Subject – Math AI(Higher Level)
Topic - Geometry and Trigonometry
Year - May 2021 – Nov 2024
Paper -2
Questions

Question 1

[Maximum mark: 17]

The following table shows the costs in US dollars (US$) of direct flights between six cities. Blank cells indicate no direct flights. The rows represent the departure cities. The columns represent the destination cities.

		Destination city					
		A	**B**	**C**	**D**	**E**	**F**
Departure city	**A**		90	150			
	B	90		80	70	140	
	C	150	80				
	D		70			100	180
	E		140		100		210
	F				180	210	

(a) Show the direct flights between the cities as a graph. [2]

(b) Write down the adjacency matrix for this graph. [2]

(c) Using your answer to part (b), find the number of different ways to travel from and return to city A in exactly 6 flights. [2]

(d) State whether or not it is possible to travel from and return to city A in exactly 6 flights, having visited each of the other 5 cities exactly once. Justify your answer. [2]

The following table shows the least cost to travel between the cities.

		Destination city					
		A	B	C	D	E	F
Departure city	A	0	90	150	160	a	b
	B	90	0	80	70	140	250
	C	150	80	0	150	220	330
	D	160	70	150	0	100	180
	E	a	140	220	100	0	210
	F	b	250	330	180	210	0

(e) Find the values of a and b. [2]

A travelling salesman has to visit each of the cities, starting and finishing at city A.

(f) Use the nearest neighbour algorithm to find an upper bound for the cost of the trip. [3]

(g) By deleting vertex A, use the deleted vertex algorithm to find a lower bound for the cost of the trip. [4]

Question 2

[Maximum mark: 14]

An aircraft's position is given by the coordinates (x, y, z), where x and y are the aircraft's displacement east and north of an airport, and z is the height of the aircraft above the ground. All displacements are given in kilometres.

The velocity of the aircraft is given as $\begin{pmatrix} -150 \\ -50 \\ -20 \end{pmatrix} \text{km h}^{-1}$.

At $13{:}00$ it is detected at a position $30\,\text{km}$ east and $10\,\text{km}$ north of the airport, and at a height of $5\,\text{km}$. Let t be the length of time in hours from $13{:}00$.

(a) Write down a vector equation for the displacement, r of the aircraft in terms of t. [2]

(b) If the aircraft continued to fly with the velocity given

(i) verify that it would pass directly over the airport;

(ii) state the height of the aircraft at this point;

(iii) find the time at which it would fly directly over the airport. [4]

When the aircraft is $4\,\text{km}$ above the ground it continues to fly on the same bearing but adjusts the angle of its descent so that it will land at the point $(0, 0, 0)$.

(c) (i) Find the time at which the aircraft is $4\,\text{km}$ above the ground.

(ii) Find the direct distance of the aircraft from the airport at this point. [5]

(d) Given that the velocity of the aircraft, after the adjustment of the angle of descent, is $\begin{pmatrix} -150 \\ -50 \\ a \end{pmatrix} \text{km h}^{-1}$, find the value of a. [3]

Question 3

[Maximum mark: 18]

An ice-skater is skating such that her position vector when viewed from above at time t seconds can be modelled by

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a\mathrm{e}^{bt}\cos t \\ a\mathrm{e}^{bt}\sin t \end{pmatrix}$$

with respect to a rectangular coordinate system from a point O, where the non-zero constants a and b can be determined. All distances are in metres.

(a) Find the velocity vector at time t. [3]

(b) Show that the magnitude of the velocity of the ice-skater at time t is given by

$$a\mathrm{e}^{bt}\sqrt{(1+b^2)}.$$ [4]

At time $t = 0$, the displacement of the ice-skater is given by $\begin{pmatrix} 5 \\ 0 \end{pmatrix}$ and the velocity of the ice-skater is given by $\begin{pmatrix} -3.5 \\ 5 \end{pmatrix}$.

(c) Find the value of a and the value of b. [3]

(d) Find the magnitude of the velocity of the ice-skater when $t = 2$. [2]

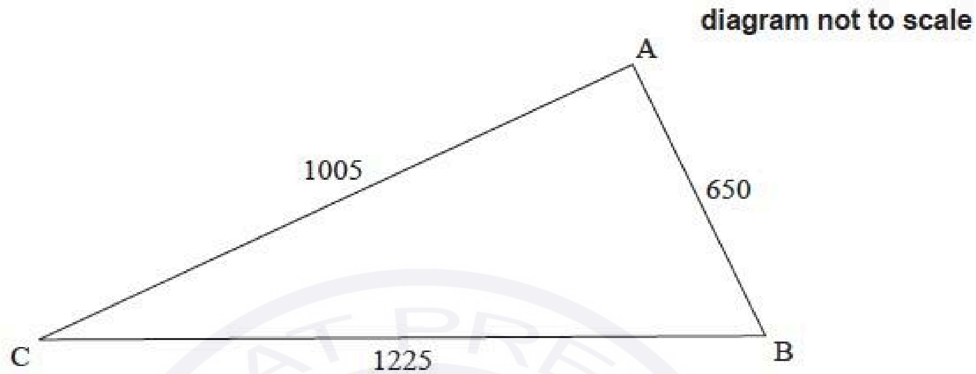
At a point P, the ice-skater is skating parallel to the y-axis for the first time.

(e) Find OP. [6]

Question 4

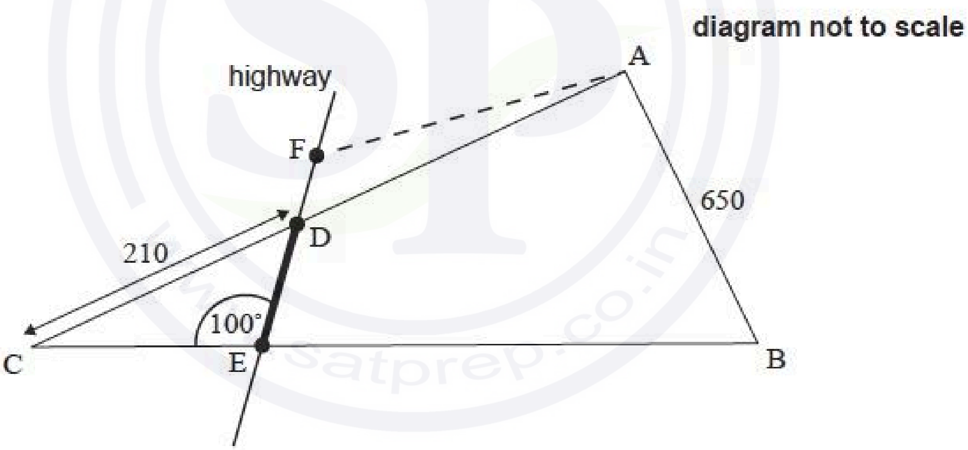
[Maximum mark: 15]

A farmer owns a field in the shape of a triangle ABC such that $AB = 650\,m$, $AC = 1005\,m$ and $BC = 1225\,m$.



