Subject – Math AI(Higher Level)
Topic - Number and Algebra
Year - May 2021 – Nov 2022
Paper -1
Answers

Question 1

$\log A = x\log B + y\log C + \log k$ ***(M1)***

$\log 5.74 = x\log 2.1 + y\log 3.4 + \log k$
$\log 2.88 = x\log 1.5 + y\log 2.4 + \log k$
$\log 0.980 = x\log 0.8 + y\log 1.9 + \log k$ ***M1A1***

Allow any consistent base, allow numerical equivalents.

attempting to solve their system of equations ***(M1)***

$x = 1.53,\ y = 0.505$ ***A1***

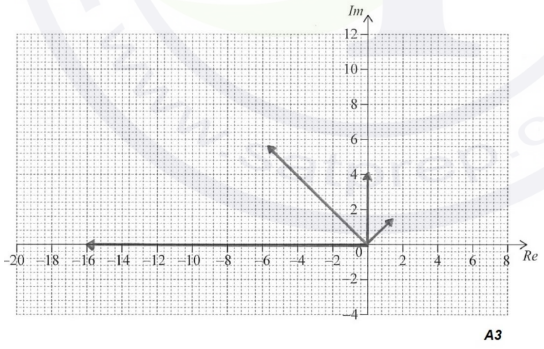
$k = 0.997$ ***A1***

Total [6 marks]

Question 2

(a) (i) $4e^{\frac{\pi}{2}i},\ 8e^{\frac{3\pi}{4}i},\ 16e^{\pi i}\ \left(= 4i,\ -4\sqrt{2} + 4\sqrt{2}i,\ -16\right)$ ***(M1)A1***

(ii)



A3

[5 marks]

(b) $2^2 + 1^2 = a^2$ ***M1***

$a = \sqrt{5}\ \ (= 2.24)$ ***A1***

[2 marks]

Total [7 marks]

Question 3

(Model A)

$R = 3pe^{-0.5p}$ ***M1***

predicted values

p	R
1	1.8196
2	2.2073
3	2.0082

(A1)

$SS_{res} = (1.8196 - 1.5)^2 + (2.2073 - 1.8)^2 + (2.0082 - 1.5)^2$ ***(M1)***

$= 0.5263\ldots$ ***A1***

(Model B)

$R = 2.5pe^{-0.6p}$

predicted values

p	R
1	1.372
2	1.506
3	1.2397

(A1)

$SS_{res} = 0.170576\ldots$ ***A1***

chose model B ***A1***

Total [7 marks]

Question 4

(a) (i) $i^2 = -1$ ***(M1)***

$w = -2 + 1 = -1$ ***A1***

(ii) $w = -1 + i + 1 = i$ ***A1***

[3 marks]

(b) **EITHER**

rotation of 90° (anticlockwise, centre at the origin) ***A1A1***

Note: Award ***A1*** for "rotation" and ***A1*** for "90°".

followed by a translation of $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ***A1***

OR

translation of $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$ ***A1***

followed by rotation of 90° (anticlockwise, centre at the origin) ***A1A1***

Note: Award ***A1*** for "rotation" and ***A1*** for "90°".

[3 marks]

(c) **EITHER**

move 1 to left to $1 - i$ ***(M1)***

then rotate by -90° to

$-1 - i$ ***A1***

OR

$iz + 1 = 2 - i$

$iz = 1 - i$

$z = \dfrac{1 - i}{i}$ ***(M1)***

$-1 - i$ ***A1***

[2 marks]

Total [8 marks]

Question 5

(a) (i) attempt to find u_{20} using an arithmetic sequence ***(M1)***

e.g. $u_1 = 500$ and $d = 100$ **OR** $u_{20} = 500 + 1900$ **OR** $500, 600, 700, \ldots$

(Charlie ran) $2400\ \text{m}$ ***A1***

(ii) $(r =)\ 1.02$ ***(A1)***

attempt to find u_{20} using a geometric sequence ***(M1)***

e.g. identifying $u_1 = 500$ and a value for r **OR** $500 \times r^{19}$ **OR** $500,\ 510,\ 520.2, \ldots$

(Daniella ran) $728\ \text{m}\ (728.405...)$ ***A1***

[5 marks]

(b) $500 \times 1.02^{n-1} > 500 + (n-1) \times 100$ ***(M1)***

attempt to solve inequality ***(M1)***

$n > 184.215...$

$n = 185$ ***A1***

[3 marks]

Total [8 marks]

Question 6

(a) (i) $z_1^3 = 27e^{\frac{i\pi}{4}} (= 27e^{0.785398...i})$ ***A1A1***

Note: Award ***A1*** for 27 and ***A1*** for the angle in the correct form.

