ 0.2 & 0.4 \end{pmatrix}$ **(A1)**

$A^{180} = \begin{pmatrix} 0.75 & 0.75 \\ 0.25 & 0.25 \end{pmatrix}$ **(M1)**

0.75 **A1**

**[3 marks]**
**Total: [5 marks]**

## Question 11

(a) $10 = \frac{2}{1-r}$ *(M1)*

$r = 0.8$ *A1*

*[2 marks]*

(b) $2\times(0.8)^{n-1} < 0.5$ **OR** $2\times(0.8)^{n-1} = 0.5$ *(M1)*

$(n >)\ 7.212.....$ *(A1)*

$n = 8$ *A1*

**Note:** If $n = 7$ is seen, with or without seeing the value 7.212… then award ***M1A1A0***.

*[3 marks]*

***Total: [5 marks]***

## Question 12

(a) $N = 360$
$I\% = 3.8$
$PV = (\pm)170\,000$
$FV = 0$
$P/Y = 12$
$C/Y = 12$ *(M1)(A1)*

$(PMT =)\ 792.13\ \text{AUD}$ *A1*

*[3 marks]*

(b) (i) $N = 120$
$I\% = 3.8$
$PV = (\pm)170\,000$
$PMT = (\mp)792.13$
$P/Y = 12$
$C/Y = 12$ *(M1)(A1)*

$(FV =)\ 133019.94\ \text{AUD}$ *A1*

(ii) amount of money paid: $120\times 792.13\ (=95055.60)$ *(M1)*

loan paid off: $170\,000 - 133019.94\ (=36980.06)$ *(M1)*

interest paid: $(95055.60 - 36980.06 =)\ 58075.54\ \text{AUD}$ *A1*

*[6 marks]*

***Total: [9 marks]***

## Question 13

(a) attempt at using trapezoidal rule formula *(M1)*

$\frac{1}{2}\left(\frac{2-0}{5}\right)(30+50+2(50+60+40+20))$ *A1*

(total carbon =) 84 tonnes *A1*

*[3 marks]*

(b) $\left|\frac{84-72}{72}\right|\times 100\%$ *(M1)*

**Note:** Award *(M1)* for correct substitution of final answer in part (a) into percentage error formula.

$=16.7\%$ $(16.6666...\%)$ *A1*

*[2 marks]*

*Total: [5 marks]*

## Question 14

(a) $\log_{10} 100 = a-3$ *(M1)*

$a=5$ *A1*

*[2 marks]*

(b) **EITHER**

$N=10^{5-M}$ *(M1)*

$=\frac{10^5}{10^M}\left(=\frac{100000}{10^M}\right)$

**OR**

$100=\frac{b}{10^3}$ *(M1)*

**THEN**

$b=100000$ $(=10^5)$ *A1*

*[2 marks]*

(c) $N=\frac{10^5}{10^{7.2}}=0.00631$ $(0.0063095...)$ *A1*

## Question 15

(a) $\lambda = 1$

$\begin{pmatrix} -0.8 & 0.7 \\ 0.8 & -0.7 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ **OR** $\begin{pmatrix} 0.2 & 0.7 \\ 0.8 & 0.3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ ***(M1)***

$0.8x = 0.7y$ ***(A1)***

an eigenvector is $\begin{pmatrix} 7 \\ 8 \end{pmatrix}$ (or equivalent with integer values) ***A1***

***[3 marks]***

(b) **EITHER**

(the long-term probability matrix is given by the eigenvector corresponding to the eigenvalue equal to $1$, scaled so that the sum of the entries is $1$)

$8 + 7 = 15$ ***(M1)***

**OR**

$\begin{pmatrix} 0.2 & 0.7 \\ 0.8 & 0.3 \end{pmatrix}\begin{pmatrix} p \\ 1-p \end{pmatrix} = \begin{pmatrix} p \\ 1-p \end{pmatrix}$ ***(M1)***

**OR**

considering high powers of the matrix *e.g.* $\begin{pmatrix} 0.2 & 0.7 \\ 0.8 & 0.3 \end{pmatrix}^{50}$ ***(M1)***

$\begin{pmatrix} \frac{7}{15} & \frac{7}{15} \\ \frac{8}{15} & \frac{8}{15} \end{pmatrix}$

**THEN**

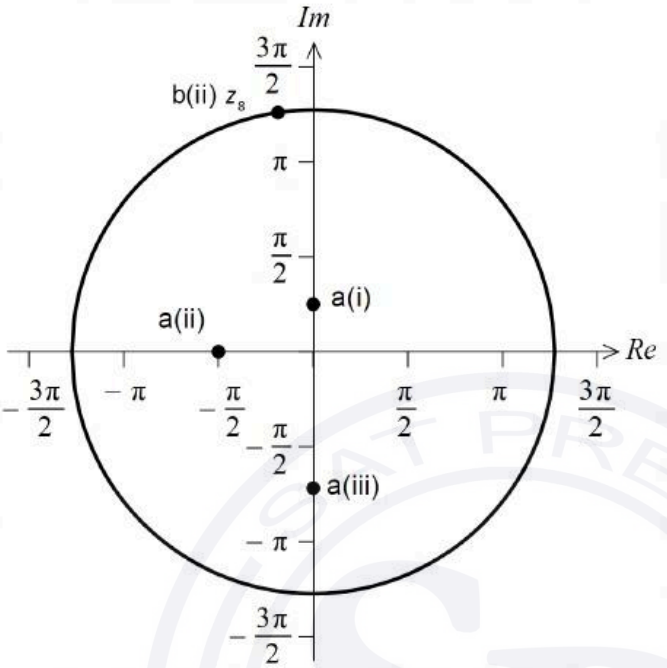
probability of being in state $\mathrm{A}$ is $\frac{7}{15}$ ***A1***

***[2 marks]***

***[Total 5 marks]***

**Question 16**

(a)




***A1A1A1***

| **Note:** | Award ***A1*** for correct modulus and ***A1*** for correct argument for part (a)(i), and ***A1*** for other two points correct.<br>The points may not be labelled, and they may be shown by line segments. |
|---|---|

***[3 marks]***

(b) (i) $\frac{1}{2}\theta = 4$ ***(M1)***

$\Rightarrow \theta = 8$ ***A1***

(ii) $z_8$ is shown in the diagram above ***A1A1***

| **Note:** | Award ***A1*** for a point plotted on the circle and ***A1*** for a point plotted in the second quadrant. |
|---|---|

***[4 marks]***
***[Total 7 marks]***

**Question 17**

(a) use of geometric sequence with $r = 0.85$ ***M1***

**EITHER**

$(0.85)^6(1.8)$ **OR** $0.678869\ldots$ **OR** $(0.85)^5(1.53)$ ***A1***

$= 0.68$ m

$= 68$ cm ***AG***

**OR**

$(0.85)^6(180)$ **OR** $(0.85)^5(153)$ ***A1***

$= 68$ cm ***AG***

***[2 marks]***

(b) **EITHER**

$(0.85)^n(1.8) > 0.1$ **OR** $(0.85)^{n-1}(1.53) > 0.1$ ***(M1)***

**Note:** If $1.8$ m (or $180$ cm) is used then ***(M1)*** only awarded for use of $n$ in $(0.85)^n(1.8) > 0.1$.
If $1.53$ m (or $153$ cm) is used then ***(M1)*** only awarded for use of $n-1$ in $(0.85)^{n-1}(1.53) > 0.1$.