(a) Find the size of $A\hat{C}B$. [3]

The local town is planning to build a highway that will intersect the borders of the field at points D and E, where $DC = 210\,m$ and $C\hat{E}D = 100^\circ$, as shown in the diagram below.



(b) Find DE. [3]

The town wishes to build a carpark here. They ask the farmer to exchange the part of the field represented by triangle DCE. In return the farmer will get a triangle of equal area ADF, where F lies on the same line as D and E, as shown in the diagram above.

(c) Find the area of triangle DCE. [5]

(d) Estimate DF. You may assume the highway has a width of zero. [4]

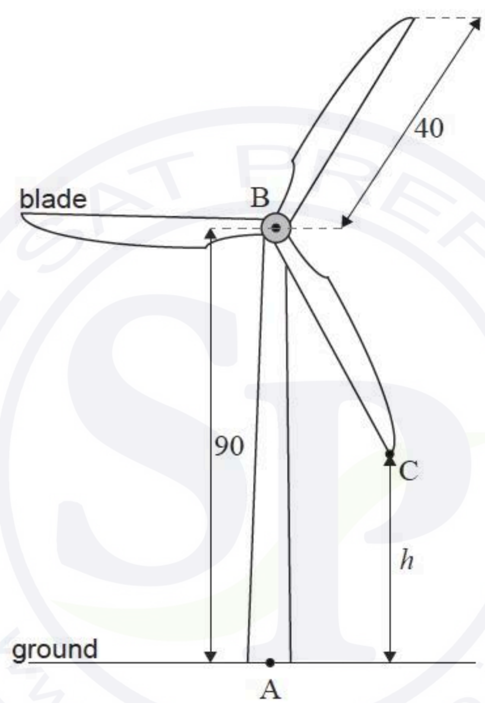
Question 5

[Maximum mark: 20]

A wind turbine is designed so that the rotation of the blades generates electricity. The turbine is built on horizontal ground and is made up of a vertical tower and three blades.

The point A is on the base of the tower directly below point B at the top of the tower. The height of the tower, AB, is $90\,\text{m}$. The blades of the turbine are centred at B and are each of length $40\,\text{m}$. This is shown in the following diagram.

diagram not to scale



The end of one of the blades of the turbine is represented by point C on the diagram. Let h be the height of C above the ground, measured in metres, where h varies as the blade rotates.

(a) Find the

(i) maximum value of h.

(ii) minimum value of h. [2]

The blades of the turbine complete 12 rotations per minute under normal conditions, moving at a constant rate.

(b) (i) Find the time, in seconds, it takes for the blade $[\text{BC}]$ to make one complete rotation under these conditions.

(ii) Calculate the angle, in degrees, that the blade $[\text{BC}]$ turns through in one second. [3]

The height, h, of point C can be modelled by the following function. Time, t, is measured from the instant when the blade [BC] first passes [AB] and is measured in seconds.

$$h(t) = 90 - 40\cos(72t^{\circ}),\ t \geq 0$$

(c) (i) Write down the amplitude of the function.

(ii) Find the period of the function. [2]

(d) Sketch the function $h(t)$ for $0 \leq t \leq 5$, clearly labelling the coordinates of the maximum and minimum points. [3]

(e) (i) Find the height of C above the ground when $t = 2$.

(ii) Find the time, in seconds, that point C is above a height of $100\,\text{m}$, during each complete rotation. [5]

The wind speed increases and the blades rotate faster, but still at a constant rate.

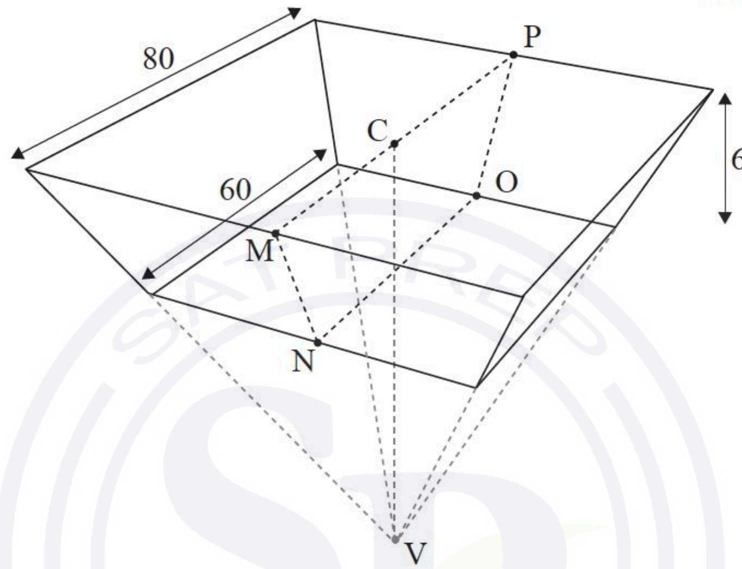
(f) Given that point C is now higher than $110\,\text{m}$ for 1 second during each complete rotation, find the time for one complete rotation. [5]

Question 6

[Maximum mark: 14]

A large water reservoir is built in the form of part of an upside-down right pyramid with a horizontal square base of length 80 metres. The point C is the centre of the square base and point V is the vertex of the pyramid.

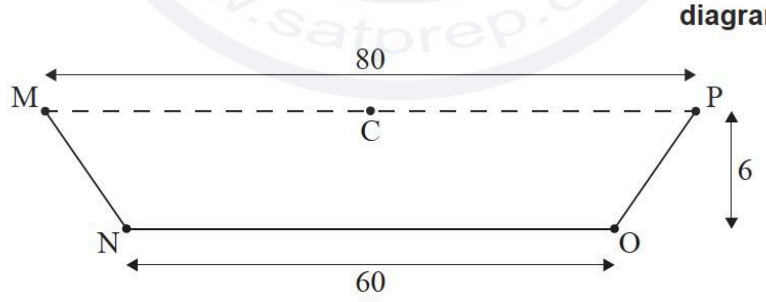
diagram not to scale



The bottom of the reservoir is a square of length 60 metres that is parallel to the base of the pyramid, such that the depth of the reservoir is 6 metres as shown in the diagram.