(ii) $\left(\frac{z_1}{z_2}\right)^4 = \left(\frac{81}{16}\right)e^{\frac{i\pi}{2}} (= 5.0625e^{1.57079...i})$ ***A1A2***

Note: Award ***A1*** for $\frac{81}{16}$, ***A2*** for the angle in the correct form and ***A1*** for the angle in incorrect form e.g. $\text{cis}\frac{\pi}{2}$ and/or $\frac{5\pi}{2}$.

Award ***A1*** if i is given in place of $\text{cis}\frac{\pi}{2}$.

[5 marks]

(b) $z_1 z_2 = 6\,\text{cis}\left(\frac{3\pi}{4} + \frac{n\pi}{16}\right)$ ***(M1)***

$= 6\,\text{cis}\left(\frac{12\pi + n\pi}{16}\right)$

$12\pi + n\pi = 32\pi$ ***(M1)***

$n = 20$ ***A1***

[3 marks]

Total [8 marks]

Question 7

(a) recognition of geometric sequence *eg* $r = 0.82$ **(M1)**

$S_{10} = \dfrac{450\left(1-0.82^{10}\right)}{1-0.82}$ **(A1)**

$= 2160 \text{ m}$ $(2156.37...)$ **A1**

[3 marks]

(b) $S_{\infty} = \dfrac{450}{1-0.82}$ **(M1)**

$= 2500 < 2520$ so the balloon will not reach the required height. **A1**

[2 marks]

(c) horizontal motion not taken into account,
rate of cooling will not likely be linear,
balloon is considered a point mass / size of balloon not considered,
effects of wind/weather unlikely to be consistent,
a discrete model has been used, whereas a continuous one may offer greater accuracy **R1**

Note: Accept any other sensible answer.

[1 mark]

Total [6 marks]

Question 8

(a) $£495 \times 0.9^5 = £292$ $(£292.292...)$ **(M1)A1**

[2 marks]

(b) $495 \times 0.9^k = 2200 \times 0.85^k$ **(M1)**

$k = 26.1$ $(26.0968...)$ **A1**

Note: Award ***M1A0*** for $k-1$ in place of k.

[2 marks]

(c) depreciation rates unlikely to be constant (especially over a long time period) **R1**

Note: Accept reasonable answers based on the magnitude of k or the fact that "value" depends on factors other than time.

[1 mark]

Total [5 marks]

Question 9

(a) $m = -0.695\ (-0.695383...)$; $b = 4.63\ (4.62974...)$ **A1A1**

[2 marks]

(b) $\ln x = -0.695(\ln 25) + 4.63$ **M1**

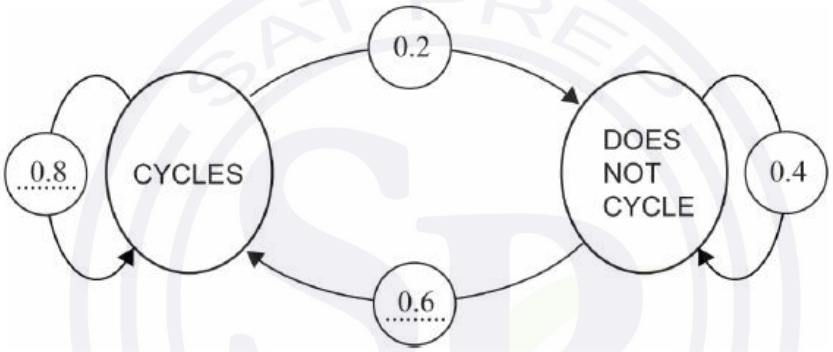
$\ln x = 2.39288...$ **(A1)**

$x = 10.9\%$ **A1**

[3 marks]
Total: [5 marks]

Question 10

(a)



A1A1

[2 marks]