17 ***A1***

**OR**

$(0.85)^{17}(1.8) = 0.114$ m and $(0.85)^{18}(1.8) = 0.0966$ m ***(M1)***

17 ***A1***

**OR**

solving $(0.85)^n(1.8) = 0.1$ to find $n = 17.8$ ***(M1)***

17 ***A1***

**Note:** Evidence of solving may be a graph **OR** the "solver" function **OR** use of logs to solve the equation. Working may use cm.

***[2 marks]***

(c) **EITHER**

distance (in one direction) travelled between first and fourth bounce

$= \frac{(1.8\times 0.85)(1-0.85^3)}{1-0.85}$ $(=3.935925)$ ***(A1)***

recognizing distances are travelled twice except first distance ***(M1)***

$1.8+2(3.935925)$

$=9.67$ m $(9.67185...$ m$)$ ***A1***

**OR**

distance (in one direction) travelled between drop and fourth bounce

$= \frac{(1.8)(1-0.85^4)}{1-0.85}$ $(=5.735925)$ ***(A1)***

recognizing distances are travelled twice except first distance ***(M1)***

$2(5.735925)-1.8$

$=9.67$ m $(9.67185...$ m$)$ ***A1***

**OR**

distance (in one direction) travelled between first and fourth bounce

$(0.85)(1.8)+(0.85)^2(1.8)+(0.85)^3(1.8)$ $(=3.935925...)$ ***(A1)***

recognizing distances are travelled twice except first distance ***(M1)***

$1.8+2(0.85)(1.8)+2(0.85)^2(1.8)+2(0.85)^3(1.8)$

$=9.67$ m $(9.67185...$ m$)$ ***A1***

**Note:** Answers may be given in cm.

***[3 marks]***

***[Total 7 marks]***

## Question 18

(a) $x+y+z=600$ ***A1***

$15x+10y+12z=7816$ ***A1***

$x=2y$ ***A1***

**Note:** Condone other labelling if clear, e.g. $a$ (adult), $c$ (child) and $s$ (student). Accept equivalent, distinct equations e.g. $2y+y+z=600$.

***[3 marks]***

(b) $x=308,\ y=154,\ z=138$ ***A1A1***

**Note:** Award ***A1*** for all three correct values seen, ***A1*** for correctly labelled as $x$, $y$ or $z$. Accept answers written in words: e.g. 308 adult tickets.

***[2 marks]***

***[Total 5 marks]***

## Question 19

(a) (one vector to the line is $\begin{pmatrix}0\\c\end{pmatrix}$ therefore) $\boldsymbol{a}=\begin{pmatrix}0\\c\end{pmatrix}$ **A1**

the line goes $m$ up for every $1$ across

(so the direction vector is) $\boldsymbol{b}=\begin{pmatrix}1\\m\end{pmatrix}$ **A1**

**Note:** Although these are the most likely answers, many others are possible.

***[2 marks]***

(b) (from GDC **OR** $6\times2-4\times3$) $|M|=0$ **A1**

***[1 mark]***

(c) **METHOD 1**

$\begin{pmatrix}X\\Y\end{pmatrix}=\begin{pmatrix}6&3\\4&2\end{pmatrix}\begin{pmatrix}x\\mx+c\end{pmatrix}=\begin{pmatrix}6x+3mx+3c\\4x+2mx+2c\end{pmatrix}$ **M1A1**

$=\begin{pmatrix}3(2x+mx+c)\\2(2x+mx+c)\end{pmatrix}$ **A1**

therefore the new line has equation $3Y=2X$ **A1**

which is independent of $m$ or $c$ **AG**

**Note:** The ***AG*** line (or equivalent) must be seen for the final ***A1*** line to be awarded.

**METHOD 2**

take two points on the line, e.g $(0,\ c)$ and $(1,\ m+c)$ **M1**

these map to $\begin{pmatrix}6&3\\4&2\end{pmatrix}\begin{pmatrix}0\\c\end{pmatrix}=\begin{pmatrix}3c\\2c\end{pmatrix}$

and $\begin{pmatrix}6&3\\4&2\end{pmatrix}\begin{pmatrix}1\\m+c\end{pmatrix}=\begin{pmatrix}6+3m+3c\\4+2m+2c\end{pmatrix}$ **A1**

therefore a direction vector is $\begin{pmatrix}6+3m\\4+2m\end{pmatrix}=(2+m)\begin{pmatrix}3\\2\end{pmatrix}$

(since $m\neq-2$) a direction vector is $\begin{pmatrix}3\\2\end{pmatrix}$

the line passes through $\begin{pmatrix}3c\\2c\end{pmatrix}-c\begin{pmatrix}3\\2\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$ therefore it always has the origin as a jump-on vector **A1**

the vector equation is therefore $\boldsymbol{r}=\mu\begin{pmatrix}3\\2\end{pmatrix}$ **A1**

which is independent of $m$ or $c$ **AG**

**METHOD 3**

$\boldsymbol{r} = \begin{pmatrix} 6 & 3 \\ 4 & 2 \end{pmatrix}\left(\begin{pmatrix} 0 \\ c \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ m \end{pmatrix}\right) = \begin{pmatrix} 3c \\ 2c \end{pmatrix} + \lambda\begin{pmatrix} 6+3m \\ 4+2m \end{pmatrix}$ **M1A1**

$= c\begin{pmatrix} 3 \\ 2 \end{pmatrix} + (2+m)\lambda\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ **A1**

$= \mu\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

where $\mu = c + (2+m)\lambda$ is an arbitrary parameter. **A1**

which is independent of $m$ or $c$ (as $\mu$ can take any value) **AG**

**Note:** The ***AG*** line (or equivalent) must be seen for the final ***A1*** line to be awarded.

***[4 marks]***

***Total [7 marks]***

**METHOD 2**

converting given expressions to cos form ***(M1)***

$V_T = 2\cos 3t + 5\cos(3t+4)$

(from graph) $A = 3.99$ $(3.99088\ldots)$ ***A1***

$V_T = 3.99\cos(Bt + C)$

either by considering transformations or inserting points

$B = 3$ ***A1***

$C = -1.89$ $(-1.89418\ldots)$ ***A1***

**Note:** Accept arguments differing by $2\pi$ e.g. $4.38900\ldots$.

(so, $V_T = 3.99\cos(3t - 1.89)$ $\left(3.99088\ldots\cos(3t - 1.89418\ldots)\right)$)

**Note:** It is possible to have $A = 3.99$, $B = -3$ with $C = 1.89$ **OR** $A = -3.99$, $B = 3$ with $C = 1.25$ **OR** $A = -3.99$, $B = -3$ with $C = -1.25$ due to properties of the cosine curve.

