

Subject - Math AI(Higher Level)
Topic - Function
Year - May 2021 - Nov 2024
Paper -2
Questions

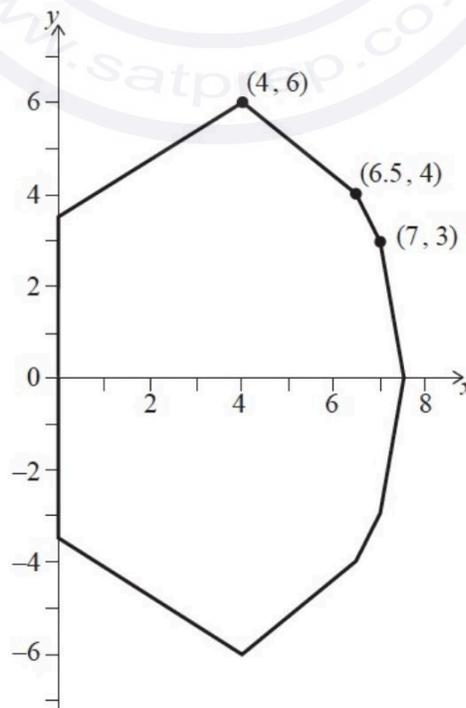
Question 1

[Maximum mark: 14]

Charlotte decides to model the shape of a cupcake to calculate its volume.



From rotating a photograph of her cupcake she estimates that its cross-section passes through the points $(0, 3.5)$, $(4, 6)$, $(6.5, 4)$, $(7, 3)$ and $(7.5, 0)$, where all units are in centimetres. The cross-section is symmetrical in the x -axis, as shown below:



She models the section from $(0, 3.5)$ to $(4, 6)$ as a straight line.

- (a) Find the equation of the line passing through these two points. [2]

Charlotte models the section of the cupcake that passes through the points $(4, 6)$, $(6.5, 4)$, $(7, 3)$ and $(7.5, 0)$ with a quadratic curve.

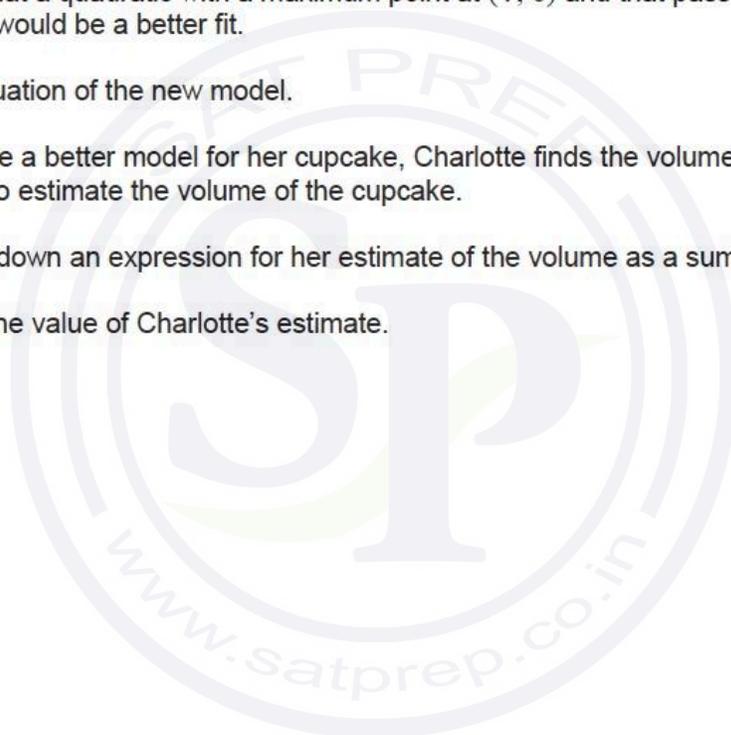
- (b) (i) Find the equation of the least squares regression quadratic curve for these four points.
(ii) By considering the gradient of this curve when $x = 4$, explain why it may not be a good model. [3]

Charlotte thinks that a quadratic with a maximum point at $(4, 6)$ and that passes through the point $(7.5, 0)$ would be a better fit.

- (c) Find the equation of the new model. [4]

Believing this to be a better model for her cupcake, Charlotte finds the volume of revolution about the x -axis to estimate the volume of the cupcake.

- (d) (i) Write down an expression for her estimate of the volume as a sum of two integrals.
(ii) Find the value of Charlotte's estimate. [5]



Question 2

[Maximum mark: 12]

A scientist is conducting an experiment on the growth of a certain species of bacteria.

The population of the bacteria, P , can be modelled by the function

$$P(t) = 1200 \times k^t, \quad t \geq 0,$$

where t is the number of hours since the experiment began, and k is a positive constant.

(a) (i) Write down the value of $P(0)$.

(ii) Interpret what this value means in this context. [2]

3 hours after the experiment began, the population of the bacteria is 18 750.

(b) Find the value of k . [2]

(c) Find the population of the bacteria 1 hour and 30 minutes after the experiment began. [2]

The scientist conducts a second experiment with a different species of bacteria. The population of this bacteria, S , can be modelled by the function

$$S(t) = 5000 \times 1.65^t, \quad t \geq 0,$$

where t is the number of hours since both experiments began.

(d) Find the value of t when the two populations of bacteria are equal. [2]

It takes 2 hours and m minutes for the number of bacteria in the second experiment to reach 19 000.

(e) Find the value of m , giving your answer as an integer value. [4]