Subject – Math AI(Higher Level) Topic - Statistics and Probability Year - May 2021 – Nov 2022 Paper -1 Answers

(a) $P(X = 8)$		(M1)
	or evidence of recognizing binomial probability.	
<i>eg</i> , $P(X = 8)$	$, X \sim \mathrm{B}\left(20, \frac{6}{15}\right).$	
= 0.180 (0.1797	705)	A1
		[2 marks]
(b) let x be the nur	mber of male students	
recognize that p	probability of selecting a male is equal to $\frac{x}{80}$	(A1)
set up equation	on ${}^{20}C_8 \left(\frac{x}{80}\right)^8 \left(\frac{80-x}{80}\right)^{12} = 0.153357$	(M1)
number of male	e students = 37	(M1)A1
Note: Award (M1)A	9 for 27.	
		[4 marks]
		Total [6 marks]
Question 2		
(a) $H_0: m = 3, H_1: m$	1<3 Satore?	A1
	nber of fish caught)	
$P(X \le 1 \mid m = 3) =$		M1A1
(c) $P(X \ge 2 m = 2.$	5) $(=1-P(X \le 1 m = 2.5))$	M1A1
= 0.713		A1
		[3 marks]
		Total [6 marks]

let T be the time to serve both customers and T_i the time to serve the i th customer

assuming independence of T_1 and T_2	R1
<i>T</i> is normally distributed and $T = T_1 + T_2$	(M1)
E(T) = 1.5 + 1.5 = 3	A1
$Var(T) = 0.4^2 + 0.4^2 = 0.32$	M1A1
P(T < 4) = 0.961	A1
	Total [6 marks]

Question 4

(b)

(1.463, 1.493)

(a)
$$s_{n-1} = \sqrt{\frac{10}{9}} \times 0.0196 = 0.02066...$$
 (M1)A1

[2 marks]

(M1)A1

Note: If s_n used answer is (1.464, 1.492), award **M1A0.**

(c) 95% of the time these results would be produced by a population with mean of less than $1.5 \, \text{kg}$, so it is likely the mean weight is less than $1.5 \, \text{kg}$

[2 marks]

R1 [1 mark]

Total [5 marks]

(a)
$$\frac{4}{18} \left(\frac{2}{9}\right)$$
 A1 [1 mark]

(b)
$$-3 \times \frac{1}{18} + (-1) \times \frac{4}{18} + 0 \times \frac{3}{18} + \dots + 5 \times \frac{7}{18}$$
 (M1)

Note: Award (M1) for their correct substitution into the formula for expected value.

$$=1.83\left(\frac{33}{18}, 1.83333...\right)$$
 A1 [2 marks]

(c) $2 \times \frac{1}{18} \times \frac{3}{18}$

Note: Award **(M1)** for $\frac{1}{18} \times \frac{3}{18}$, award **(M1)** for multiplying their product by 2.

$$=\frac{1}{54}\left(\frac{6}{324}, 0.0185185..., 1.85\%\right)$$

[3 marks]

Total [6 marks]

(M1)(M1)

A1

(a) discrete	A1	[1 mark]
(b) $\frac{24+60+3k+40+15+6}{88+k} = 2$	M1A1	
Note: Award <i>M1</i> for substitution into the formula for the mean, award <i>A1</i> for a correct equation.		
attempt to solve their equation	(M1)	
<i>k</i> = 31	A1	
		[4 marks]
(c) systematic	A1	
		[1 mark]
	Total	[6 marks]

(a)	$158 \times 6 = 948(g)$	(M1)A1	[2 marks]
(b)	variance 6×13^2	(M1)	
	$SD = 31.8(g) (13\sqrt{6}, 31.8433)$	A1	[2 marks]
(C)	$X \sim N(948, 31.8433^2)$		
	P(X > 1000) = 0.0512 (0.0512350)	(M1)A1	[2 marks]
		Total	[6 marks]
Ques	stion 8		
(a)	Convenience	A1	[1 mark]
(b)	H_0 : 1% of the toys produced are faulty	A1	
	H_1 : More than 1% are faulty	A1	
			[2 marks]
(C)	$X \sim B(200, 0.01)$	(M1)	
(-)	$P(X \ge 4) = 0.142$	A1	
Not	te: Any attempt using Normal approximation to find <i>p</i> -value is awarded A	NOAO.	
			[2 marks]
(d)	14% > 10%	R1	
(u)	so there is insufficient evidence to reject H_0 .	A1	
Not	te: Do not award <i>R0A1</i> . Accept "fail to reject H_0 " or "accept H_0 ".		
			[2 marks]