The second diagram shows a vertical cross section, MNOPC, of the reservoir.

diagram not to scale



(a) Find the angle of depression from M to N. [2]

(b) (i) Find CV.

(ii) Hence or otherwise, show that the volume of the reservoir is $29\,600\,\text{m}^3$. [5]

Every day $80\,m^3$ of water from the reservoir is used for irrigation.

Joshua states that, if no other water enters or leaves the reservoir, then when it is full there is enough irrigation water for at least one year.

(c) By finding an appropriate value, determine whether Joshua is correct. [2]

To avoid water leaking into the ground, the five interior sides of the reservoir have been painted with a watertight material.

(d) Find the area that was painted. [5]

Question 7

[Maximum mark: 21]

At an archery tournament, a particular competition sees a ball launched into the air while an archer attempts to hit it with an arrow.

The path of the ball is modelled by the equation

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} + t\begin{pmatrix} u_x \\ u_y - 5t \end{pmatrix}$$

where x is the horizontal displacement from the archer and y is the vertical displacement from the ground, both measured in metres, and t is the time, in seconds, since the ball was launched.

- u_x is the horizontal component of the initial velocity
- u_y is the vertical component of the initial velocity.

In this question both the ball and the arrow are modelled as single points. The ball is launched with an initial velocity such that $u_x = 8$ and $u_y = 10$.

(a) (i) Find the initial speed of the ball.

(ii) Find the angle of elevation of the ball as it is launched. [4]

(b) Find the maximum height reached by the ball. [3]

(c) Assuming that the ground is horizontal and the ball is not hit by the arrow, find the x coordinate of the point where the ball lands. [3]

(d) For the path of the ball, find an expression for y in terms of x. [3]

An archer releases an arrow from the point $(0, 2)$. The arrow is modelled as travelling in a straight line, in the same plane as the ball, with speed $60\,\text{m}\,\text{s}^{-1}$ and an angle of elevation of 10°.

(e) Determine the two positions where the path of the arrow intersects the path of the ball. [4]

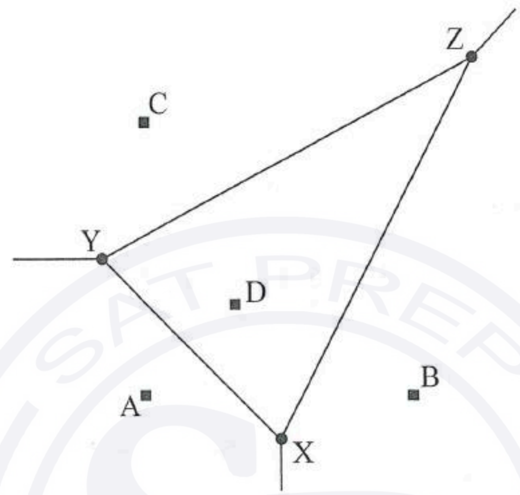
(f) Determine the time when the arrow should be released to hit the ball before the ball reaches its maximum height. [4]

Question 8

[Maximum mark: 18]

The Voronoi diagram below shows four supermarkets represented by points with coordinates $A(0, 0)$, $B(6, 0)$, $C(0, 6)$ and $D(2, 2)$. The vertices X, Y, Z are also shown. All distances are measured in kilometres.

diagram not to scale



(a) Find the midpoint of [BD]. [2]

(b) Find the equation of (XZ). [4]

The equation of (XY) is $y = 2 - x$ and the equation of (YZ) is $y = 0.5x + 3.5$.

(c) Find the coordinates of X. [3]

The coordinates of Y are $(-1, 3)$ and the coordinates of Z are $(7, 7)$.

(d) Determine the exact length of [YZ]. [2]

(e) Given that the exact length of [XY] is $\sqrt{32}$, find the size of $X\hat{Y}Z$ in degrees. [4]

(f) Hence find the area of triangle XYZ. [2]

A town planner believes that the larger the area of the Voronoi cell XYZ, the more people will shop at supermarket D.

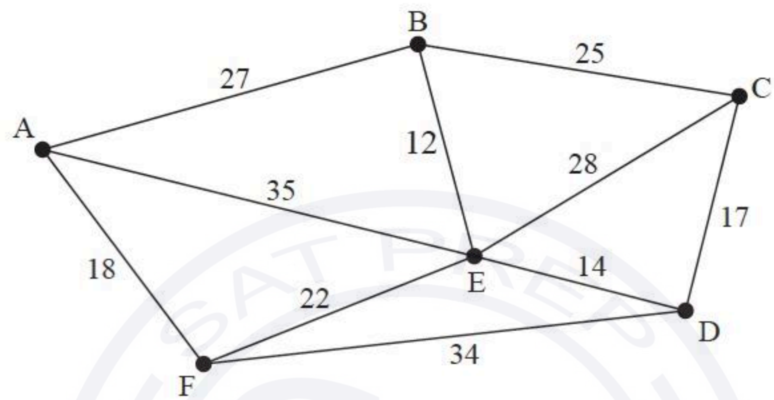
(g) State one criticism of this interpretation. [1]

Question 9

[Maximum mark: 14]

A company has six offices, A, B, C, D, E and F. One of the company managers, Nanako, needs to visit the offices. She creates the following graph that shows the distances, in kilometres, between some of the offices.

diagram not to scale



(a) Write down a Hamiltonian cycle for this graph. [1]

(b) State, with a reason, whether the graph contains an Eulerian circuit. [1]

Nanako wishes to find the shortest cycle to visit all the offices. She decides to complete a weighted adjacency table, showing the least distance between each pair of offices.

	A	B	C	D	E	F
A		27	52	p	35	18
B			25	26	12	q
C				17	28	r
D					14	34
E						22
F						

(c) Write down the value of

(i) p.

(ii) q.