(b) $A = \begin{pmatrix} 0.8 & 0.6 \\ 0.2 & 0.4 \end{pmatrix}$ **(A1)**

$A^{180} = \begin{pmatrix} 0.75 & 0.75 \\ 0.25 & 0.25 \end{pmatrix}$ **(M1)**

0.75 **A1**

[3 marks]
Total: [5 marks]

Question 11

(a) $10 = \frac{2}{1-r}$ ***(M1)***

$r = 0.8$ ***A1***

[2 marks]

(b) $2\times(0.8)^{n-1} < 0.5$ **OR** $2\times(0.8)^{n-1} = 0.5$ ***(M1)***

$(n >)\ 7.212.....$ ***(A1)***

$n = 8$ ***A1***

Note: If $n = 7$ is seen, with or without seeing the value 7.212… then award ***M1A1A0***.

[3 marks]

Total: [5 marks]

Question 12

(a) $N = 360$

$I\% = 3.8$

$PV = (\pm)170\,000$

$FV = 0$

$P/Y = 12$

$C/Y = 12$ ***(M1)(A1)***

$(PMT =)\ 792.13\ \text{AUD}$ ***A1***

[3 marks]

(b) (i) $N = 120$

$I\% = 3.8$

$PV = (\pm)170\,000$

$PMT = (\mp)792.13$

$P/Y = 12$

$C/Y = 12$ ***(M1)(A1)***

$(FV =)\ 133019.94\ \text{AUD}$ ***A1***

(ii) amount of money paid: $120\times 792.13\ (=95055.60)$ ***(M1)***

loan paid off: $170\,000 - 133019.94\ (=36980.06)$ ***(M1)***

interest paid: $(95055.60 - 36980.06 =)\ 58075.54\ \text{AUD}$ ***A1***

[6 marks]

Total: [9 marks]

Question 13

(a) attempt at using trapezoidal rule formula *(M1)*

$\frac{1}{2}\left(\frac{2-0}{5}\right)(30+50+2(50+60+40+20))$ *A1*

(total carbon =) 84 tonnes *A1*

[3 marks]

(b) $\left|\frac{84-72}{72}\right|\times 100\%$ *(M1)*

Note: Award *(M1)* for correct substitution of final answer in part (a) into percentage error formula.

$=16.7\%$ $(16.6666...\%)$ *A1*

[2 marks]

Total: [5 marks]

Question 14

(a) $\log_{10} 100 = a-3$ *(M1)*

$a=5$ *A1*

[2 marks]

(b) **EITHER**

$N=10^{5-M}$ *(M1)*

$=\frac{10^5}{10^M}\left(=\frac{100000}{10^M}\right)$

OR

$100=\frac{b}{10^3}$ *(M1)*

THEN

$b=100000$ $(=10^5)$ *A1*

[2 marks]

(c) $N=\frac{10^5}{10^{7.2}}=0.00631$ $(0.0063095...)$ *A1*

Question 15

(a) $\lambda = 1$

$\begin{pmatrix} -0.8 & 0.7 \\ 0.8 & -0.7 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ **OR** $\begin{pmatrix} 0.2 & 0.7 \\ 0.8 & 0.3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ ***(M1)***

$0.8x = 0.7y$ ***(A1)***

an eigenvector is $\begin{pmatrix} 7 \\ 8 \end{pmatrix}$ (or equivalent with integer values) ***A1***

[3 marks]

(b) **EITHER**

(the long-term probability matrix is given by the eigenvector corresponding to the eigenvalue equal to 1, scaled so that the sum of the entries is 1)

$8 + 7 = 15$ ***(M1)***

OR

$\begin{pmatrix} 0.2 & 0.7 \\ 0.8 & 0.3 \end{pmatrix}\begin{pmatrix} p \\ 1-p \end{pmatrix} = \begin{pmatrix} p \\ 1-p \end{pmatrix}$ ***(M1)***

OR

considering high powers of the matrix *e.g.* $\begin{pmatrix} 0.2 & 0.7 \\ 0.8 & 0.3 \end{pmatrix}^{50}$ ***(M1)***

$\begin{pmatrix} \frac{7}{15} & \frac{7}{15} \\ \frac{8}{15} & \frac{8}{15} \end{pmatrix}$