***[4 marks]***

(b) maximum voltage is $3.99$ $(3.99088\ldots)$ (units) ***A1***

***[1 mark]***

***Total [5 marks]***

## Question 20

(a) **METHOD 1**

recognizing that the real part is distributive ***(M1)***

$V_T = \mathrm{Re}\left(2\mathrm{e}^{3\mathrm{i}t} + 5\mathrm{e}^{3\mathrm{i}t+4\mathrm{i}}\right)$

$= \mathrm{Re}\left(\mathrm{e}^{3\mathrm{i}t}\left(2 + 5\mathrm{e}^{4\mathrm{i}}\right)\right)$ ***(A1)***

(from the GDC) $2 + 5\mathrm{e}^{4\mathrm{i}} = 3.99088\ldots\mathrm{e}^{-1.89418\ldots\mathrm{i}}$ ***(A1)***

therefore $V_T = 3.99\cos(3t - 1.89)$ $\left(3.99088\ldots\cos(3t - 1.89418\ldots)\right)$ ***A1***

**METHOD 2**

converting given expressions to cos form **(M1)**

$V_T = 2\cos 3t + 5\cos(3t+4)$

(from graph) $A = 3.99 \ \ (3.99088\ldots)$ **A1**

$V_T = 3.99\cos(Bt+C)$

either by considering transformations or inserting points

$B = 3$ **A1**

$C = -1.89 \ \ (-1.89418\ldots)$ **A1**

(so, $V_T = 3.99\cos(3t-1.89) \ \ (3.99088\ldots\cos(3t-1.89418\ldots))$ )

***[4 marks]***

(b) maximum voltage is $3.99 \ \ (3.99088\ldots)$ (units) **A1**

***[1 mark]***

***Total [5 marks]***

**Question 21**

(a) **METHOD 1 –** *(With FV=4000)*

**EITHER**

N= 10
I=1.5
FV= 4000
P/Y= 1
C/Y= 1 ***(A1)(M1)***

**Note:** Award ***A1*** for $(3.5-2=)\ 1.5$ seen and ***M1*** for all other entries correct.

**OR**

$4000 = A(1+0.015)^{10}$ ***(A1)(M1)***

**Note:** Award ***A1*** for $1.5$ or $0.015$ seen, ***M1*** for attempt to substitute into compound interest formula **and** equating to $4000$.

**THEN**

(PV =) $3447 ***A1***

**Note:** Award ***A0*** if not rounded to a whole number or a negative sign given.

**METHOD 2 –** *(With FV including inflation)*

calculate FV with inflation

$4000\times1.02^{10}$ ***(A1)***

(=4875.977…)

**EITHER**

$4000\times1.02^{10} = \text{PV}\times1.035^{10}$ ***(A1)***

**OR**

N= 10
I= 3.5
FV= 4875.977…
P/Y= 1
C/Y= 1 ***(M1)***

**Note:** Award ***M1*** for *their* FV and all other entries correct.

**THEN**

(PV =) $3457 ***A1***

**Note:** Award ***A0*** if not rounded to a whole number or a negative sign given.

**METHOD 3 –** *(Using formula to calculate real rate of return)*
(real rate of return =) 1.47058…(%) ***(A1)***

**EITHER**
$4000 = \text{PV} \times 1.0147058\ldots^{10}$ ***(A1)***
**OR**
N= 10
I= 1.47058…
FV= 4000
P/Y= 1
C/Y= 1 ***(M1)***

**Note:** Award ***M1*** for all entries correct.

**THEN**
(PV =) $3457 ***A1***

***[3 marks]***

(b) **METHOD 1 –** *(Finding the future value of the investment using PV from part (a))*
N= 10
I=3.5
PV= 3446.66…(from Method 1) **OR** 3456.67…(from Methods 2, 3)
P/Y= 1
C/Y= 1 ***(M1)***

**Note:** Award ***M1*** for interest rate 3.5 **and** answer to part (a) as PV.

(FV=) $4861.87 **OR** $4875.97 ***(A1)***
so payment required (from TVM) will be $294 **OR** $295 ***A1***

**Note:** Award ***A0*** if a negative sign given, unless already penalized in part (a).

**METHOD 2 –** *(Using FV)*
N= 10
I=3.5
PV= –1000
FV= 4875.977…
P/Y= 1
C/Y= 1 ***(A1)(M1)***

**Note:** Award ***A1*** for I=3.5 **and** FV= ±4875.977…, ***M1*** for all other entries correct **and** opposite PV and FV signs.

(PMT =) $295 (295.393) ***A1***

**Note:** Correct 3sf answer is 295, however accept an answer of 296 given that the context supports rounding up. Award ***A0*** if a negative sign given, unless already penalized in part (a).

***[3 marks]***
***Total [6 marks]***

## Question 22

(a) $\pi \times 2^2 \times \frac{30}{360}$ ***(M1)***

$= 1.047 \text{ cm}^2$ ***A1***

**Note:** Do not award the final mark if the answer is not correct to 4 sf.

***[2 marks]***

(b) attempt to substitute any two values from 1.5, 2.5, 25 or 35 into area of sector formula ***(M1)***

$\left(\text{upper bound} = \pi \times 2.5^2 \times \frac{35}{360} = \right) \ 1.91 \ \text{cm}^2 \ \ (1.90895\ldots)$ ***A1***

$\left(\text{lower bound} = \pi \times 1.5^2 \times \frac{25}{360} = \right) \ 0.491 \ \text{cm}^2 \ \ (0.490873\ldots)$ ***A1***

**Note:** Given the nature of the question, accept correctly rounded **OR** correctly truncated 3 significant figure answers.

***[3 marks]***

(c) $\left(\left|\frac{1.047 - 1.90895\ldots}{1.90895\ldots}\right| \times 100 = \right) \ 45.2 \ (\%) \ \ (45.1532\ldots)$ ***A1***

$\left(\left|\frac{1.047 - 0.490873\ldots}{0.490873\ldots}\right| \times 100 = \right) \ 113 \ (\%) \ \ (113.293\ldots)$ ***A1***

so the largest percentage error is 113 % ***A1***

**Note:** Accept 45.1 (%) (45.1428), from use of full accuracy answers. Given the nature of the question, accept correctly rounded **OR** correctly truncated 3 significant figure answers. Award ***A0A1A0*** if 113% is the only value found.

***[3 marks]***

***Total [8 marks]***

## Question 23

**METHOD 1**

$\frac{u_1}{1-r}=9$ **A1**

therefore $u_1=9-9r$

$u_1=4+u_1r$ **A1**

substitute or solve graphically: **M1**

$9-9r=4+(9-9r)r$ **OR** $\frac{4}{(1-r)^2}=9$

$9r^2-18r+5=0$

$r=\frac{1}{3}$ or $r=\frac{5}{3}$

only $r=\frac{1}{3}$ is possible as the sum to infinity exists **R1**

then $u_1=9-\left(9\times\frac{1}{3}\right)=6$

$u_3=6\times\frac{1}{3}^2=\frac{2}{3}$ **A1**

**METHOD 2**

$\frac{u_1}{1-r}=9$ **A1**

$r=\frac{u_1-4}{u_1}$ **A1**

attempt to solve **M1**

$\frac{u_1}{1-\left(\frac{u_1-4}{u_1}\right)}=9$

$\frac{u_1}{\left(\frac{4}{u_1}\right)}=9$

$(u_1)^2=36$

$u_1=\pm6$

attempting to solve both possible sequences

6, 2, ... or $-6, -10$ …

$r=\frac{1}{3}$ or $r=\frac{5}{3}$

only $r=\frac{1}{3}$ is possible as the sum to infinity exists **R1**

$u_3=6\times\left(\frac{1}{3}\right)^2=\frac{2}{3}$ **A1**

***Total [5 marks]***

## Question 24

(a) Gradient $= \frac{14.9+1.3}{6}$ $(=2.7)$ **(M1)**

$\log_{10} Q = 2.7P - 1.3$ **(A1)**

$Q = 10^{2.7P-1.3}$ **OR** $Q = 0.0501\times 10^{2.7P}$ $(=0.0501187...\times 10^{2.7P})$ **A1**