(iii) r. [3]

(d) Starting at vertex E, use the nearest neighbour algorithm to find an upper bound for Nanako's cycle. [3]

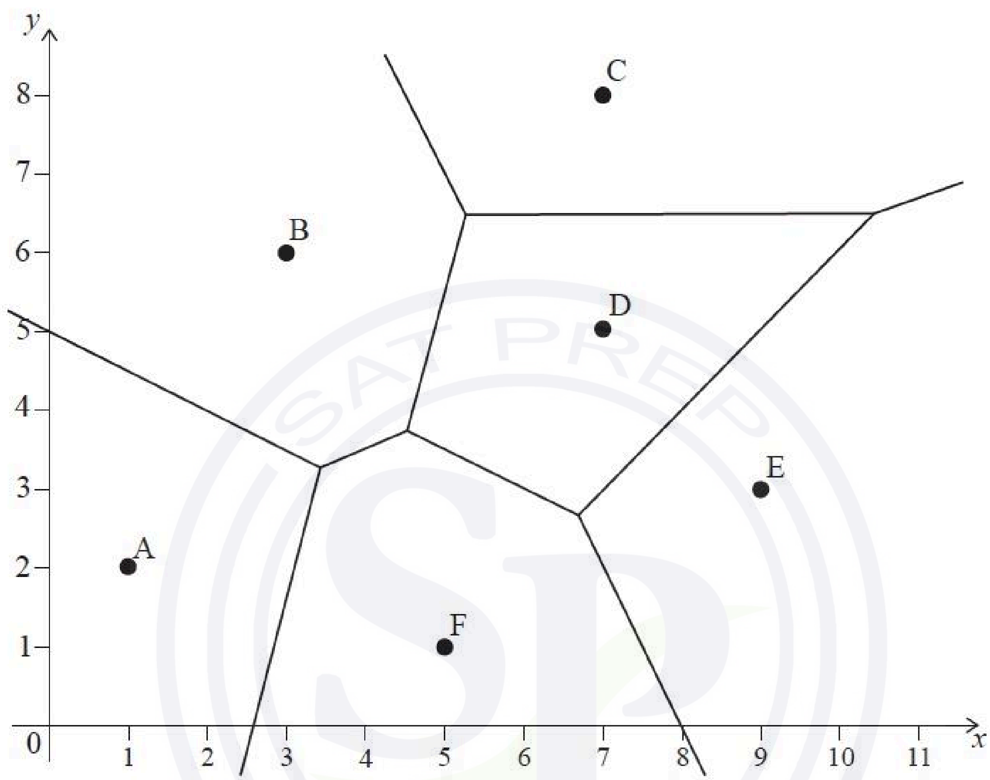
(e) By deleting vertex F, find a lower bound for Nanako's cycle. [4]

(f) Explain, with a reason, why the answer to part (e) might not be the best lower bound. [2]

Question 10

[Maximum mark: 13]

Six restaurant locations (labelled A, B, C, D, E and F) are shown, together with their Voronoi diagram. All distances are measured in kilometres.



(a) Elena wants to eat at the closest restaurant to her. Write down the restaurant she should go to, if she is at

(i) $(2, 7)$.

(ii) $(0, 1)$, when restaurant A is closed. [2]

Restaurant C is at $(7, 8)$ and restaurant D is at $(7, 5)$.

(b) Find the equation of the perpendicular bisector of CD. [2]

Restaurant B is at $(3, 6)$.

(c) Find the equation of the perpendicular bisector of BC. [5]

(d) Hence find

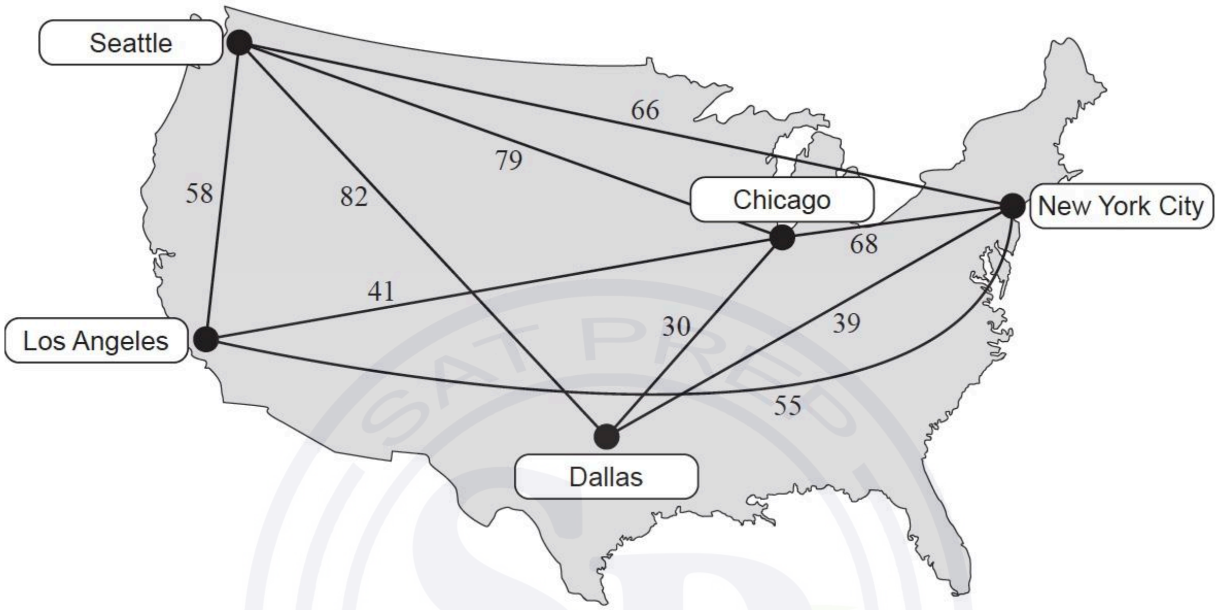
(i) the coordinates of the point which is of equal distance from B, C and D.

(ii) the distance of this point from D. [4]

Question 11

. [Maximum mark: 19]

The following graph shows five cities of the USA connected by weighted edges representing the cheapest direct flights in dollars ($) between cities.



(a) Explain why the graph can be described as “connected”, but not “complete”. [2]

(b) Find a minimum spanning tree for the graph using Kruskal’s algorithm.

State clearly the order in which your edges are added, and draw the tree obtained. [3]

(c) Using only the edges obtained in your answer to part (b), find an upper bound for the travelling salesman problem. [2]

Ronald lives in New York City and wishes to fly to each of the other cities, before finally returning to New York City. After some research, he finds that there exists a direct flight between Los Angeles and Dallas costing $\$26$. He updates the graph to show this.

(d) By using the nearest neighbour algorithm and starting at Los Angeles, determine a better upper bound than that found in part (c).

State clearly the order in which you are adding the vertices. [3]

(e) (i) By deleting the vertex which represents Chicago, use the deleted vertex algorithm to determine a lower bound for the travelling salesman problem.

