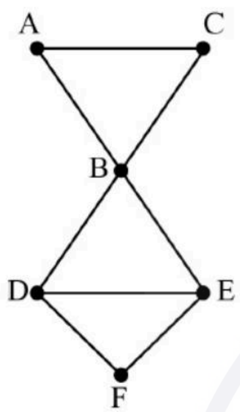
Subject – Math AI(Higher Level)
Topic - Geometry and Trigonometry
Year - May 2021 – Nov 2024
Paper -2
Answers

Question 1

(a)



A2

[2 marks]

(b) attempt to form an adjacency matrix *M1*

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$ *A1*

[2 marks]

(c) raising the matrix to the power six *(M1)*

50 *A1*

[2 marks]

(d) not possible *A1*

because you must pass through B twice *R1*

[2 marks]

(e) $a = 230$, $b = 340$ **A1A1**

[2 marks]

(f) $\mathrm{A} \to \mathrm{B} \to \mathrm{D} \to \mathrm{E} \to \mathrm{F} \to \mathrm{C} \to \mathrm{A}$ **(M1)**

$90 + 70 + 100 + 210 + 330 + 150$ **(A1)**

(US$) 950 **A1**

[3 marks]

Question 2

(a) $\boldsymbol{r} = \begin{pmatrix} 30 \\ 10 \\ 5 \end{pmatrix} + t\begin{pmatrix} -150 \\ -50 \\ -20 \end{pmatrix}$ **A1A1**

[2 marks]

(b) (i) when $x = 0,\ t = \frac{30}{150} = 0.2$ **M1**

EITHER

when $y = 0,\ t = \frac{10}{150} = 0.2$ **A1**

since the two values of t are equal the aircraft passes directly over the airport

OR

$t = 0.2,\ y = 0$ **A1**

(ii) height $= 5 - 0.2 \times 20 = 1\text{km}$ **A1**

(iii) time 13:12 **A1**

[4 marks]

(c) (i) $5 - 20t = 4 \Rightarrow t = \frac{1}{20}$ (3 minutes) **(M1)**

time 13:03 **A1**

(ii) displacement is $\begin{pmatrix} 22.5 \\ 7.5 \\ 4 \end{pmatrix}$ **A1**

distance is $\sqrt{22.5^2 + 7.5^2 + 4^2}$ **(M1)**

$= 24.1\text{km}$ **A1**

[5 marks]

(d) ***METHOD 1***

time until landing is $12 - 3 = 9$ minutes **M1**

height to descend $= 4\,\text{km}$

$a = \dfrac{-4}{\frac{9}{60}}$ **M1**

$= -26.7$ **A1**

METHOD 2

$\begin{pmatrix} -150 \\ -50 \\ a \end{pmatrix} = s\begin{pmatrix} 22.5 \\ 7.5 \\ 4 \end{pmatrix}$ **M1**

$-150 = 22.5s \Rightarrow s = -\dfrac{20}{3}$ **M1**

$a = -\dfrac{20}{3} \times 4$

$= -26.7$ **A1**

[3 marks]

Total [14 marks]

Question 3

(a) use of product rule **(M1)**

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} abe^{bt}\cos t - ae^{bt}\sin t \\ abe^{bt}\sin t + ae^{bt}\cos t \end{pmatrix}$$ ***A1A1***

[3 marks]

(b) $|\boldsymbol{v}|^2 = \dot{x}^2 + \dot{y}^2 = \left[abe^{bt}\cos t - ae^{bt}\sin t\right]^2 + \left[abe^{bt}\sin t + ae^{bt}\cos t\right]^2$ ***M1***

> **Note:** It is more likely that an expression for $|\boldsymbol{v}|$ is seen.
> $\sqrt{\dot{x}^2 + \dot{y}^2}$ is not sufficient to award the ***M1***, their part (a) must be substituted.

$= \left[a^2\sin^2 t - 2a^2 b\sin t\cos t + a^2b^2\cos^2 t + a^2\cos^2 t + 2a^2 b\sin t\cos t + a^2b^2\sin^2 t\right]e^{2bt}$ ***A1***

use of $\sin^2 t + \cos^2 t = 1$ within a factorized expression that leads to the final answer ***M1***

$= a^2\left(b^2+1\right)e^{2bt}$ ***A1***

magnitude of velocity is $a\,e^{bt}\sqrt{\left(1+b^2\right)}$ ***AG***

[4 marks]

(c) when $t = 0$, $ae^{bt}\cos t = 5$

$a = 5$ ***A1***

$abe^{bt}\cos t - ae^{bt}\sin t = -3.5$ ***(M1)***

$b = -0.7$ ***A1***

> **Note:** Use of $a\,e^{bt}\sqrt{\left(1+b^2\right)}$ result from part (b) is an alternative approach.