***[3 marks]***

(b) $\ln R$ on one axis and $Q$ on the other axis **A1**

***[1 mark]***

(c) $\log_{10}(4.3\ln R + 12.1) = 2.7P - 1.3$ **OR** $10^{2.7P-1.3} = 4.3\ln R + 12.1$ **(M1)**

$P = \frac{\log_{10}(4.3\ln R + 12.1) + 1.3}{2.7}$ **A1**

***[2 marks]***

***Total [6 marks]***

## Question 25

(a) $m = 1 - 2.5\log_{10}(0.0525)$ **(M1)**

$= 4.20$ $(4.19960\ldots)$ **A1**

***[2 marks]***

(b) attempt to solve $7 = 1 - 2.5\log_{10}(b)$ **(M1)**

**Note:** Accept a sketch from their GDC as an attempt to solve $7 = 1 - 2.5\log_{10}(b)$.

$b = 0.00398$ $(0.00398107...)$ **A1**

***[2 marks]***

(c) $-3.2 = \left(1 - 2.5\log_{10}(b_n)\right) - \left(1 - 2.5\log_{10}(b_p)\right)$ **M1**

$-3.2 = -2.5\log_{10}\left(\frac{b_n}{b_p}\right)$ **A1**

$\frac{b_n}{b_p} = 19.1$ $(19.0546...)$ **A1**

***[3 marks]***

***Total [7 marks]***

**Question 26**

(a) attempt to find $\det(\boldsymbol{M})$ ***(M1)***

$= 14$

$(12 \times 14) = 168 \text{ cm}^2$ ***A1***

***[2 marks]***

(b) let X have coordinates $(x, y)$

**METHOD 1**

$$\boldsymbol{M}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2t-3 \\ 6-5t \end{pmatrix}$$ ***(M1)***

$$\begin{pmatrix} x \\ y \end{pmatrix} = \boldsymbol{M}^{-1}\begin{pmatrix} 2t-3 \\ 6-5t \end{pmatrix}$$ ***(A1)***

$$\boldsymbol{M}^{-1} = \frac{1}{14}\begin{pmatrix} 1 & 4 \\ -3 & 2 \end{pmatrix}$$ ***A1***

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{14}\begin{pmatrix} 2t-3+24-20t \\ -6t+9+12-10t \end{pmatrix}$$ ***(M1)***

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{14}\begin{pmatrix} 21-18t \\ 21-16t \end{pmatrix} \quad \textbf{OR} \quad \left(\frac{21-18t}{14}, \frac{21-16t}{14}\right)$$ ***A1A1***

**METHOD 2**

writing two simultaneous equations ***(M1)***

$2x - 4y = 2t - 3$ ***(A1)***

$3x + y = 6 - 5t$ ***(A1)***

attempting to solve the equations ***(M1)***

$$(x, y) = \left(\frac{3}{2} - \frac{9t}{7}, \frac{3}{2} - \frac{8t}{7}\right)$$ ***A1A1***

***[6 marks]***

***Total [8 marks]***

## Question 27

(a) $I\% = 7.5$
$PV = \mp 800$
$PMT = \mp 500$
$FV = \pm 10\,000$
$P/Y = 12$
$C/Y = 12$ ***(M1)(A1)***

**Note:** Award ***M1*** for an attempt to use a financial app in their technology (e.g. at least four rows seen, but not necessarily correct), award ***A1*** for $\text{PMT} = -500$ or $\text{PMT} = 500$, with same sign to PV and opposite sign to FV.

$17.3070\ldots$ ***(A1)***
$(k =)\ 18$ ***A1***

**Note:** Award ***(M0)(A0)(A0)A0*** for a final answer of 17 with no working. The final answer must be an integer.

***[4 marks]***

(b) $10389 - (18 \times 500 + 800)$ **OR** $10389 - (9800)$ ***(A1)(M1)***

**Note:** Award ***(A1)*** for $10389$ $(10389.38\ldots)$ seen. Award ***(M1)*** for subtraction of their $(18 \times 500 + 800)$ from FV. ***FT*** from their value of $k$. Award ***A0M1A0*** for $10\,000 - (18 \times 500 + 800)$. Do not award the final ***A1FT*** if their answer is negative.

589 EUR ***A1***

**Note:** Final answer must be to the nearest euro.

***[3 marks]***
***Total [7 marks]***

## Question 28

recognizing that the linear equation must be expressed in log form **(M1)**

$\log y = m\log x + \log c$ (or $\log y = m\log x + C$)

**EITHER**

use of slope formula (must involve logs) **(M1)**

$$m = \frac{\log(34.822) - \log(13.1951)}{\log(4) - \log(2)} = 1.4$$ **A1**

attempt to substitute a value **(M1)**

$\log c = \log(13.1951) - 1.4\log 2 (= 0.69897...)$

$\Rightarrow c = 5$ **A1**

**OR**

$y = c.x^m$ **(A1)**

attempt to set up two equations involving power functions **(M1)**

$13.1951 = c \times 2^m$ and $34.822 = c \times 4^m$

$$2^m = \frac{34.822}{13.1951} = 2.639... \Rightarrow m = \log_2 2.639... = 1.4$$ **A1**

$$c = \frac{13.1951}{2.639...} = 5$$ **A1**

**THEN**

(so the equation is) $y = 5 \times x^{1.4}$ **A1**

## Question 29

attempt to express $V_1$ or $V_2$ in exponential form **(M1)**

*e.g.* $V_1 = \text{Im}(6\,\text{e}^{\text{i}\left(at+\frac{\pi}{6}\right)})$, $V_2 = \text{Im}(6\,\text{e}^{\text{i}\left(at+\frac{\pi}{2}\right)})$

**Note:** Accept angles in radians or degrees.

$(V_1 + V_2 =)\ 6\text{e}^{\text{i}\times\frac{\pi}{6}} + 6\text{e}^{\text{i}\times\frac{\pi}{2}}$ **(A1)**

**Note:** This mark can be awarded even if seen as part of a correct larger expression.

$= 10.4\text{e}^{1.05\text{i}}\ \left(6\sqrt{3}\,\text{e}^{\frac{\text{i}\pi}{3}}\right)$ **(A1)**

so $V$ is $10.4$ $(10.3923..., 6\sqrt{3})$ and $\theta$ is $60$ (degrees) **A1A1**

***[Total: 5 marks]***

## Question 30

(a) $N = 24$

$I = 4$

$PV = \pm 1000$

$PMT = \pm 100$

$P/Y = 12$

$C/Y = 12$ ***(M1)(A1)***

**Note:** Award ***M1*** for an attempt to use a financial app in their technology (i.e. at least three entries seen, but not necessarily correct).
Approaches that use the compound interest formula receive no marks.
Award ***A1*** for correct values of $PV$ and $PMT$ (signs must be the same) **and** a correct value of $N$.

$FV = (\$)3577.43$ ***A1***

**Note:** Award at most ***(M1)(A1)A0*** if the final answer is negative or not rounded to $2$ dp.

***[3 marks]***

(b) $N = 36.5 \ (36.4689\ldots)$ ***(A1)***

$N = 37$ (months) ***A1***

**Note:** Allow ***FT*** from incorrect GDC inputs seen in part (a) for the first ***A1*** providing that $PV$ and $FV$ have opposite signs and the resulting value of $N$ is positive.