THEN

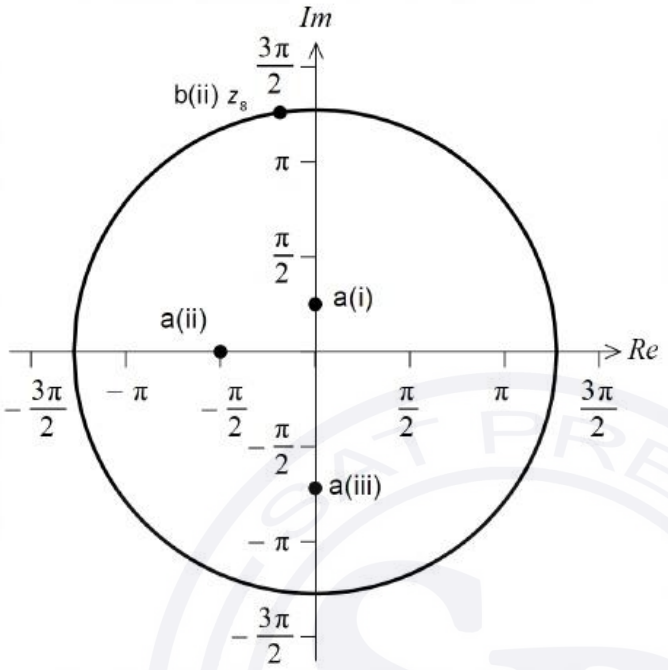
probability of being in state A is $\frac{7}{15}$ ***A1***

[2 marks]

[Total 5 marks]

Question 16

(a)



A1A1A1

Note:	Award ***A1*** for correct modulus and ***A1*** for correct argument for part (a)(i), and ***A1*** for other two points correct. The points may not be labelled, and they may be shown by line segments.

[3 marks]

(b) (i) $\frac{1}{2}\theta = 4$ ***(M1)***

$\Rightarrow \theta = 8$ ***A1***

(ii) z_8 is shown in the diagram above ***A1A1***

Note:	Award ***A1*** for a point plotted on the circle and ***A1*** for a point plotted in the second quadrant.

[4 marks]
[Total 7 marks]

Question 17

(a) use of geometric sequence with $r = 0.85$ ***M1***

EITHER

$(0.85)^6(1.8)$ **OR** $0.678869\ldots$ **OR** $(0.85)^5(1.53)$ ***A1***

$= 0.68$ m

$= 68$ cm ***AG***

OR

$(0.85)^6(180)$ **OR** $(0.85)^5(153)$ ***A1***

$= 68$ cm ***AG***

[2 marks]

(b) **EITHER**

$(0.85)^n(1.8) > 0.1$ **OR** $(0.85)^{n-1}(1.53) > 0.1$ ***(M1)***

Note: If 1.8 m (or 180 cm) is used then ***(M1)*** only awarded for use of n in $(0.85)^n(1.8) > 0.1$.
If 1.53 m (or 153 cm) is used then ***(M1)*** only awarded for use of $n-1$ in $(0.85)^{n-1}(1.53) > 0.1$.

17 ***A1***

OR

$(0.85)^{17}(1.8) = 0.114$ m and $(0.85)^{18}(1.8) = 0.0966$ m ***(M1)***

17 ***A1***

OR

solving $(0.85)^n(1.8) = 0.1$ to find $n = 17.8$ ***(M1)***

17 ***A1***

Note: Evidence of solving may be a graph **OR** the "solver" function **OR** use of logs to solve the equation. Working may use cm.

[2 marks]

(c) **EITHER**

distance (in one direction) travelled between first and fourth bounce

$= \frac{(1.8 \times 0.85)(1-0.85^3)}{1-0.85}$ $(= 3.935925)$ ***(A1)***

recognizing distances are travelled twice except first distance ***(M1)***

$1.8 + 2(3.935925)$

$= 9.67$ m $(9.67185...$ m$)$ ***A1***

OR

distance (in one direction) travelled between drop and fourth bounce

$= \frac{(1.8)(1-0.85^4)}{1-0.85}$ $(= 5.735925)$ ***(A1)***

recognizing distances are travelled twice except first distance ***(M1)***

$2(5.735925) - 1.8$

$= 9.67$ m $(9.67185...$ m$)$ ***A1***

OR

distance (in one direction) travelled between first and fourth bounce

$(0.85)(1.8) + (0.85)^2(1.8) + (0.85)^3(1.8)$ $(= 3.935925...)$ ***(A1)***

recognizing distances are travelled twice except first distance ***(M1)***

$1.8 + 2(0.85)(1.8) + 2(0.85)^2(1.8) + 2(0.85)^3(1.8)$

$= 9.67$ m $(9.67185...$ m$)$ ***A1***

Note: Answers may be given in cm.