[3 marks]

(d) $5e^{-0.7\times 2}\sqrt{\left(1+(-0.7)^2\right)}$ ***(M1)***

$1.51 \ (1.50504\ldots)$ ***A1***

[2 marks]

(e) $\dot{x} = 0$ ***(M1)***

$a\,e^{bt}(b\cos t - \sin t) = 0$

$\tan t = b$

$t = 2.53 \ (2.53086...)$ ***(A1)***

correct substitution of their t to find x or y ***(M1)***

$x = -0.697 \ (-0.696591...)$ **and** $y = 0.488 \ (0.487614...)$ ***(A1)***

use of Pythagoras / distance formula ***(M1)***

$\text{OP} = 0.850 \text{ m} \ (0.850297\ldots)$ ***A1***

[6 marks]

Total [18 marks]

Question 4

(a) use of cosine rule **(M1)**

$\hat{ACB} = \cos^{-1}\left(\frac{1005^2 + 1225^2 - 650^2}{2 \times 1005 \times 1225}\right)$ **(A1)**

$= 32.0^\circ$ $(31.9980...)$ **OR** 0.558 $(0.558471...)$ **A1**

[3 marks]

(b) use of sine rule **(M1)**

$\frac{\text{DE}}{\sin 31.9980...^\circ} = \frac{210}{\sin 100^\circ}$ **(A1)**

$(\text{DE} =)$ 113 m $(112.9937...)$ **A1**

[3 marks]

(c) **METHOD 1**

$180^\circ - (100^\circ +$ their part $(a))$ **(M1)**

$= 48.0019...^\circ$ **OR** $0.837791...$ **(A1)**

substituted area of triangle formula **(M1)**

$\frac{1}{2} \times 112.9937... \times 210 \times \sin 48.002^\circ$ **(A1)**

8820 m^2 $(8817.18...)$ **A1**

METHOD 2

$\frac{\text{CE}}{\sin(180 - 100 - \text{their part } (a))} = \frac{210}{\sin 100}$ **(M1)**

$(\text{CE} =)$ $158.472...$ **(A1)**

substituted area of triangle formula **(M1)**

EITHER

$\frac{1}{2} \times 112.993... \times 158.472... \times \sin 100$ **(A1)**

OR

$\frac{1}{2} \times 210... \times 158.472... \times \sin(\text{their part } (a))$ **(A1)**

THEN

8820 m^2 $(8817.18...)$ **A1**

(d) $1005 - 210$ **OR** 795 **(A1)**

equating answer to part (c) to area of a triangle formula **(M1)**

$8817.18... = \frac{1}{2} \times \text{DF} \times (1005 - 210) \times \sin 48.002...^\circ$ **(A1)**

$(\text{DF} =)$ 29.8 m $(29.8473...)$ **A1**

[4 marks]

Total [15 marks]

Question 5

(a) (i) maximum $h = 130$ metres **A1**

(ii) minimum $h = 50$ metres **A1**

[2 marks]

(b) (i) $(60 \div 12 =)$ 5 seconds **A1**

(ii) $360 \div 5$ ***(M1)***

Note: Award ***(M1)*** for 360 divided by their time for one revolution.

$= 72^\circ$ **A1**

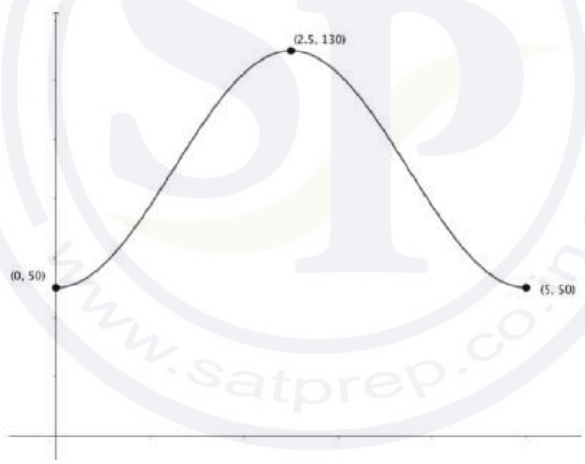
[3 marks]