***[2 marks]***

***[Total: 5 marks]***

## Question 31

(a) **METHOD 1 (find product of matrices first)**

$T \to T'$ is represented by $\boldsymbol{QP} = \begin{pmatrix} -4 & 1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}$ ***(M1)***

$= \begin{pmatrix} -12 & -2 \\ 3 & 7 \end{pmatrix}$ ***(A1)***

recognizing need to find their $(\boldsymbol{QP})^{-1}$ ***(M1)***

$(\boldsymbol{QP})^{-1} = \begin{pmatrix} -12 & -2 \\ 3 & 7 \end{pmatrix}^{-1}$

$= -\frac{1}{78}\begin{pmatrix} 7 & 2 \\ -3 & -12 \end{pmatrix}$ **OR** $= \begin{pmatrix} -0.0897435... & -0.0256410... \\ 0.0384615... & 0.153846... \end{pmatrix}$ ***A1***

**METHOD 2 (find inverses of both matrices first)**

recognizing need to find inverse of both $\boldsymbol{P}$ and $\boldsymbol{Q}$ ***(M1)***

$\boldsymbol{P}^{-1} = \begin{pmatrix} \frac{1}{3} & -\frac{1}{6} \\ 0 & \frac{1}{2} \end{pmatrix}$ **AND** $\boldsymbol{Q}^{-1} = \begin{pmatrix} -\frac{3}{13} & \frac{1}{13} \\ \frac{1}{13} & \frac{4}{13} \end{pmatrix}$ ***(A1)***

$T' \to T$ is represented by $\boldsymbol{P}^{-1}\boldsymbol{Q}^{-1} = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}^{-1}\begin{pmatrix} -4 & 1 \\ 1 & 3 \end{pmatrix}^{-1}$ ***(M1)***

$= -\frac{1}{78}\begin{pmatrix} 7 & 2 \\ -3 & -12 \end{pmatrix}$ **OR** $= \begin{pmatrix} -0.0897435... & -0.0256410... \\ 0.0384615... & 0.153846... \end{pmatrix}$ ***A1***

**Note:** In METHOD 1, award ***M1A0M1A0*** if they multiply the matrices in the wrong order.
In METHOD 2, award ***M1A1M1A0*** if they multiply the matrices in the wrong order.

***[4 marks]***

(b) $\left(\det\left[-\frac{1}{78}\begin{pmatrix} 7 & 2 \\ -3 & -12 \end{pmatrix}\right] =\right) -\frac{1}{78}$ **OR** $\left(\det\begin{pmatrix} -12 & -2 \\ 3 & 7 \end{pmatrix} =\right) -78$ ***(A1)***

area of $T' = \left|\det \boldsymbol{QP}\right| \times$ area of $T$ **OR** area of $T = \left|\det(\boldsymbol{QP})^{-1}\right| \times$ area of $T'$ ***(M1)***

$\Rightarrow$ area of $T = 273 \times \frac{1}{78}$

$= 3.5$ (cm$^2$) ***A1***

**Note:** Award ***(A1)(M0)A0*** for an answer of $-3.5$ (cm$^2$) with or without working. Accept an answer of $4.04$ (cm$^2$) from use of 3sf values in their answer to part (a).

***[3 marks]***

***Total [7 marks]***

## Question 32

(a) **METHOD 1 (use of financial app in GDC)**

| | | |
|---|---|---|
| $N = 5$ | **OR** | $N = 20$ |
| $I\% = 1.2$ | | $I\% = 1.2$ |
| $PV = \pm 520$ | | $PV = \pm 520$ |
| $P/Y = 1$ | | $P/Y = 4$ |
| $C/Y = 4$ | | $C/Y = 4$ |

***(M1)(A1)***

**Note:** Award ***M1*** for evidence of using the financial app on the calculator, ***A1*** for all correct entries.

($) 552.11 ***A1***

**Note:** Award at most ***(M1)(A1)A0*** if correct answer is not given to two decimal places.

**METHOD 2 (use of formula)**

attempt to substitute into compound interest formula ***(M1)***

$$520 \times \left(1 + \frac{1.2}{100 \times 4}\right)^{5 \times 4} \quad \textit{(A1)}$$

($) 552.11 ***A1***

**Note:** Award at most ***(M1)(A1)A0*** if correct answer is not given to two decimal places.

***[3 marks]***

(b) **EITHER**

$N = 5$

$I\% = 43.5$ (43.4772...(%))

$PV = \pm 520$

$FV = \mp 30$ ***(M1)(A1)A1***

**Note:** Award ***M1*** for evidence of using the finance app on the calculator, ***A1*** for all correct entries, ***A1*** for correct final answer. Condone missing -/+ sign if the correct final answer is seen.

**OR**

$$30 = 520\left(1 - \frac{r}{100}\right)^{5} \text{ (or equivalent)} \quad \textit{(M1)(A1)}$$

$(r =)$ 43.5% (43.477...%) ***A1***

**Note:** Award ***M1*** for using the compound interest formula, ***A1*** for correct substitutions and for equating to 30, ***A1*** for correct final answer. Accept $(r =) -43.5\%$.
Award ***M1A1A0*** for a final answer of 56.5%.

***[3 marks]***

***Total [6 marks]***

## Question 33

(a) probability of non veg remaining non veg **A1**

***[1 mark]***

(b) attempt to use $\det(\boldsymbol{A}-\lambda\boldsymbol{I})=0$ **(M1)**

$$\begin{vmatrix} 0.8-\lambda & 0.1 \\ 0.2 & 0.9-\lambda \end{vmatrix} = 0$$

$(0.8-\lambda)(0.9-\lambda)-0.1\times 0.2=0$ **(A1)**

$\lambda=1;\ \lambda=0.7$ **A1**

***[3 marks]***

(c) $-2a+b=0$ **M1**

$\boldsymbol{v}_1=\begin{pmatrix}1\\2\end{pmatrix}$ (accept any multiples of this answer) **A1**

$\boldsymbol{v}_1$ means that in the long term the ratio of veg to non-veg is $1:2$

(in the long term one-third of students will be veg and two-thirds will not) **A1**

***[3 marks]***

***[Total 7 marks]***

## Question 34

(a) $z = 2e^{-0.524i} \left( = 2e^{-\frac{\pi}{6}i} \right)$ **A1A1**

**Note:** Award ***A1*** for the correct modulus and ***A1*** for the correct argument.

***[2 marks]***

(b) **METHOD 1**

$z_1 + z_2 = e^{2ti}\left(1 + 2e^{-\frac{\pi}{6}i}\right)$ ***(M1)***

$= e^{2ti} \times 2.90931... e^{-0.350879...\,i}$ ***(A1)***

$\text{Im}(z_1 + z_2) = 2.91\sin(2t - 0.351)$ ***A1A1***

**METHOD 2**

attempt to find the modulus of $\left(1 + 2e^{-\frac{\pi}{6}i}\right)$ ***(M1)***

$p = \left|1 + 2e^{-\frac{\pi}{6}i}\right|$

attempt to find the argument of $\left(1 + 2e^{-\frac{\pi}{6}i}\right)$ ***(M1)***