[3 marks]

[Total 7 marks]

Question 18

(a) $x + y + z = 600$ ***A1***

$15x + 10y + 12z = 7816$ ***A1***

$x = 2y$ ***A1***

Note: Condone other labelling if clear, e.g. a (adult), c (child) and s (student). Accept equivalent, distinct equations e.g. $2y + y + z = 600$.

[3 marks]

(b) $x = 308,\ y = 154,\ z = 138$ ***A1A1***

Note: Award ***A1*** for all three correct values seen, ***A1*** for correctly labelled as x, y or z. Accept answers written in words: e.g. 308 adult tickets.

[2 marks]

[Total 5 marks]

Question 19

(a) (one vector to the line is $\begin{pmatrix}0\\c\end{pmatrix}$ therefore) $\boldsymbol{a}=\begin{pmatrix}0\\c\end{pmatrix}$ ***A1***

the line goes m up for every 1 across

(so the direction vector is) $\boldsymbol{b}=\begin{pmatrix}1\\m\end{pmatrix}$ ***A1***

Note: Although these are the most likely answers, many others are possible.

[2 marks]

(b) (from GDC **OR** $6\times2-4\times3$) $|M|=0$ ***A1***

[1 mark]

(c) **METHOD 1**

$\begin{pmatrix}X\\Y\end{pmatrix}=\begin{pmatrix}6&3\\4&2\end{pmatrix}\begin{pmatrix}x\\mx+c\end{pmatrix}=\begin{pmatrix}6x+3mx+3c\\4x+2mx+2c\end{pmatrix}$ ***M1A1***

$=\begin{pmatrix}3(2x+mx+c)\\2(2x+mx+c)\end{pmatrix}$ ***A1***

therefore the new line has equation $3Y=2X$ ***A1***

which is independent of m or c ***AG***

Note: The ***AG*** line (or equivalent) must be seen for the final ***A1*** line to be awarded.

METHOD 2

take two points on the line, e.g $(0,\ c)$ and $(1,\ m+c)$ ***M1***

these map to $\begin{pmatrix}6&3\\4&2\end{pmatrix}\begin{pmatrix}0\\c\end{pmatrix}=\begin{pmatrix}3c\\2c\end{pmatrix}$

and $\begin{pmatrix}6&3\\4&2\end{pmatrix}\begin{pmatrix}1\\m+c\end{pmatrix}=\begin{pmatrix}6+3m+3c\\4+2m+2c\end{pmatrix}$ ***A1***

therefore a direction vector is $\begin{pmatrix}6+3m\\4+2m\end{pmatrix}=(2+m)\begin{pmatrix}3\\2\end{pmatrix}$

(since $m\neq-2$) a direction vector is $\begin{pmatrix}3\\2\end{pmatrix}$

the line passes through $\begin{pmatrix}3c\\2c\end{pmatrix}-c\begin{pmatrix}3\\2\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$ therefore it always has the origin as a jump-on vector ***A1***

the vector equation is therefore $\boldsymbol{r}=\mu\begin{pmatrix}3\\2\end{pmatrix}$ ***A1***

which is independent of m or c ***AG***

METHOD 3

$\boldsymbol{r} = \begin{pmatrix} 6 & 3 \\ 4 & 2 \end{pmatrix}\left(\begin{pmatrix} 0 \\ c \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ m \end{pmatrix}\right) = \begin{pmatrix} 3c \\ 2c \end{pmatrix} + \lambda\begin{pmatrix} 6+3m \\ 4+2m \end{pmatrix}$ **M1A1**

$= c\begin{pmatrix} 3 \\ 2 \end{pmatrix} + (2+m)\lambda\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ **A1**

$= \mu\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

where $\mu = c + (2+m)\lambda$ is an arbitrary parameter. **A1**

which is independent of m or c (as μ can take any value) **AG**

Note: The ***AG*** line (or equivalent) must be seen for the final ***A1*** line to be awarded.

