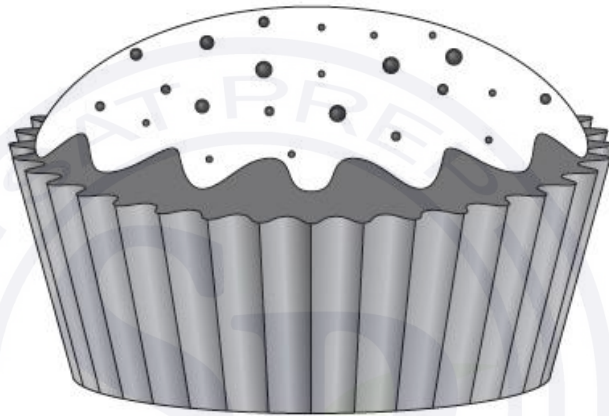


Subject - Math AI(Higher Level)
Topic - Function
Year - May 2021 - Nov 2022
Paper -2
Questions

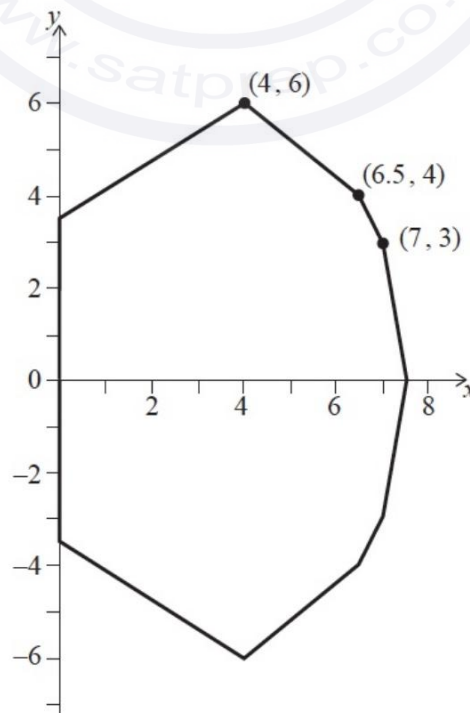
Question 1

[Maximum mark: 14]

Charlotte decides to model the shape of a cupcake to calculate its volume.



From rotating a photograph of her cupcake she estimates that its cross-section passes through the points $(0, 3.5)$, $(4, 6)$, $(6.5, 4)$, $(7, 3)$ and $(7.5, 0)$, where all units are in centimetres. The cross-section is symmetrical in the x -axis, as shown below:



She models the section from $(0, 3.5)$ to $(4, 6)$ as a straight line.

- (a) Find the equation of the line passing through these two points. [2]

Charlotte models the section of the cupcake that passes through the points $(4, 6)$, $(6.5, 4)$, $(7, 3)$ and $(7.5, 0)$ with a quadratic curve.

- (b) (i) Find the equation of the least squares regression quadratic curve for these four points.
(ii) By considering the gradient of this curve when $x = 4$, explain why it may not be a good model. [3]

Charlotte thinks that a quadratic with a maximum point at $(4, 6)$ and that passes through the point $(7.5, 0)$ would be a better fit.

- (c) Find the equation of the new model. [4]

Believing this to be a better model for her cupcake, Charlotte finds the volume of revolution about the x -axis to estimate the volume of the cupcake.

- (d) (i) Write down an expression for her estimate of the volume as a sum of two integrals.
(ii) Find the value of Charlotte's estimate. [5]