(ii) Similarly, by instead deleting the vertex which represents Seattle, determine another lower bound. [5]

(f) Hence, using your previous answers, write down your best inequality for the **least** expensive tour Ronald could take. Let the variable C represent the total cost, in dollars, for the tour. [2]

(g) Write down a tour that is strictly greater than your lower bound and strictly less than your upper bound. [2]

Question 12

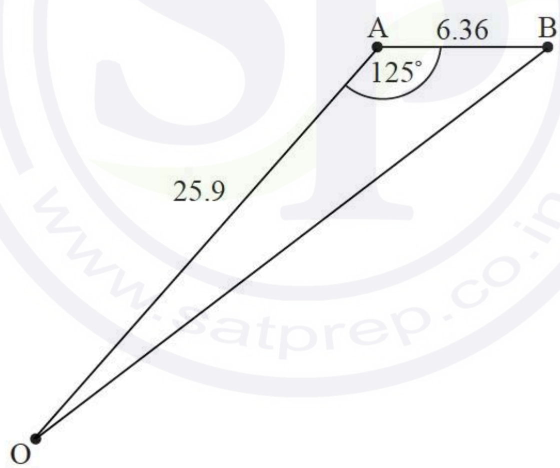
[Maximum mark: 13]

The diagram shows points in a park viewed from above, at a specific moment in time.

The distance between two trees, at points A and B, is 6.36 m.

Odette is playing football in the park and is standing at point O, such that $OA = 25.9\text{ m}$ and $O\hat{A}B = 125^{\circ}$.

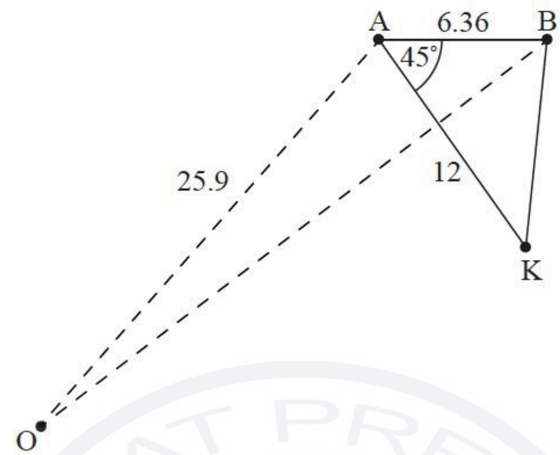
diagram not to scale



(a) Calculate the area of triangle AOB. [3]

Odette's friend, Khemil, is standing at point K such that he is $12\,\mathrm{m}$ from A and $K\hat{A}B = 45^\circ$.

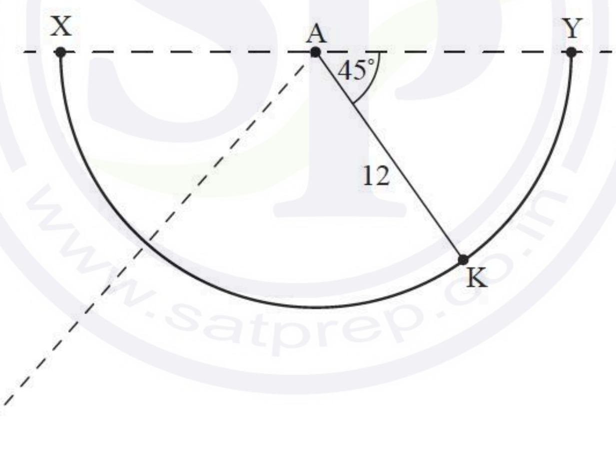
diagram not to scale



(b) Calculate Khemil's distance from B. [3]

XY is a semicircular path in the park with centre A, such that $K\hat{A}Y = 45^\circ$. Khemil is standing on the path and Odette's football is at point X. This is shown in the diagram below.

diagram not to scale



The length $KX = 22.2\,\mathrm{m}$, $K\hat{O}X = 53.8^\circ$ and $O\hat{K}X = 51.1^\circ$.

(c) Find whether Odette or Khemil is closer to the football. [4]

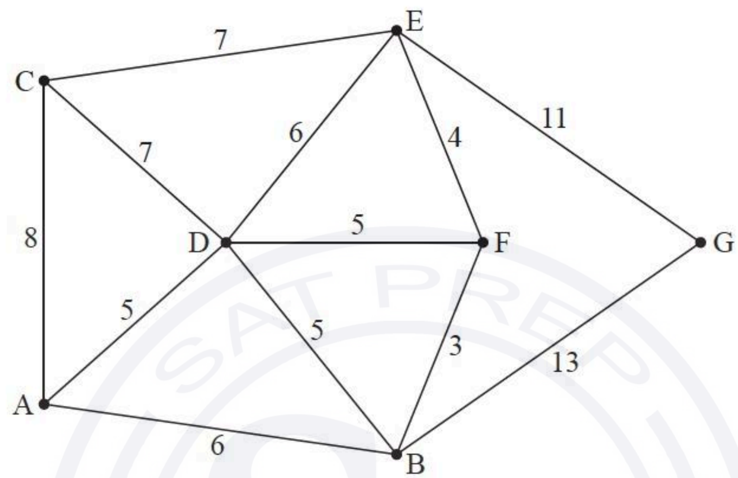
Khemil runs along the semicircular path to pick up the football.

(d) Calculate the distance that Khemil runs. [3]

Question 13

[Maximum mark: 17]

The vertices in the following graph represent seven towns. The edges represent their connecting roads. The weight on each edge represents the distance, in kilometres, between the two connected towns.



(a) Determine whether it is possible to complete a journey that starts and finishes at different towns that also uses each of the roads exactly once. Give a reason for your answer. [2]

The shortest distance, in kilometres, between any two towns is given in the table.

	A	B	C	D	E	F	G
A		6	8	5	11	9	19
B	6		12	5	7	3	13
C	8	12		7	7	***a***	***b***
D	5	5	7		6	5	***c***
E	11	7	7	6		4	11
F	9	3	***a***	5	4		***d***
G	19	13	***b***	***c***	11	***d***	

(b) Find the value of

(i) a;

(ii) b;

(iii) c;

(iv) d. [2]

(c) Use the nearest neighbour algorithm, starting at vertex G, to find an upper bound for the travelling salesman problem. [3]

(d) (i) Sketch a minimum spanning tree for the subgraph with vertices A, B, C, D, E, F.

(ii) Write down the total weight of the minimum spanning tree. [4]

(e) Hence find a lower bound for the travelling salesman problem. [2]

(f) Explain one way in which an improved lower bound could be found. [1]

It is found that the optimum solution starting at A is actually A–C–E–G–B–F–D–A.