[4 marks]

Total [7 marks]

METHOD 2

converting given expressions to cos form ***(M1)***

$V_T = 2\cos 3t + 5\cos(3t+4)$

(from graph) $A = 3.99$ $(3.99088\ldots)$ ***A1***

$V_T = 3.99\cos(Bt + C)$

either by considering transformations or inserting points

$B = 3$ ***A1***

$C = -1.89$ $(-1.89418\ldots)$ ***A1***

Note: Accept arguments differing by 2π e.g. $4.38900\ldots$.

(so, $V_T = 3.99\cos(3t - 1.89)$ $\left(3.99088\ldots\cos(3t - 1.89418\ldots)\right)$)

Note: It is possible to have $A = 3.99$, $B = -3$ with $C = 1.89$ **OR** $A = -3.99$, $B = 3$ with $C = 1.25$ **OR** $A = -3.99$, $B = -3$ with $C = -1.25$ due to properties of the cosine curve.

[4 marks]

(b) maximum voltage is 3.99 $(3.99088\ldots)$ (units) ***A1***

[1 mark]

Total [5 marks]

Question 20

(a) **METHOD 1**

recognizing that the real part is distributive ***(M1)***

$V_T = \mathrm{Re}\left(2e^{3ti} + 5e^{3ti+4i}\right)$

$= \mathrm{Re}\left(e^{3ti}\left(2 + 5e^{4i}\right)\right)$ ***(A1)***

(from the GDC) $2 + 5e^{4i} = 3.99088\ldots e^{-1.89418\ldots i}$ ***(A1)***

therefore $V_T = 3.99\cos(3t - 1.89)$ $\left(3.99088\ldots\cos(3t - 1.89418\ldots)\right)$ ***A1***

METHOD 2

converting given expressions to cos form **(M1)**

$V_T = 2\cos 3t + 5\cos(3t+4)$

(from graph) $A = 3.99 \ \ (3.99088\ldots)$ **A1**

$V_T = 3.99\cos(Bt+C)$

either by considering transformations or inserting points

$B = 3$ **A1**

$C = -1.89 \ \ (-1.89418\ldots)$ **A1**

(so, $V_T = 3.99\cos(3t-1.89) \ \ (3.99088\ldots\cos(3t-1.89418\ldots))$)

[4 marks]

(b) maximum voltage is $3.99 \ \ (3.99088\ldots)$ (units) **A1**

[1 mark]

Total [5 marks]

Question 21

(a) **METHOD 1 –** *(With FV=4000)*

EITHER

N= 10
I=1.5
FV= 4000
P/Y= 1
C/Y= 1 ***(A1)(M1)***

Note: Award ***A1*** for $(3.5-2=)\ 1.5$ seen and ***M1*** for all other entries correct.

OR

$4000 = A(1+0.015)^{10}$ ***(A1)(M1)***

Note: Award ***A1*** for 1.5 or 0.015 seen, ***M1*** for attempt to substitute into compound interest formula **and** equating to 4000.

THEN

(PV =) $3447 ***A1***

Note: Award ***A0*** if not rounded to a whole number or a negative sign given.

METHOD 2 – *(With FV including inflation)*

calculate FV with inflation

4000×1.02^{10} ***(A1)***

(=4875.977…)

EITHER

$4000\times1.02^{10} = \text{PV}\times1.035^{10}$ ***(A1)***

OR

N= 10
I= 3.5
FV= 4875.977…
P/Y= 1
C/Y= 1 ***(M1)***

Note: Award ***M1*** for *their* FV and all other entries correct.

THEN

(PV =) $3457 ***A1***

Note: Award ***A0*** if not rounded to a whole number or a negative sign given.

METHOD 3 – *(Using formula to calculate real rate of return)*
(real rate of return =) 1.47058…(%) ***(A1)***

EITHER
$4000 = \text{PV} \times 1.0147058\ldots^{10}$ ***(A1)***
OR
N= 10
I= 1.47058…
FV= 4000
P/Y= 1
C/Y= 1 ***(M1)***

Note: Award ***M1*** for all entries correct.