(c) (i) (amplitude $=$) 40 **A1**

(ii) (period $= \frac{360}{72} =$) 5 **A1**

[2 marks]

(d)



Maximum point labelled with correct coordinates. **A1**

At least one minimum point labelled. Coordinates seen for any minimum points must be correct. **A1**

Correct shape with an attempt at symmetry and "concave up" evident as it approaches the minimum points. Graph must be drawn in the given domain. **A1**

[3 marks]

(e) (i) $h = 90 - 40\cos(144^{\circ})$ ***(M1)***

$(h =)\ 122\ (\text{m})\ (122.3606.....)$ ***A1***

(ii) evidence of $h = 100$ on graph **OR** $100 = 90 - 40\cos(72t)$ ***(M1)***

t coordinates $3.55\ (3.54892...)$ **OR** $1.45\ (1.45107...)$ or equivalent ***(A1)***

Note: Award ***A1*** for either t-coordinate seen.

= 2.10 seconds $(2.09784\ldots)$ ***A1***

[5 marks]

(f) **METHOD 1**

$90 - 40\cos(at^{\circ}) = 110$ ***(M1)***

$\cos(at^{\circ}) = -0.5$

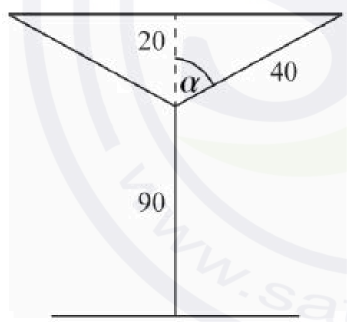
$at^{\circ} = 120,\ 240$ ***(A1)***

$1 = \dfrac{240}{a} - \dfrac{120}{a}$ ***(M1)***

$a = 120$ ***(A1)***

period $= \dfrac{360}{120} = 3$ seconds ***A1***

METHOD 2



attempt at diagram ***(M1)***

$\cos\alpha = \dfrac{20}{40}$ (or recognizing special triangle) ***(M1)***

angle made by C, $2\alpha = 120^{\circ}$ ***(A1)***

one third of a revolution in 1 second ***(M1)***

hence one revolution = 3 seconds ***A1***

METHOD 3

considering $h(t)=110$ on original function **(M1)**

$t=\frac{5}{3}$ or $\frac{10}{3}$ **(A1)**

$\frac{10}{3}-\frac{5}{3}=\frac{5}{3}$ **(A1)**

Note: Accept $t=1.67$ or equivalent.

so period is $\frac{3}{5}$ of original period **(R1)**

so new period is 3 seconds **A1**

[5 marks]

Total: [20 marks]

Question 6

(a) $\tan(\theta)=\frac{6}{10}$ **(M1)**

$(\theta=)\ 31.0^\circ\ (30.9637...^\circ)$ **OR** $0.540\ (0.540419\ldots)$ **A1**

[2 marks]

(b) (i) $(\text{CV}=)\ 40\tan(\theta)$ **OR** $(\text{CV}=)\ 4\times 6$ **(M1)**

Note: Award ***(M1)*** for an attempt at trigonometry or similar triangles (e.g. ratios).

$(\text{CV}=)\ 24\text{ m}$ **A1**

(ii) $(V=)\ \frac{1}{3}80^2\times 24-\frac{1}{3}60^2\times 18$ ***M1A1A1***

Note: Award ***M1*** for finding the difference between the volumes of two pyramids, ***A1*** for each correct volume expression. The final ***A1*** is contingent on correct working leading to the given answer.
If the correct final answer is not seen, award at most ***M1A1A0***. Award ***M0A0A0*** for any height derived from $V=29\,600$, including 18.875 or 13.875.

$(V=)\ 29\,600\text{ m}^3$ **AG**

[5 marks]

(c) **METHOD 1**

$\left(\frac{29\,600}{80}=\right)$ 370 (days) ***A1***

$(370 > 366)$ Joshua is correct ***A1***

Note: Award ***A0A0*** for unsupported answer of "Joshua is correct". Accept $1.01... > 1$ for the first ***A1*** mark.

METHOD 2

$80 \times 366 = 29280 \text{ m}^3$ **OR** $80 \times 365 = 29200 \text{ m}^3$ ***A1***

$(29280 < 29600)$ Joshua is correct ***A1***

Note: The second ***A1*** can be awarded for an answer consistent with their result.