Total [7 marks]

(a)

(a)		2				
t	1	2	3	4	5	6
	1	3	5	7	9	11
P(T=t)	36	36	36	36	36	36
	(0.027777)	(0.083333)	(0.138888)	(0.194444)	(0.25)	(0.305555)
Notes Au	and Ad if three a	ka fiyo naahahili	tion are correct			A2
Note: Av	vard A1 if three	to five probabili	ties are correct			
						[2 marks
(b) (i)	$\frac{32}{36}\left(\frac{8}{9}, 0.888\right)$	888, 88.9%)			((A1)
(ii)		or of 32 OR	denominator of	0.888888, et		M1)
	$\frac{11}{32}$ (0.34375,	34.4%)				A1
	52					[3 marks
(c) <u>1×1</u> -	$\frac{+3\times2+5\times3+}{36}$	+11×6			(M1)
$=\frac{16}{34}$	$\frac{1}{5}$ (4 $\frac{17}{36}$, 4.47, 4.	47222)				A1
50). ~					[2 marks
						Total [7 marks
uestion 10						
$X \sim Po(8.8)$)				(1	(11)
Note: Awa	ard (M1) for calc	ulating the mea	in, 8.8, of the d	istribution		
	rudini Officia					
P(X > 9) = 1	$P(X \ge 10)$ OR	P(X > 9) = 1	$-P(X \le 9)$		(1	A1)
$\mathbf{P}(X > 9) = 0$	0.386 (0.38626	0)			(M1).	A1
Note: Awa	ard (M1)(M0)(M1)A0 for finding	$P(X \ge 9) = 0.5$	518 (0.517719)	
		The second se		2.355	5054 V	
OR	$P(X \le 9) = 0.61$	4 (0.613740).			

(a)	$H_0: m = 110, H_1: m > 110$	A1
Note:	Accept other appropriate variables for the mean. Accept 22 in place of 110.	
		[1 mark]
(b)	$P(X \ge 128) = 0.05024$	(M1)(A1)
	$P(X \ge 129) = 0.04153$	(M1)
	(probability of making a type I error is) 0.0415	A1
Note:	If other probabilities are seen, the final $A1$ cannot be awarded unless 0.0415 is clearly identified as the final answer.	
		[4 marks]
(c)	$X \sim Po(110)$	
	$P(X \ge 126) = 0.072 > 0.05$ OR recognizing $126 < 129$ or ≤ 128	R1
	so there is insufficient evidence to reject H_0	A1
	(<i>ie</i> there is insufficient evidence to suggest that the number of coffe sold has increased)	ees being
Note:	Accept 'Accept H _o '.	
	Do not award R0A1 .	
		[2 marks]
		Total [7 marks]

(a)
$$\overline{x} = \frac{\sum x}{n} = \frac{2506}{30} = 83.5 \quad (83.5333...)$$
 A1

[1 mark]

(b)	$\left(s_{n-1}^{2} = \frac{\sum x^{2} - \frac{\left(\sum x\right)^{2}}{n}}{n-1} = \frac{209738 - \frac{2506^{2}}{30}}{29}\right)$	(M1)	
	=13.9 (13.9126)	A1 [2	marks]
(c)	(82.1, 84.9) (82.1405, 84.9261)	A2 [2	marks]
(d)	85 is outside the confidence interval and therefore Talha would suggest that the manufacturer's claim is incorrect	R1	[1 mark]
Ques	tion 13	Total [6 marks]
-			
(a)	$\left(\frac{74+97+91+86+112}{5}\right) = 92$	A1	
	2 .5		[1 mark]
(b)	(i) H_0 : The data satisfies the model	A1	
	H _i : The data does not satisfy the model	A1	
	Note: Do not accept " H_0 : The same number of copies will be sold each of a similar statement if the word 'expect' or 'expected' is included. Since the same number of the same number of copies will be sold each of a similar statement if the word 'expect' or 'expected' is included.		
	(ii) 4	A1	
	(iii) $\chi^2_{\text{calc}} = 8.54 \ (8.54347)$ OR <i>p</i> -value = 0.0736 (0.0735802)	A2	
	8.54 < 9.49 OR $0.0736 > 0.05$	R1	
	therefore there is insufficient evidence to reject ${\rm H}_{_0}$	A1	
	(i.e. the data satisfies the model)	[7	[marks]