THEN
(PV =) $3457 ***A1***

[3 marks]

(b) **METHOD 1 –** *(Finding the future value of the investment using PV from part (a))*
N= 10
I=3.5
PV= 3446.66…(from Method 1) **OR** 3456.67…(from Methods 2, 3)
P/Y= 1
C/Y= 1 ***(M1)***

Note: Award ***M1*** for interest rate 3.5 **and** answer to part (a) as PV.

(FV=) $4861.87 **OR** $4875.97 ***(A1)***
so payment required (from TVM) will be $294 **OR** $295 ***A1***

Note: Award ***A0*** if a negative sign given, unless already penalized in part (a).

METHOD 2 – *(Using FV)*
N= 10
I=3.5
PV= –1000
FV= 4875.977…
P/Y= 1
C/Y= 1 ***(A1)(M1)***

Note: Award ***A1*** for I=3.5 **and** FV= ±4875.977…, ***M1*** for all other entries correct **and** opposite PV and FV signs.

(PMT =) $295 (295.393) ***A1***

Note: Correct 3sf answer is 295, however accept an answer of 296 given that the context supports rounding up. Award ***A0*** if a negative sign given, unless already penalized in part (a).

[3 marks]
Total [6 marks]

Question 22

(a) $\pi \times 2^2 \times \frac{30}{360}$ ***(M1)***

$= 1.047 \text{ cm}^2$ ***A1***

Note: Do not award the final mark if the answer is not correct to 4 sf.

[2 marks]

(b) attempt to substitute any two values from 1.5, 2.5, 25 or 35 into area of sector formula ***(M1)***

$\left(\text{upper bound} = \pi \times 2.5^2 \times \frac{35}{360} = \right) \ 1.91 \ \text{cm}^2 \ \ (1.90895\ldots)$ ***A1***

$\left(\text{lower bound} = \pi \times 1.5^2 \times \frac{25}{360} = \right) \ 0.491 \ \text{cm}^2 \ \ (0.490873\ldots)$ ***A1***

Note: Given the nature of the question, accept correctly rounded **OR** correctly truncated 3 significant figure answers.

[3 marks]

(c) $\left(\left|\frac{1.047 - 1.90895\ldots}{1.90895\ldots}\right| \times 100 = \right) \ 45.2 \ (\%) \ \ (45.1532\ldots)$ ***A1***

$\left(\left|\frac{1.047 - 0.490873\ldots}{0.490873\ldots}\right| \times 100 = \right) \ 113 \ (\%) \ \ (113.293\ldots)$ ***A1***

so the largest percentage error is 113 % ***A1***

Note: Accept 45.1 (%) (45.1428), from use of full accuracy answers. Given the nature of the question, accept correctly rounded **OR** correctly truncated 3 significant figure answers. Award ***A0A1A0*** if 113% is the only value found.

[3 marks]

Total [8 marks]

Question 23

METHOD 1

$\frac{u_1}{1-r}=9$ **A1**

therefore $u_1=9-9r$

$u_1=4+u_1r$ **A1**

substitute or solve graphically: **M1**

$9-9r=4+(9-9r)r$ **OR** $\frac{4}{(1-r)^2}=9$

$9r^2-18r+5=0$

$r=\frac{1}{3}$ or $r=\frac{5}{3}$

only $r=\frac{1}{3}$ is possible as the sum to infinity exists **R1**

then $u_1=9-\left(9\times\frac{1}{3}\right)=6$

$u_3=6\times\frac{1}{3}^2=\frac{2}{3}$ **A1**

METHOD 2

$\frac{u_1}{1-r}=9$ **A1**

$r=\frac{u_1-4}{u_1}$ **A1**

attempt to solve **M1**

$\frac{u_1}{1-\left(\frac{u_1-4}{u_1}\right)}=9$

$\frac{u_1}{\left(\frac{4}{u_1}\right)}=9$

$(u_1)^2=36$

$u_1=\pm6$

attempting to solve both possible sequences

6, 2, ... or $-6, -10$ …

$r=\frac{1}{3}$ or $r=\frac{5}{3}$

only $r=\frac{1}{3}$ is possible as the sum to infinity exists **R1**

$u_3=6\times\left(\frac{1}{3}\right)^2=\frac{2}{3}$ **A1**

Total [5 marks]

Question 24

(a) Gradient $= \frac{14.9+1.3}{6}$ $(=2.7)$ **(M1)**

$\log_{10} Q = 2.7P - 1.3$ **(A1)**

$Q = 10^{2.7P-1.3}$ **OR** $Q = 0.0501\times 10^{2.7P}$ $(=0.0501187...\times 10^{2.7P})$ **A1**