[2 marks]

(d) height of trapezium is $\sqrt{10^2+6^2}$ $(=11.6619...)$ ***(M1)***

area of trapezium is $\frac{80+60}{2}\times\sqrt{10^2+6^2}$ $(=816.333...)$ ***(M1)(A1)***

$(SA=)$ $4\times\left(\frac{80+60}{2}\times\sqrt{10^2+6^2}\right)+60^2$ ***(M1)***

Note: Award ***M1*** for adding 4 times their (MNOP) trapezium area to the area of the (60×60) base.

$(SA=)$ 6870 m^2 (6865.33 m^2) ***A1***

Note: No marks are awarded if the correct shape is not identified.

[5 marks]
Total: [14 marks]

Question 7

(a) (i) $\sqrt{10^2+8^2}$ **(M1)**

$=12.8\ \ (12.8062\ldots, \sqrt{164})\ (\text{m s}^{-1})$ **A1**

(ii) $\tan^{-1}\left(\frac{10}{8}\right)$ **(M1)**

$=0.896$ **OR** $51.3\ \ (0.896055\ldots$ **OR** $51.3401\ldots°)$ **A1**

> **Note:** Accept 0.897 or 51.4 from use of $\arcsin\left(\frac{10}{12.8}\right)$.

[4 marks]

(b) $y=t(10-5t)$ **(M1)**

> **Note:** The ***M1*** might be implied by a correct graph or use of the correct equation.

METHOD 1 – graphical Method
sketch graph **(M1)**

> **Note:** The ***M1*** might be implied by correct graph or correct maximum (eg $t=1$).

max occurs when $y=5$ m **A1**

METHOD 2 – calculus
differentiating and equating to zero **(M1)**

$\frac{dy}{dt}=10-10t=0$

$t=1$

$y\ (=1(10-5))=5$ m **A1**

METHOD 3 – symmetry
line of symmetry is $t=1$ **(M1)**

$y\ (=1(10-5))=5$ m **A1**

[3 marks]

(c) attempt to solve $t(10-5t)=0$ **(M1)**

$t=2$ (or $t=0$) **(A1)**

$x\ (=5+8\times 2)=21$ m **A1**

> **Note:** Do not award the final ***A1*** if $x=5$ is also seen.

[3 marks]

Question 8

(a) $\left(\frac{2+6}{2}, \frac{2+0}{2}\right)$ **(M1)**

$(4, 1)$ **A1**

Note: Award ***A0*** if parentheses are omitted in the final answer.

[2 marks]

(b) attempt to substitute values into gradient formula **(M1)**

$\left(\frac{0-2}{6-2}=\right)-\frac{1}{2}$ **(A1)**

therefore the gradient of perpendicular bisector is 2 **(M1)**

so $y-1=2(x-4)$ $\quad(y=2x-7)$ **A1**

[4 marks]

(c) identifying the correct equations to use: **(M1)**

$y=2-x$ and $y=2x-7$

evidence of solving their correct equations or finding points of intersection graphically **(M1)**

$(3, -1)$ **A1**

Note: Accept an answer expressed as "$x=3$, $y=-1$".

[3 marks]

(d) attempt to use distance formula **(M1)**

$\text{YZ}=\sqrt{(7-(-1))^2+(7-3)^2}$

$=\sqrt{80}$ $\left(4\sqrt{5}\right)$ **A1**

[2 marks]

(e) **METHOD 1 (cosine rule)**

length of XZ is $\sqrt{80}$ $\left(4\sqrt{5}, 8.94427...\right)$ **(A1)**

Note: Accept 8.94 and 8.9.

attempt to substitute into cosine rule **(M1)**

$\cos \text{X}\hat{\text{Y}}\text{Z}=\frac{80+32-80}{2\times\sqrt{80}\sqrt{32}}$ $\quad(=0.316227\ldots)$ **(A1)**

Note: Award ***A1*** for correct substitution of XZ, YZ, $\sqrt{32}$ values in the cos rule. Exact values do not need to be used in the substitution.