Total [8 marks]

Question 14

	$H_0: \mu_c = \mu_s$	A1
	$H_1: \mu_c > \mu_s$	A1
Not	Accept an equivalent statement in words, must include mean and reference to "population mean" / "mean for all chinchilla rabbits" for the first A1 to be awarded. The terms "on average" and "generally" are also acceptable to indicate populations. Do not accept an imprecise "the means are equal".	
		[2 marks]
(b)	<i>p</i> -value = 0.0408 (0.0408065)	A2
Not	e: Award A1 for an answer of 0.041565, from "unpooled" settings on GDC	
0	APRA	[2 marks]
(c)	0.0408 < 0.05 .	R1
	(there is sufficient evidence to) reject (or not accept) H_0	A1
	(there is sufficient evidence to suggest that chinchilla rabbits are (generally) heavier
	than sable rabbits)	
Not	than sable rabbits) re: Do not award R0A1 . Accept 'accept H ₁ '.	
Not		[2 marks]
Not		
	e: Do not award R0A1 . Accept 'accept H ₁ '.	[2 marks] Total [6 marks]
ues	tion 15	
	e: Do not award R0A1 . Accept 'accept H ₁ '.	
ues	tion 15 let <i>X</i> be the random variable "the weight of a sack of potatoes"	Total [6 marks] (M1) A1
ues	tion 15 let <i>X</i> be the random variable "the weight of a sack of potatoes" $P(X < 50)$	Total [6 marks] (M1)
ues	tion 15 let X be the random variable "the weight of a sack of potatoes" P(X < 50) = 0.588 kg (0.587929) P(X < l) = 0.25	Total [6 marks] (M1) A1 [2 marks] (M1)
ues (a)	tion 15 let X be the random variable "the weight of a sack of potatoes" P(X < 50) = 0.588 kg (0.587929)	Total [6 marks] (M1) A1 [2 marks]
ues (a)	tion 15 let X be the random variable "the weight of a sack of potatoes" P(X < 50) = 0.588 kg (0.587929) P(X < l) = 0.25	Total [6 marks] (M1) A1 [2 marks] (M1) A1
ues (a) (b)	tion 15 let X be the random variable "the weight of a sack of potatoes" P(X < 50) = 0.588 kg (0.587929) P(X < l) = 0.25 49.2 kg (49.1929)	Total [6 marks] (M1) A1 [2 marks] (M1) A1 [2 marks]
ues (a) (b)	tion 15 let <i>X</i> be the random variable "the weight of a sack of potatoes" P(X < 50) = 0.588 kg (0.587929) P(X < l) = 0.25 49.2 kg (49.1929) attempt to sum 10 independent random variables	Total [6 marks] (M1) A1 [2 marks] (M1) A1 [2 marks] (M1)

(a) 75	A1
	[1 mark]
(b) recognition that all entries add up to 120	(M1)
a = 120 - 6 - 13 - 26 - b OR $a = 75 - b$	A1 [2 marks]
(c) (i) $\frac{6 \times 1 + 13 \times 2 + 26 \times 3 + (75 - b) \times 4 + b \times 5}{120} = 3.65$	(M1)(A1)
Note: Award (<i>M1</i>) for attempt to substitute into mean formula, LHS ex sufficient for the <i>M</i> mark. Award (<i>A1</i>) for correct substitutions in OR in two variables, followed by evidence of solving simultaneo $a+b=75$.	one variable
(<i>b</i> =) 28	A1
(ii) 120 – their part (c)(i) seen (e.g. 92 indicated on graph)	(M1)
84	A1
	[5 marks] Total: [8 marks]
Question 17	
(a) $X \sim Po(324)$	A1
Note: Both distribution and mean must be seen for A1 to be awarded	
	[1 mark]
(b) $P(X \le 300)$	(M1)
= 0.0946831 ≈ 0.0947	A1
	[2 marks]
(c) (mean number of cars =) $4.5 \times 60 = 270$	(A1)
$P(X > 300 \mid \lambda = 270)$	(M1)
Note: Award <i>M1</i> for using $\lambda = 270$ to evaluate a probability.	
$P(X \ge 301)$ OR $1 - P(X \le 300)$	(M1)
= 0.0334207 ≈ 0.0334	A1
	[4 marks] [Total 7 marks]