$q = \arg\left(1 + 2e^{-\frac{\pi}{6}i}\right)$

$\text{Im}(z_1 + z_2) = 2.91\sin(2t - 0.351)$ ***A1A1***

**METHOD 3**

sketching $\text{Im}(z_1 + z_2)$ ***(M1)***

max is $(0.961, 2.91)$ ***(A1)***

first root is $0.1754$ ***(A1)***

$\text{Im}(z_1 + z_2) = 2.91\sin(2t - 0.351)$ ***A1***

***[4 marks]***

***[Total 6 marks]***

## Question 35

(a) attempt to use the term formula for a geometric sequence *(M1)*

$4\times(0.8)^2$

$=2.56\ (\text{mm})\ \left(\frac{64}{25}\right)$ *A1*

*[2 marks]*

(b) attempt to use the sum formula for a geometric sequence *(M1)*

$\frac{4\times(1-0.8^{12})}{1-0.8}$

$=18.6\ (\text{mm})\ \ (18.6256...)$ *A1*

*[2 marks]*

(c) attempt to use infinite geometric sum *(M1)*

*e.g.* $\frac{4}{1-0.8}$ **OR** $20$

adding $25$ to their expression or value *(M1)*

(maximum width $=25+20$)

$45\ (\text{mm})$ *A1*

*[3 marks]*

*[Total 7 marks]*

## Question 36

(a) $\sqrt{2}\,e^{\frac{3\pi i}{4}}$ **A1A1**

**[2 marks]**

(b) (i) enlargement SF $\sqrt{2}$, $(1.41,\ 1.41421\ldots)$, (centre $(0,\ 0)$) **A1**

rotation of $\frac{3\pi}{4}$, $(2.36,\ 2.35619\ldots)$, (centre $(0,\ 0)$) **A1**

(ii) $\begin{pmatrix}\sqrt{2} & 0\\ 0 & \sqrt{2}\end{pmatrix}\begin{pmatrix}\cos\left(\frac{3\pi}{4}\right) & -\sin\left(\frac{3\pi}{4}\right)\\ \sin\left(\frac{3\pi}{4}\right) & \cos\left(\frac{3\pi}{4}\right)\end{pmatrix}$ **(A1)(A1)**

**Note:** The order of matrix multiplication does not matter in this case.

$=\begin{pmatrix}-1 & -1\\ 1 & -1\end{pmatrix}$ **A1**

**Note:** Do not ***FT*** if their matrices in part (b)(i) are not representative of a dilation AND a rotation.

**[5 marks]**

(c) 8 **A1**

**Note:** Since $\frac{3\pi n}{4}$ must be a multiple of $2\pi$.

**[1 mark]**

**[Total: 8 marks]**

## Question 37

(a) recognizing that the growth in one year is the difference in the two heights *(M1)*

$5 \times 0.8 = 4$ (m) *A1*

*[2 marks]*

(b) recognizing geometric sequence, $r = 0.8$ *(M1)*

attempt to find total height by adding initial height to a term in series *(M1)*

**EITHER**

$42 + \frac{4(1-(0.8)^6)}{1-(0.8)}$ *(A1)*

**OR**

$37 + \frac{5(1-(0.8)^7)}{1-(0.8)}$ *(A1)*

**THEN**

$= 56.7571\ldots$

$= 56.76$ (m) **OR** 5676 (cm) *A1*

**Note:** Accept an answer in cm or in m to two decimal places.

*[4 marks]*

(c) attempt to use infinite geometric series to find the total growth of the tree *(M1)*

e.g. $\frac{5}{1-0.8}$ **OR** $\frac{4}{1-0.8}$

$\left( S_\infty = 37 + \frac{5}{1-0.8},\ S_\infty = 42 + \frac{4}{1-0.8} \right)$

$k = 62$ *A1*

*[2 marks]*

*[Total: 8 marks]*

## Question 38

(a) **EITHER**

| | | |
|---|---|---|
| $N = 72$ | | $N = 6$ |
| $I = 1.25$ | | $I = 1.25$ |
| $PV = -3000$ | **OR** | $PV = -3000$ |
| $P/Y = 12$ | | $P/Y = 1$ |
| $C/Y = 12$ | | $C/Y = 12$ |

***(M1)(A1)***

**Note:** Award ***M1*** for attempt to use finance app on GDC, ***A1*** for all entries correct.

**OR**

$$3000\left(1+\frac{1.25}{1200}\right)^{72}$$ ***(M1)(A1)***

**Note:** Award ***M1*** for using CI formula, ***A1*** for correct entries.

**THEN**

$FV = (\$)\ 3233.53$ ***A1***

**Note:** Answer must be correct to 2 dp for the final ***A1*** to be awarded.

***[3 marks]***

(b) **EITHER**

| | | |
|---|---|---|
| $I = 1.25$ | | $I = 1.25$ |
| $PV = -3000$ | | $PV = -3000$ |
| $FV = 3550$ | **OR** | $FV = 3550$ |
| $P/Y = 12$ | | $P/Y = 1$ |
| $C/Y = 12$ | | $C/Y = 12$ |

***(A1)***

**Note:** Award ***A1*** for all entries correct and opposite signs for PV and FV values.

**OR**

$$3550 = 3000\left(1+\frac{1.25}{1200}\right)^{12N}$$ ***(A1)***

**Note:** Award ***A1*** for all entries correct.

**THEN**

$N = 162\ (161.686\ldots)$ (months) ***A1***

***[2 marks]***

(c) 18450 ***A1***

***[1 mark]***

(d) **EITHER**

| | | |
|---|---|---|
| $N = 96$ | | $N = 96$ |
| $I = 12.6$ | | $I = 1.05$ |
| $PV = -18450$ | **OR** | $PV = -18450$ |
| $FV = 0$ | | $FV = 0$ |
| $P/Y = 12$ | | $P/Y = 1$ |
| $C/Y = 12$ | | $C/Y = 1$ |

***(M1)(A1)***

**Note:** Award ***M1*** for attempt to use finance app on GDC, ***A1*** for $N = 96$, $PV = -18450$ and $FV = 0$,

**THEN**

$PMT = (\$)\ 306$ ***A1***

**Note:** The answer must be correct to the nearest dollar for the final ***A1*** to be awarded.

***[3 marks]***

***[Total: 9 marks]***

## Question 39

(a) $\begin{pmatrix} 0.25 & 0.25 \\ 0.1 & 0.1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ ***(M1)***

**Note:** Accept equivalent methods including only using one line of the matrix.

$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ (or any multiple) ***A1***

***[2 marks]***

(b) $D^n = \begin{pmatrix} 0.65^n & 0 \\ 0 & 1 \end{pmatrix}$ ***A1***

***[1 mark]***

(c) $\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 0.65^n & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}^{-1}$ ***(M1)***

**EITHER**

multiplying by the initial state ***(M1)***

$\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 0.65^n & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}^{-1}\begin{pmatrix} 7000 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 0.65^n & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2000 \\ 1000 \end{pmatrix}$ ***(A1)***

$\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 2000\times 0.65^n \\ 1000 \end{pmatrix}$ ***(A1)***

$\begin{pmatrix} 2000\times 0.65^n + 5000 \\ -2000\times 0.65^n + 2000 \end{pmatrix}$ ***(A1)***

**OR**

$\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}^{-1} = \frac{1}{7}\begin{pmatrix} 2 & -5 \\ 1 & 1 \end{pmatrix}$ ***A1***

$\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 0.65^n & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0.65^n & 5 \\ -0.65^n & 2 \end{pmatrix}$ ***A1***

**Note:** The preceding ***A1*** marks can be awarded independently.

$\frac{1}{7}\begin{pmatrix} 5+2\times0.65^n & 5-5\times0.65^n \\ 2-2\times0.65^n & 2+5\times0.65^n \end{pmatrix}$ ***A1***

**Note:** Award ***A0*** if any term in the matrix is incorrect.

multiplying by the initial state ***(M1)***

$\frac{1}{7}\begin{pmatrix} 5+2\times0.65^n & 5-5\times0.65^n \\ 2-2\times0.65^n & 2+5\times0.65^n \end{pmatrix}\begin{pmatrix} 7000 \\ 0 \end{pmatrix}$

**THEN**

$2000-2000\times0.65^n \quad \left(=2000\left(1-0.65^n\right)\right)$ ***A1***

**Note:** For the final ***A1***, follow through within the question part from the bottom-left entry of their 2x2 matrix or the bottom entry of their 2x1 matrix but only if "in terms of $n$".