[3 marks]

(b) $\ln R$ on one axis and Q on the other axis **A1**

[1 mark]

(c) $\log_{10}(4.3\ln R + 12.1) = 2.7P - 1.3$ **OR** $10^{2.7P-1.3} = 4.3\ln R + 12.1$ **(M1)**

$P = \frac{\log_{10}(4.3\ln R + 12.1) + 1.3}{2.7}$ **A1**

[2 marks]

Total [6 marks]

Question 25

(a) $m = 1 - 2.5\log_{10}(0.0525)$ **(M1)**

$= 4.20$ $(4.19960\ldots)$ **A1**

[2 marks]

(b) attempt to solve $7 = 1 - 2.5\log_{10}(b)$ **(M1)**

Note: Accept a sketch from their GDC as an attempt to solve $7 = 1 - 2.5\log_{10}(b)$.

$b = 0.00398$ $(0.00398107...)$ **A1**

[2 marks]

(c) $-3.2 = \left(1 - 2.5\log_{10}(b_n)\right) - \left(1 - 2.5\log_{10}(b_p)\right)$ **M1**

$-3.2 = -2.5\log_{10}\left(\frac{b_n}{b_p}\right)$ **A1**

$\frac{b_n}{b_p} = 19.1$ $(19.0546...)$ **A1**

[3 marks]

Total [7 marks]

Question 26

(a) attempt to find $\det(\boldsymbol{M})$ ***(M1)***

$= 14$

$(12 \times 14) = 168 \text{ cm}^2$ ***A1***

[2 marks]

(b) let X have coordinates (x, y)

METHOD 1

$$\boldsymbol{M}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2t-3 \\ 6-5t \end{pmatrix}$$ ***(M1)***

$$\begin{pmatrix} x \\ y \end{pmatrix} = \boldsymbol{M}^{-1}\begin{pmatrix} 2t-3 \\ 6-5t \end{pmatrix}$$ ***(A1)***

$$\boldsymbol{M}^{-1} = \frac{1}{14}\begin{pmatrix} 1 & 4 \\ -3 & 2 \end{pmatrix}$$ ***A1***

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{14}\begin{pmatrix} 2t-3+24-20t \\ -6t+9+12-10t \end{pmatrix}$$ ***(M1)***

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{14}\begin{pmatrix} 21-18t \\ 21-16t \end{pmatrix} \quad \textbf{OR} \quad \left(\frac{21-18t}{14}, \frac{21-16t}{14}\right)$$ ***A1A1***

METHOD 2

writing two simultaneous equations ***(M1)***

$2x - 4y = 2t - 3$ ***(A1)***

$3x + y = 6 - 5t$ ***(A1)***

attempting to solve the equations ***(M1)***

$$(x, y) = \left(\frac{3}{2} - \frac{9t}{7}, \frac{3}{2} - \frac{8t}{7}\right)$$ ***A1A1***

[6 marks]

Total [8 marks]

Question 27

(a) $I\% = 7.5$
$PV = \mp 800$
$PMT = \mp 500$
$FV = \pm 10\,000$
$P/Y = 12$
$C/Y = 12$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology (e.g. at least four rows seen, but not necessarily correct), award ***A1*** for $\text{PMT} = -500$ or $\text{PMT} = 500$, with same sign to PV and opposite sign to FV.

$17.3070\ldots$ ***(A1)***
$(k =)\ 18$ ***A1***

Note: Award ***(M0)(A0)(A0)A0*** for a final answer of 17 with no working. The final answer must be an integer.

[4 marks]

(b) $10389 - (18 \times 500 + 800)$ **OR** $10389 - (9800)$ ***(A1)(M1)***

Note: Award ***(A1)*** for 10389 $(10389.38\ldots)$ seen. Award ***(M1)*** for subtraction of their $(18 \times 500 + 800)$ from FV. ***FT*** from their value of k. Award ***A0M1A0*** for $10\,000 - (18 \times 500 + 800)$. Do not award the final ***A1FT*** if their answer is negative.

589 EUR ***A1***

Note: Final answer must be to the nearest euro.

[3 marks]
Total [7 marks]