Ques	tion 18		
(a)	$\log_{10} 100 = a - 3$	(M1)	
	a=5	A1	
			[2 marks]
(b)	EITHER	(1.4.4)	
	$N = 10^{5-M}$	(M1)	
	$=\frac{10^5}{10^M}\left(=\frac{100000}{10^M}\right)$		
	OR		
	$100 = \frac{b}{10^3}$	(M1)	
	10'		
	THEN		
	$b = 100000 \ (=10^5)$	A1	10 montrol
			[2 marks]
(\mathbf{c})	$N = \frac{10^5}{10^{72}} = 0.00631 (0.0063095)$	A1	
(C)	$N = \frac{10^{7.2}}{10^{7.2}} = 0.00031 (0.0003095)$	~'	
Note	: Do not accept an answer of $10^{-2.2}$.		
			[1 mark]
(d)	METHOD 1		
(u)	$Y > 100 \Rightarrow$ no earthquakes in the first 100 years	(M1)	
	EITHER		
	let X be the number of earthquakes of at least magnitude 7.2 in a year $X \sim Po(0.0063095)$		
	$(P(X=0))^{100}$	(M1)	
		()	
	OR		
	let X be the number of earthquakes in 100 years $X \sim Po(0.0063095\times100)$	(M1)	
	P(X=0)	(111)	
	· · ·		
	THEN 0.532 (0.532082)	A1	
	METHOD 2		
	$Y > 100 \Longrightarrow$ no earthquakes in the first 100 years	(M1)	
	let X be the number of earthquakes in 100 years		
	since n is large and p is small $X \sim B(100, 0.0063095)$	(M1)	
	P(X=0)	()	
	0.531 (0.531019)	A1	1223 View 18.
		ITO	[3 marks] tal 8 marks]
		[10	a o martoj

(a) let X be the weight of sugar in the bag

 $P(X < 950) = 0.308537... \approx 0.309$ (M1)A1 [2 marks]

(b) METHOD 1

let \overline{X} be the mean weight of 5 bags of sugar

$E(\overline{X}) = 1000$	(A1)
use of $\operatorname{Var}(\overline{X}) = \frac{\sigma^2}{n}$	(M1)

Var
$$(\bar{X}) = \frac{100^2}{5}$$
 (= 2000) (A1)
 $\bar{X} \sim N(1000, 2000)$

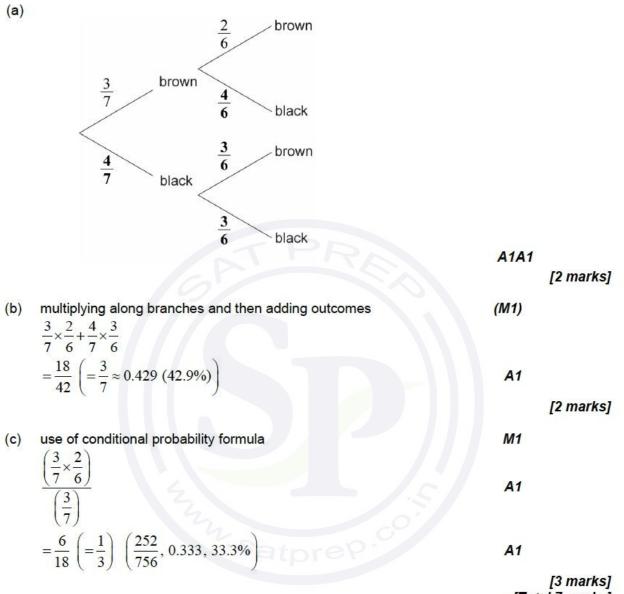
 $P(\overline{X} > 950) = 0.868223... \approx 0.868 \ (86.8\%)$ A1

METHOD 2

let T be the total weight of 5 bags of sugar

E(T) = 5000	(A1)
use of $\operatorname{Var}(X_1 + X_2) = \operatorname{Var}(X_1) + \operatorname{Var}(X_2)$ for independent random	
variables	(M1)
$\operatorname{Var}(T) = 5 \times 100^2 \ (= 50000)$	(A1)
$T \sim N(5000, 50000)$	
$P(T > 4750) = 0.868223 \approx 0.868$ (86.8%)	A1

[4 marks] [Total 6 marks]



[3 marks] [Total 7 marks]

Question 21 (a) H_0 : The die is fair OR P(any number) = $\frac{1}{6}$ OR probabilities are equal