If any mistake in the matrices is seen that DOES NOT affect the correct final answer, do not award the penultimate ***A1*** mark.

***[6 marks]***
***[Total 9 marks]***

## Question 40

(a) (i) $\left(\left|z_1\right| = \sqrt{4^2+5^2} =\right) \quad 6.40 \quad (6.40312..., \sqrt{41})$ ***A1***

(ii) $\left(\arg\left(z_1\right) =\right) \quad 0.896 \quad \left(0.896055..., 51.3401...^\circ, \arctan\left(\frac{5}{4}\right)\right)$ ***A1***

***[2 marks]***

(b) angle in triangle is $2-0.896055\ldots$ **OR** $114.591^\circ - 51.3401...^\circ$ ***(A1)***
use of area of triangle formula ***(M1)***

$\frac{1}{2}\times6.40312...\times3\times\sin\left(2-0.896055...\right)$ ***(A1)***

$8.58 \ (8.57688\ldots)$ ***A1***

**Note:** Accept methods that use Cartesian form or vector product.

***[4 marks]***
***[Total 6 marks]***

**Question 41**

(a) $\begin{pmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{pmatrix}$ ***A1A1***

**Note:** Award ***A1*** for selecting the correct matrix, ***A1*** for substituting $2\alpha$ into a rotation matrix
Award ***A1A0*** for clockwise rotation. These marks can be awarded independently.
Condone the use of a different symbol.

***[2 marks]***

(b) $\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$

$= \begin{pmatrix} \cos^2\alpha - \sin^2\alpha & -2\cos\alpha\sin\alpha \\ 2\cos\alpha\sin\alpha & \cos^2\alpha - \sin^2\alpha \end{pmatrix}$ ***M1A1***

**Note:** Award ***M1*** for an attempt to multiply matrices, e.g. at least one entry correct.

***[2 marks]***

(c) (i) (Because matrix multiplication represents the composition of transformations)
two rotations of $\alpha$ are equivalent to a rotation of $2\alpha$ ***R1***
so the two matrices are equal (so each of the entries are also equal) ***R1***
$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$ ***AG***

(ii) $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$ ***A1***

replacing $\cos^2\alpha$ with $1-\sin^2\alpha$ ***M1***
$= 1 - \sin^2\alpha - \sin^2\alpha$

$= 1 - 2\sin^2\alpha$ ***AG***

***[4 marks]***
***[Total 8 marks]***

## Question 42

(a) (i) $m = ah^3$ ***(M1)***

$64 = a \times 0.8^3 \quad (\Rightarrow a = 125)$ ***(A1)***

$m = 125h^3$ ***A1***

**Note:** The final answer must be written as $m$ in terms of $h$ to award the final ***A1***.

(ii) $m = (125\,(0.75)^3 =)\ 52.7\ (\text{kg})\ \ (52.7343...)$ ***A1***

***[4 marks]***

(b) **EITHER (finding the height of the lion)**

$(220 = 125h^3)$

$h = \sqrt[3]{1.76}\ \ (1.20736...)$ ***(A1)***

$E = k_1h^2$ (seen anywhere) ***(A1)***

$k = \dfrac{\left(\sqrt[3]{1.76}\right)^2}{0.8^2}$ ***(M1)***

$= 2.28\ \ (2.27769\ldots)$ ***A1***

**OR (finding a formula for E in terms of m)**

$\left(m = 125h^3\right)$

$E = k_1h^2$ ***(A1)***

$E = k_2m^{\frac{2}{3}}$ ***(A1)***

$k = \dfrac{220^{\frac{2}{3}}}{64^{\frac{2}{3}}}$ ***(M1)***

$= 2.28\ \ (2.27769\ldots)$ ***A1***

***[4 marks]***

***[Total 8 marks]***

## Question 43

(a) attempt to substitute into geometric sequence formula for twelfth term **OR** at least three correct terms of the sequence ***(M1)***

$u_{12} = 40 \times 1.1^{12-1}$ **OR** 40, 44, 48.4, …

114 (114.124…) ***A1***

***[2 marks]***

(b) (i) attempt to substitute into geometric series formula **OR** a sum of at least the first three terms ***(M1)***

$S_{12} = \dfrac{40\left(1.1^{12}-1\right)}{1.1-1}$ **OR** $\sum_{1}^{12}\left(40 \times 1.1^{n-1}\right)$ **OR** $40+44+48.4+\ldots$

**Note:** Award ***M1*** for $u_1 = 40$ and $r = 1.1$ seen as part of a geometric series formula, or ***M1*** for sigma notation and their $u_n$ formula (condone missing limits), or ***M1*** for the sum of at least the **correct** first three terms of the sequence.

$S_{12} = 855$ (855.371…) ***A1***

(ii) finding $S_{24} = 3539.89...$ or attempt to find the sum between $u_{13}$ and $u_{24}$ ***(M1)***

**Note:** Award ***M1*** for $S_{24} = 3539.89\ldots$ **or** sigma notation that includes correct limits and their $u_n$ formula **or** a substituted geometric series formula that includes $125.537\ldots$ and $n = 12$ **or** a list of terms that includes at least the 13th term and the 24th term.

$3539.89\ldots - 855.371\ldots$ **OR** $\sum_{13}^{24}\left(40 \times 1.1^{n-1}\right)$ **OR**

$\left(S_{13\text{ to }24} =\right)\dfrac{125.537\ldots\left(1.1^{12}-1\right)}{1.1-1}$ **OR** $125.537 + \ldots + 358.172\ldots$ ***(A1)***

**Note:** Accept a calculation using $u_{13} = 125$ or 126.

2680 $\left(2684.52\ldots, 2684, 2685\right)$ ***A1***

**Note:** For $u_{13} = 125$, the sum is $2673.03\ldots$ and for $u_{13} = 126$, the sum is $2694.41\ldots$

***[5 marks]***

***[Total: 7 marks]***

## Question 44

(a) **EITHER**

$N = 96$
$PV = \mp 100000$
$FV = \pm 150000$
$P/Y = 12$
$C/Y = 12$ ***(M1)(A1)***

**OR**

$N = 8$
$PV = \mp 100000$
$FV = \pm 150000$
$P/Y = 1$
$C/Y = 12$ ***(M1)(A1)***

**OR**

$$150000 = 100000\left(1 + \frac{I}{100 \times 12}\right)^{12 \times 8}$$ ***(M1)(A1)***

**Note:** Award ***M1*** for an attempt to use a financial app (at least 3 entries, not necessarily correct) or an attempt to use a compound interest formula.
Award ***A1*** for all entries correct in financial app or correct substitution in compound interest formula.