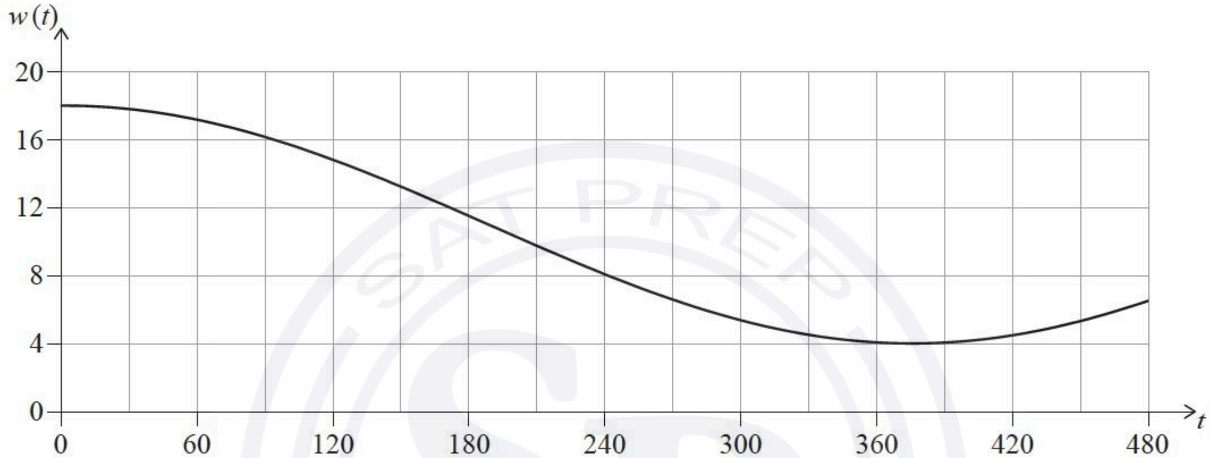
(g) Given that the length of each road shown on the graph is given to the nearest kilometre, find the lower bound for the total distance in the optimal solution. [3]

Question 14

2. [Maximum mark: 15]

The depth of water, w metres, in a particular harbour can be modelled by the function $w(t) = a\cos(bt^{\circ}) + d$ where t is the length of time, in minutes, after 06:00.

On 20 January, the first high tide occurs at 06:00, at which time the depth of water is 18 m. The following low tide occurs at 12:15 when the depth of water is 4 m. This is shown in the diagram.



(a) Find the value of a. [2]

(b) Find the value of d. [2]

(c) Find the period of the function in minutes. [3]

(d) Find the value of b. [2]

Naomi is sailing to the harbour on the morning of 20 January. Boats can enter or leave the harbour only when the depth of water is at least 6 m.

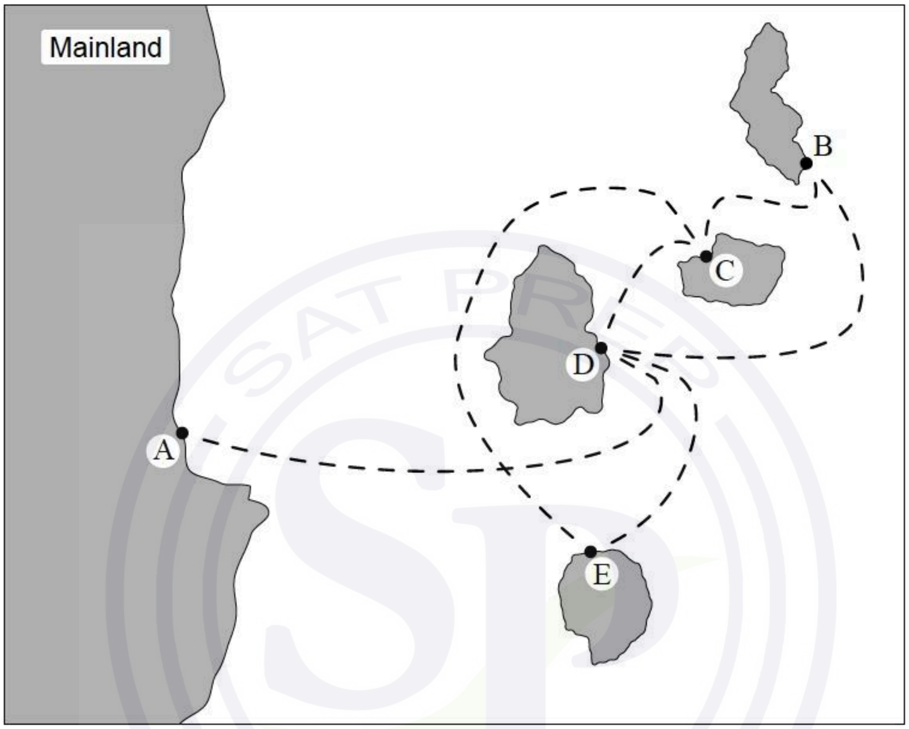
(e) Find the latest time before 12:00, to the nearest minute, that Naomi can enter the harbour. [4]

(f) Find the length of time (in minutes) between 06:00 and 15:00 on 20 January during which Naomi **cannot** enter or leave the harbour. [2]

Question 15

[Maximum mark: 18]

The following diagram is a map of a group of four islands and the closest mainland. Travel from the mainland and between the islands is by boat. The scheduled boat routes between the ports A, B, C, D and E are shown as dotted lines on the map.



Let the undirected graph G represent the boat routes between the ports A, B, C, D and E.

(a) Draw graph G. [1]

(b) Graph G can be represented by an adjacency matrix $\boldsymbol{P}$, where the rows and columns represent the ports in alphabetical order.

(i) Given that $\boldsymbol{P}^3 = \begin{pmatrix} 0 & 1 & 2 & 4 & 1 \\ 1 & 2 & 5 & a & 2 \\ 2 & 5 & 4 & 6 & 5 \\ 4 & a & 6 & 4 & 6 \\ 1 & 2 & 5 & 6 & 2 \end{pmatrix}$, find the value of a.

(ii) Hence, write down the number of different ways that someone could start at port B and end at port C, using three boat route journeys. [3]

(c) Find a possible Eulerian trail in G, starting at port A. [2]

The cost of a journey on the different boat routes is given in the following table; all prices are given in USD. The cost of a journey is the same in either direction between two ports.

	A	**B**	**C**	**D**	**E**
A				10	
B			20	25	
C		20		50	45
D	10	25	50		30
E			45	30	

Sofia wants to make a trip where she travels on each of the boat routes at least once, beginning and ending at port A.

(d) Find the minimum cost of Sofia's trip. [3]

The boat company decides to add an additional boat route to make it possible to travel on each boat route **exactly** once, starting and ending at the same port.