 $H_1: \text{The die is not fair OR } P(\text{any number}) \neq \frac{1}{6} \text{ OR probabilities are not equal} \textbf{A1}$ [1 mark]

			[i many
(b)	5	A1	[1 mark]
(c)	10	A1	[1 mark]
(d)	(p-value =) 0.287 (0.28724163)	A2	[2 marks]
(e)	0.287 > 0.05	R1	
	EITHER Insufficient evidence to reject the null hypothesis	A1	
	OR Insufficient evidence to reject that the die is fair	A1	
Ques	tion 22		
(a)	P(Type I error) = P(stating female when male)		
	$= P(W_{Male} > 11.5)$	(M1)	
	= 0.00135 (0.00134996)	A1	
			[2 marks]
(b)	P(Type II error) = P(stating male when female)		
	$= P(W_{Female} < 11.5)$	(M1)	
	= 0.309 (0.308537)	A1	
			[2 marks]
(c)	attempt to use the total probability	(M1)	
	$P(error) = 0.9 \times 0.00134996 + 0.1 \times 0.308537$		
	= 0.0321 (0.0320687)	A1	[2 marks]
		Total	[2 marks] [6 marks]

ucst	1011 /			
(a)	$\overline{x} =$	4.63 (4.62686)	A1	F 4
				[1 mai
(b)	<i>S</i> _{<i>n</i>-1}	=1.098702	(A1)	
	s_{n-1}^2	=1.21 (1.207146)	A1	
Note	: Aw	vard A0A0 for an answer of 1.19 from biased estimate.		
			[2	2 mark
(c)	(i)	$H_1: \mu > 4.4$	A1	
	(ii)	METHOD 1		
	. ,	using a <i>z</i> -test	(M1)	
		p = 0.0454992	A1	
		p < 0.05	R1	
		reject null hypothesis (therefore there is significant evidence that the IB HL r of π than the population in general)	A1 math students know more	e digits
Note		o not award R0A1 . Allow R1A1 for consistent conclusion eir <i>p</i> -value.	n following on from	
		METHOD 2		
		using a <i>t</i> -test	(M1)	
		p = 0.0478584	A1	
		<i>p</i> < 0.05	R1	
		reject null hypothesis (therefore there is significant evidence that the IB HL r of π than the population in general)	A1 math students know more	e di <mark>git</mark> s
Note		o not award R0A1 . Allow R1A1 for consistent conclusion eir <i>p</i> -value.	n following on from	
			[!	5 mark
			Total [8	8 mark

[5 marks] Total [8 marks]

(a)	Accept any one of the following (or equivalent): one minimum and one maximum point three <i>x</i> -intercepts or three roots (or zeroes) one point of inflexion	R1
No	te: Do not accept "S shape" as a justification.	
(1)		[1 mark]
(b)	(i) $(d =) -5$	A1
	(ii) $8 = a + b + c$	
	4 = 8a + 4b + 2c	
	0 = 27a + 9b + 3c	A2
	te: Award A2 if all three equations are correct. Award A1 if at least one is correct. Award A1 for three correct include the letter "d".	
	(iii) $a = 2, b = -12, c = 18$	A1
		[4 marks]
(c)		(M1)
	$0 = 2t^3 - 12t^2 + 18t - 5$	
	t = 0.358216, 1.83174, 3.81003	(A1)
	(so total time in debt is 3.81003…−1.83174…+0.358216 ≈)	
	2.34 (2.33650) years	A1
		[3 marks]
		Total [8 marks]
lues	tion 25	
(a)	$(E(X) =) 10 \times 0.8$	(M1)
199223	8 (people)	A1
	o (people)	[2 marks]
(b)	recognition of binomial probability	(M1)
	0.0881 (0.0880803)	A1
		[2 marks]
c)	0.8 and 6 seen OR 0.2 and 3 seen	(A1)
	attempt to use binomial probability	(M1)
	0.121 (0.120873)	A1
		[3 marks]

[3 marks] Total [7 marks]

(a)
$$\left(\frac{17+25}{130}\right) = \frac{42}{130} \left(\frac{21}{65}, 0.323076...\right)$$
 A1
[1 mark]

(b)
$$\left(\frac{17}{17+25}\right) = \frac{17}{42}$$
 (0.404761...) **A1A1**

Note: Award A1 for correct numerator and A1 for correct denominator. Award A1A0 for working of $\frac{17/130}{\text{their answer to (a)}}$ if followed by an incorrect answer.