**THEN**

$I = 5.08\ (5.07903\ldots)$ ***A1***

***[3 marks]***

(b) $N = 120$
$I\% = 6.1$
$PV = \mp 150000$
$PMT = \pm 1000$
$P/Y = 12$
$C/Y = 12$ ***(M1)(A1)***

**Note:** Award ***M1*** for an attempt to use a financial app (at least 3 entries, not necessarily correct). Award ***A1*** for all entries correct in financial app (condone missing -/+ sign if the correct final answer is seen).

$FV = (\$)\ 110867$ ***A1***

**Note:** Answer must be correct to nearest dollar to award the final ***A1***. Award ***(M1)(A1)A0*** for an unsupported final answer to a greater degree of accuracy eg. (\$) 110866.70…
Award ***M1A1A0*** for a truncated answer of 110866 if no working is shown.

***[3 marks]***
***[Total: 6 marks]***

## Question 45

(a)

| Bounce number, $n$ | Maximum height, $h$, according to the model |
|---|---|
| 1 | 0.613 |
| 2 | 0.514 |
| 3 | **0.431 (0.430988…)** |
| 4 | **0.361 (0.361383…)** |

***A1A1***
***[2 marks]***

(b) $(0.430988... - 0.439)^2 + (0.361383... - 0.377)^2$ seen ***(A1)***

$3.08 \times 10^{-4}$ $(3.08055\ldots \times 10^{-4})$ ***A1***

***[2 marks]***

(c) (i) $h = 0.716 \times 0.851^n$ $(0.715766\ldots, 0.850774\ldots)$ ***A2***

**Note:** Award ***A2*** for $h = 0.716\mathrm{e}^{-0.162n}$ $(h = 0.715766\ldots\mathrm{e}^{-0.161608\ldots n})$
Award at most ***A1A0*** for correct coefficients, with any error in notation (e.g. $x$ instead of $n$, missing $h=$)
Award at most ***A1A0*** for $h = 0.731 \times 0.838^n$ from using the values from the model.

(ii) $(h=)$ 0.716 (metres) ***A1***

***[3 marks]***
***[Total 7 marks]***

**Question 46**

(a) attempt to find $|\boldsymbol{M}-\lambda\boldsymbol{I}|$ ***(M1)***

$(-4-\lambda)(3-\lambda)+6 \ (=0)$ ***(A1)***

$2, -3$ ***A1***

***[3 marks]***

(b) (i) $\begin{pmatrix}2 & 0\\ 0 & -3\end{pmatrix}$ **OR** $\begin{pmatrix}-3 & 0\\ 0 & 2\end{pmatrix}$ ***A1***

(ii) **EITHER**

attempt to solve $\boldsymbol{M}\begin{pmatrix}x\\ y\end{pmatrix}=2\begin{pmatrix}x\\ y\end{pmatrix}$ **OR** $\boldsymbol{M}\begin{pmatrix}x\\ y\end{pmatrix}=-3\begin{pmatrix}x\\ y\end{pmatrix}$ ***(M1)***

**OR**

attempt to use $(\boldsymbol{M}-\lambda\boldsymbol{I})\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix}$ ***(M1)***

**THEN**

$\begin{pmatrix}1\\ 3\end{pmatrix}$ and $\begin{pmatrix}2\\ 1\end{pmatrix}$ (or any multiple) ***(A1)(A1)***

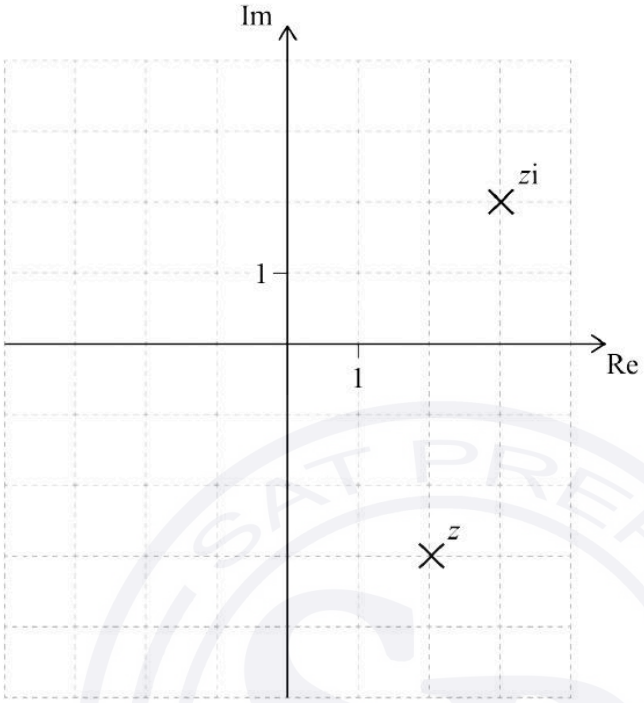
$\boldsymbol{P}=\begin{pmatrix}1 & 2\\ 3 & 1\end{pmatrix}$ **OR** $\boldsymbol{P}=\begin{pmatrix}2 & 1\\ 1 & 3\end{pmatrix}$ ***A1***

**Note:** Award ***A1*** for correct eigenvectors in a matrix in the correct position for their $\boldsymbol{D}$.

***[5 marks]***
***[Total 8 marks]***

**Question 47**

(a)




***A1***
***[1 mark]***

(b) (i) $3.61\ (3.60555\ldots,\ \sqrt{13})$ ***A1***

(ii) $-0.983\ \ (-0.982793\ldots)$

***A1***
***[2 marks]***

(c) $3+2\mathrm{i}$ ***A1***
***[1 mark]***

(d) **EITHER**

rotation, (anticlockwise) $\frac{\pi}{2}$ (or 90°) (about the origin) ***A1***

**OR**

translation $\begin{pmatrix}1\\5\end{pmatrix}$ ***A1***

***[1 mark]***
***[Total 5 marks]***

## Question 48

(a) $N = 60,\ I = 4.5,\ PV = +400\,000,\ PMT = -3600,\ P/Y = 12,\ C/Y = 12$ ***(M1)(A1)***

**Note:** Award ***M1*** for use of Finance solver with any 2 entries correct, Award ***A1*** for all correct.

$FV = \$259\,000 (\$258\,994)$ ***A1***

***[3 marks]***

(b) $I = 4.5,\ PV = +400\,000,\ PMT = -3600,\ FV = 0,\ P/Y = 12,\ C/Y = 12$ ***(A1)(A1)***

**Note:** Award ***A1*** for $FV = 0$, award ***A1*** for all correct.

$N = 144$ ***A1***

***[3 marks]***

***[Total 6 marks]***

## Question 49

(a) (i) $a = 30$ ***A1***

(ii) $p = 11$ ***A1***

***[2 marks]***

(b) **EITHER**

substituting $n = 11$ and equating to $230$ (or equivalent) ***(M1)***

$30(11) + b = 230$

**OR**

using an arithmetic sequence ***(M1)***

$200 + 30(n - 10)$

**THEN**

$b = -100$ ***A1***

***[2 marks]***

***[Total 4 marks]***

## Question 50

(a) recognition that % increase gives the common difference ***(M1)***

$25000 \times 0.05 = 1250$ ***A1***

***[2 marks]***

(b) $25000 + 1250n > 44000$ **OR** $25000 + 1250(n-1) > 44000$ (accept equality) ***(A1)***

attempt to solve their inequality **OR** equality (sketch **OR** cross-over values **OR** algebraic manipulation) ***(M1)***

$k = 16$ ***A1***

| **Note:** Award ***(A1)(M1)A0*** for 15.2, 16.2 or 17 seen. |
|---|

***[3 marks]***

***[Total: 5 marks]***