(e) (i) Identify between which two ports the additional boat route should be added.

(ii) Determine the cost of the additional boat route such that the overall cost of the trip is the same as your answer to part (d). [2]

The boat company plans to redesign which ports are connected by boat routes. Their aim is to have a single boat trip that visits all the islands and minimizes the total distance travelled, starting and finishing at the mainland, A.

The following table shows the distances in kilometres between the ports A, B, C, D and E.

	A	**B**	**C**	**D**	**E**
A		80	90	50	60
B	80		30	70	120
C	90	30		45	100
D	50	70	45		55
E	60	120	100	55	

(f) (i) Use the nearest neighbour algorithm to find an upper bound for the minimum total distance.

(ii) Use the deleted vertex algorithm on port A to find a lower bound for the minimum total distance. [7]

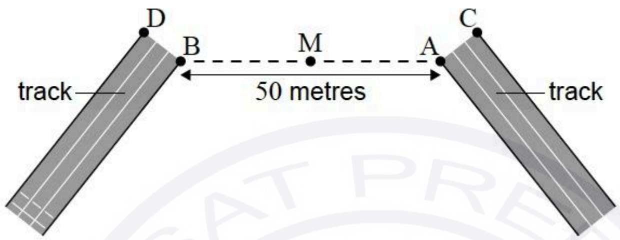
Question 16

[Maximum mark: 15]

Madhu is designing a jogging track for the campus of her school. The following diagram shows an incomplete portion of the track.

Madhu wants to design the track such that the inner edge is a smooth curve from point A to point B, and the other edge is a smooth curve from point C to point D. The distance between points A and B is 50 metres.

diagram not to scale

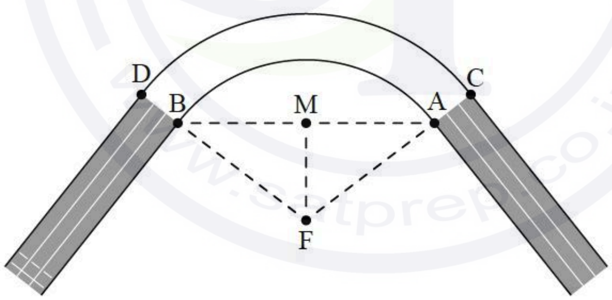


To create a smooth curve, Madhu first walks to M, the midpoint of [AB].

(a) Write down the length of [BM]. [1]

Madhu then walks 20 metres in a direction perpendicular to [AB] to get from point M to point F. Point F is the centre of a circle whose arc will form the smooth curve between points A and B on the track, as shown in the following diagram.

diagram not to scale



(b) (i) Find the length of [BF].

(ii) Find $\mathrm{B\hat{F}M}$. [4]

(c) Hence, find the length of arc AB. [3]

The outer edge of the track, from C to D, is also a circular arc with centre F, such that the track is 2 metres wide.

(d) Calculate the area of the curved portion of the track, ABDC. [4]

The base of the track will be made of concrete that is 12 cm deep.

(e) Calculate the volume of concrete needed to create the curved portion of the track. [3]

Question 17

[Maximum mark: 12]

A hygiene inspector lives in Town A and must visit restaurants in five towns $(\mathrm{B}-\mathrm{F})$, before returning to A. The inspector must not repeat any of the towns. The distances, in km, between the six towns are shown in the table.

	A	**B**	**C**	**D**	**E**	**F**
A		31	28	26	22	23
B	31		25	20	27	25
C	28	25		19	22	24
D	26	20	19		21	22
E	22	27	22	21		24
F	23	25	24	22	24	

(a) Starting at A, use the nearest neighbour algorithm to find an upper bound for the length of the journey the inspector must take. State the order in which the towns are to be visited. [4]

(b) By deleting A, use Prim's algorithm starting at B to find a lower bound for the length of the inspector's journey. [5]

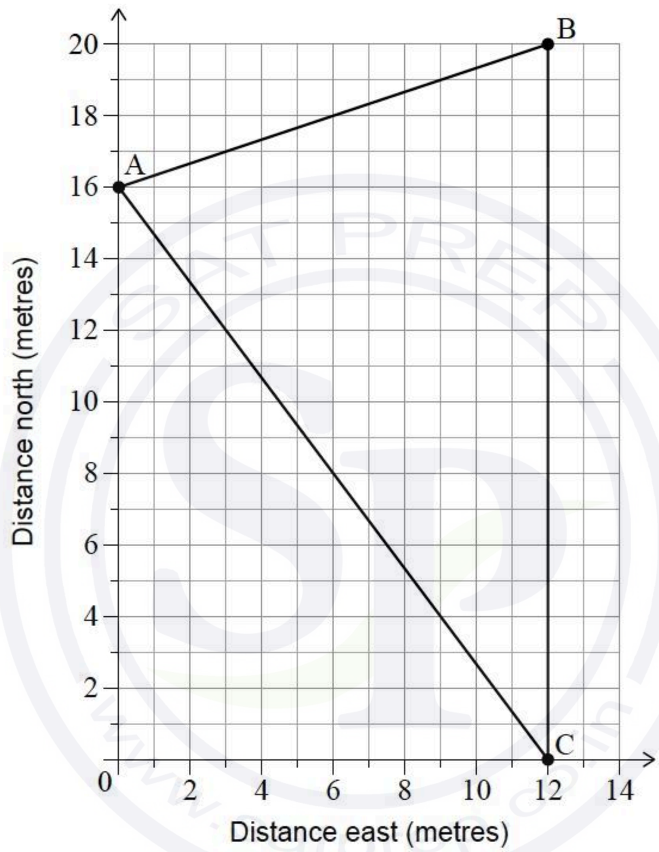
(c) By considering the minimum spanning tree found in part (b), determine whether the journey given by this lower bound is an achievable solution. [3]

Question 18

[Maximum mark: 14]

Mai is at an amusement park. A map of part of the amusement park is represented on the following coordinate axes.

Mai's favourite three attractions are positioned at A(0, 16), B(12, 20) and C(12, 0).
All measurements are in metres.



(a) Write down the distance between B and C. [1]

(b) Calculate the distance between A and B. [2]

Mai is standing at the attraction at B and wants to walk directly to the attraction at A.

(c) Calculate the bearing of A from B. [3]

A drinking fountain is to be installed at a point that is an equal distance from each of the attractions at A, B and C.

(d) (i) Write down the gradient of [AC].

(ii) Write down the mid-point of [AC].