(c)
$$\frac{41}{130} \times \frac{40}{129}$$

Note: Award A1 for two correct fractions seen, M1 for multiplying their fractions.

 $=\frac{1640}{16770} \approx 0.0978 \left(0.0977936..., \frac{164}{1677} \right)$ [3 marks] Total [6 marks]

[2 marks]

A1M1

(a) D = S - R

METHOD 1

$= \operatorname{Re}(1.15 e^{(0.0165t-2.97)i}) - \operatorname{Re}(1.08 e^{(0.0165t+0.413)i}) (+18.9-4.94)$	(M1)(A1)
$= \operatorname{Re}\left(e^{0.0165ti}\left(1.15e^{-2.97i}-1.08e^{0.413i}\right)\right) \ (+13.96)$	(M1)
$= \operatorname{Re}\left(e^{0.0165ti}(2.21379e^{-2.85310i})\right) (+13.96)$	(A1)
$= 2.21\cos(0.0165t - 2.85) + 13.96 (2.21379\cos(0.0165t - 2.8531))$	10)+13.96)
(a = 2.21, b = -2.85, c = 13.96)	A1A1

Note: Award A1 for 2.21cos (0.0165t - 2.85) and A1 for "+13.96". The A1 for 13.96 is independent of the previous marks.

METHOD 2

c = 13.9 - 4.94 = 13.96	A1
using a graph of D	M1
maximum (172.915, 16.1738)	(A1)
minimum (-17.4842, 11.7462)	(A1)

EITHER

amplitude 16.173811.7462= 4.4276	
$a = 2.21 \ (2.2138)$	A1
OR	
a = 16.1738 13.96 = 2.21 (2.2138)	A1

THEN

EITHER when t = 0, D = 11.8377... $11.8377... = 2.2138\cos(b) + 13.96$ b = -2.85 (2.85309...) A1 OR $b = -0.0165 \times 172.915... = -2.85$ (2.85309...) A1

(b) 16.2 (16.1737...) hours on day 173

Note: Accept an answer of "day 172" for the second A1.

[2 marks] Total [8 marks]

A1A1

[6 marks]

a) (let p be the probability of a student choosing healthy options) $H_0: p = 0.3$	A1
$H_0: p = 0.3$ $H_1: p > 0.3$	A1
$n_1. p > 0.5$	Al
Note: Award A0A1 for correct hypotheses with μ in place of p . Accept equivalent hypotheses in words.	
en-reserver •	[2 marks]
b) a type I error is rejecting H_0 when H_0 is true	(M1)
(let N = number of students choosing a healthy option) $N \sim B(80, 0.3)$	
$P(31 \le N \le 80)$ OR $P(N \ge 31)$ OR $1 - P(N \le 30)$	(M1)
lote: Do not accept the use of the Normal approximation.	
0.0587 (0.0587481)	A1
	[3 marks]
c) a type II error is accepting H_0 when H_0 is not true	(M1)
$N \sim B(80, 0.4)$	(114)
$P(0 \le N \le 30)$ OR $P(N \le 30)$	(M1) A1
0.369 (0.368726)	A1 [3 marks] Total [8 marks]
lestion 29	Total [0 marks]
(a) $0.5 \times 0.1 + 0.4 \times 0.4 + 0.1 \times 0.5$	(M1)(M1)(M1)
(a) 0.5×0.1+0.4×0.4+0.1×0.5	
Note: Award M1 for 0.5×0.1 or 0.1×0.5 , M1 for 0.4×0.4 , M1 for three correct products.	adding
0.26	A1
	[4 marks]
(b) $0 = -8 \times 0.5 + 4 \times 0.4 + 0.1k$	(M1)(M1)
Note: Award M1 for correct substitution into the formula for expected award M1 for the expected value formula equated to zero.	ed value,
(k =) 24 (points)	A1

(a) The favourite breakfast/berry (of adults) is independent of (their) income (level). A1

[1 mark]

(b) $\chi^2 = 2.27$ (2.26821...)

A2 [2 marks]

(c) EITHER 2.27 < 7.78 OR 2.27 < critical valueOR 0.687 > 0.1 (using *p*-value)

R1

THEN

(Do not reject H_0)

Insufficient evidence (at the 10% significance level) that the favourite berry depends on income level.

[2 marks] Total [5 marks]