(iii) Hence calculate the coordinates of the drinking fountain. [8]

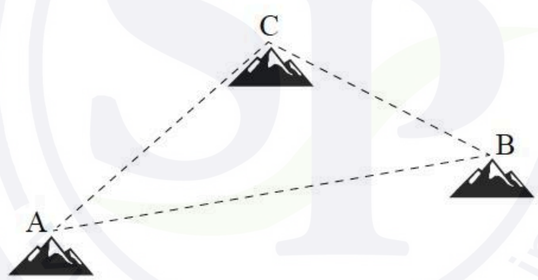
Question 19

[Maximum mark: 12]

A national park contains three mountains whose summits are at points A, B and C.

According to a coordinate system, the position of A is $(0, 0, 2.8)$ and the position of B is $(7.2, 5.1, 2.4)$. All the values are in kilometres.

diagram not to scale



(a) (i) Find the vector $\overrightarrow{AB}$.

(ii) Hence find AB, the distance between A and B. [3]

The vector $\overrightarrow{AC}$ is parallel to the vector $\begin{pmatrix} 1.1 \\ 8.4 \\ 0.2 \end{pmatrix}$.

(b) Find the angle between $\begin{pmatrix} 1.1 \\ 8.4 \\ 0.2 \end{pmatrix}$ and $\overrightarrow{AB}$. [5]

The angle between $\overrightarrow{BA}$ and $\overrightarrow{BC}$ is 55.2°.

(c) Use the sine rule to find AC. [4]

Question 20

[Maximum mark: 15]

In this question, all distances are in kilometres and t is in hours.

Let $\begin{pmatrix}1\\0\\0\end{pmatrix}$ be a displacement of 1 km due east, $\begin{pmatrix}0\\1\\0\end{pmatrix}$ be a displacement of 1 km due north,

and $\begin{pmatrix}0\\0\\1\end{pmatrix}$ be a vertical displacement of 1 km.

Highway 85 in Saudi Arabia is a long, straight, flat road.

Relative to the centre of the town Arar, point O, the position vector of a car, C, travelling along this road is given by:

$$\vec{\text{OC}} = \begin{pmatrix}10\\-5\\0\end{pmatrix} + t\begin{pmatrix}50\\-33\\0\end{pmatrix}.$$

(a) Find the speed of the car. [2]

The police are testing a long-range drone, D, to monitor cars travelling along this road. The drone is launched at $t = 0$ from the point with position vector $\begin{pmatrix}200\\-100\\0.02\end{pmatrix}$ and flies in a straight line with a constant height of 0.02 km and a constant velocity of $\begin{pmatrix}-15\\-20\\0\end{pmatrix}$.

(b) Find the angle between the path of the car and the path of the drone. [3]

(c) Write down the position vector, $\vec{\text{OD}}$, of the drone at time t. [1]

(d) At time t_1, the drone passes through the point with position vector $\begin{pmatrix} 152 \\ p \\ 0.02 \end{pmatrix}$.

Find the value of

(i) t_1

(ii) p. [3]

(e) (i) Find an expression for $\overrightarrow{\mathrm{CD}}$, the relative position of the drone from the car.

(ii) Hence, find the shortest distance between the car and the drone. [6]

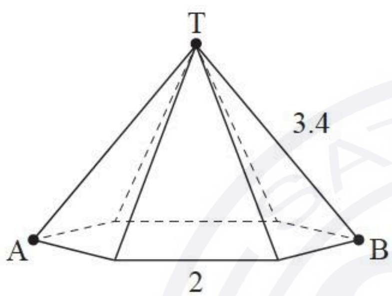
Question 21

[Maximum mark: 17]

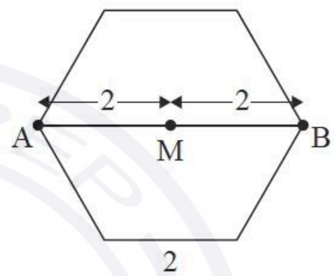
Gaurika is designing a tent in the shape of a right pyramid with a regular hexagonal base, centre M. The length of each side of the base is $2\,\mathrm{m}$, the length of each sloping edge is $3.4\,\mathrm{m}$, and the distance between each vertex on the base and M is $2\,\mathrm{m}$, as shown in the diagrams.

diagrams not to scale

3D view of tent



2D view of base

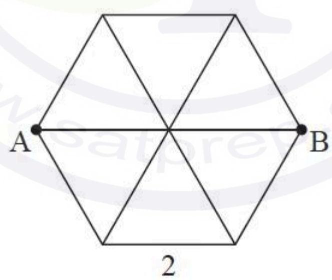


The top of the tent, T, will be supported by a vertical pole from M.

(a) Find the length of the pole, MT. [2]

The hexagonal base can be divided into six equilateral triangles.

diagram not to scale



(b) Find

(i) the area of the base

(ii) the volume of the tent. [5]

(c) Find the value of $\mathrm{M\hat{A}T}$. [2]

For extra support, Gaurika decides to attach a rope, with length $2.6\,\text{m}$, to the tent at a point, X, on the edge AT.

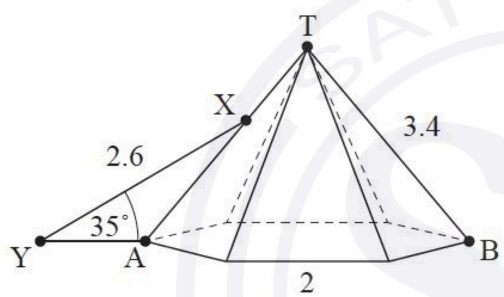
The rope will be fixed to the ground at point Y, such that:

- the rope, $[XY]$, is straight
- points Y, A and B lie on a straight line
- $A\hat{Y}X = 35^\circ$.

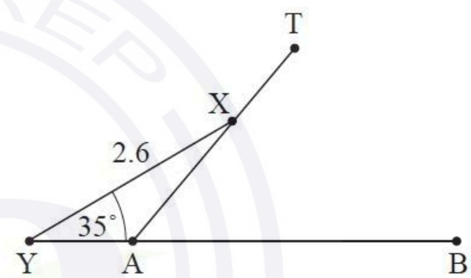
This is shown in the diagrams.

diagrams not to scale

3D view of tent and rope



2D view of triangle AXY



(d) Find AY. [4]

For decoration at night, Gaurika wants to fix a strip of lights from point A to a point, Z, along the rope $[XY]$.

The strip of lights, $[AZ]$, is straight and has length $0.9\,\text{m}$.

(e) Find the two possible values of YZ. [4]