$\left(\text{X}\hat{\text{Y}}\text{Z}=\right)$ $71.6°$ $\quad(71.5650\ldots°)$ **A1**

METHOD 2 (splitting isosceles triangle in half)

length of XZ is $\sqrt{80}$ $\left(4\sqrt{5}, 8.94427...\right)$ ***(A1)***

Note: Accept 8.94 and 8.9.

required angle is $\cos^{-1}\left(\frac{\sqrt{32}}{2\sqrt{80}}\right)$ ***(M1)(A1)***

Note: Award ***A1*** for correct substitution of XZ (or YZ), $\frac{\sqrt{32}}{2}$ values in the cos rule. Exact values do not need to be used in the substitution.

$\left(X\hat{Y}Z=\right)$ $71.6°$ $(71.5650°)$ ***A1***

Note: Last ***A1*** mark may be lost if prematurely rounded values of XZ, YZ and/or XY are used.

[4 marks]

(f) (area =) $\frac{1}{2}\sqrt{80}\sqrt{32}\sin 71.5650\ldots$ **OR** (area =) $\frac{1}{2}\sqrt{32}\sqrt{72}$ ***(M1)***

$=24\ \text{km}^2$ ***A1***

[2 marks]

(g) *Any sensible answer such as:*
There might be factors other than proximity which influence shopping choices.
A larger area does not necessarily result in an increase in population.
The supermarkets might be specialized / have a particular clientele who visit even if other shops are closer.
Transport links might not be represented by Euclidean distances.
etc. ***R1***

[1 mark]

Total [18 marks]

Question 9

(a)	any correct Hamiltonian cycle e.g. ABCDEFA	***A1***	
			[1 mark]
(b)	no, since not all vertices have an even degree (or equivalent)	***R1***	
			[1 mark]
(c)	(i) 49	***A1***	
	(ii) 34	***A1***	
	(iii) 50	***A1***	
			[3 marks]
(d)	cycle is EBCDFAE		
	$UB = 12 + 25 + 17 + 34 + 18 + 35$	***(M1)(A1)***	
	Note: Award ***M1*** for $12 + 25 + 17 + \ldots$ **OR** EBCD.		
	$= 141$	***A1***	
			[3 marks]
(e)	attempt to find MST for vertices A, B, C, D and E	***M1***	
	$12 + 14 + 17 + 27 \ (= 70)$	***A1***	
	$LB = 70 + 18 + 22$	***(M1)***	
	$= 110$	***A1***	
			[4 marks]
(f)	**EITHER**		
	deleting a different vertex	***A1***	
	might give a higher value (and hence a better lower bound).	***R1***	
	OR		
	the edges selected in part (e) do not form a cycle.	***A1***	
	so a higher value is possible	***R1***	
			[2 marks]
			[14 marks]

Question 10

(a) (i) B — *A1*

(ii) F — *A1*

[2 marks]

(b) correct substitution into the midpoint formula — *(M1)*

$$\frac{8+5}{2}$$

$y = 6.5$ — *A1*

Note: Answer must be an equation for the ***A1*** to be awarded.

[2 marks]

(c) midpoint $= (5, 7)$ — *(A1)*

correct use of gradient formula — *(M1)*

$$\frac{8-6}{7-3}$$

gradient of $\text{BC} = 0.5$ — *(A1)*

negative reciprocal of gradient — *(M1)*

perpendicular gradient $= -2$

$y - 7 = -2(x-5)$ (or $y = -2x + 17$) — *A1*

Note: Do not follow through within the part for the final ***A1***.

[5 marks]

(d) (i) attempt to find the intersection of two perpendicular bisectors (BC & CD) — *(M1)*

Note: This may be seen graphically or algebraically.

$6.5 - 7 = -2(x-5)$ **OR** $6.5 = -2x + 17$

Note: Accept equivalent methods using the perpendicular bisector of BD, $y - 5.5 = 4(x-5)$ **OR** $y = 4x - 14.5$

$x = 5.25,\ y = 6.5$ **OR** $(5.25, 6.5)$ — *A1*

Note: The x-coordinate must be exact or expressed to at least 3 sf.

(ii) their correct substitution into distance formula — *(M1)*

$$\sqrt{(5.25-7)^2 + (6.5-5)^2}$$

$= 2.30\text{ km}\ \left(2.30488\ldots,\ \frac{\sqrt{85}}{4}\right)$ — *A1*

[4 marks]

Total [13 marks]

Question 11

(a) any city can be travelled to or from any other city (so is connected) ***R1***

EITHER
but there is no direct flight between Los Angeles and Dallas (for example) ***R1***

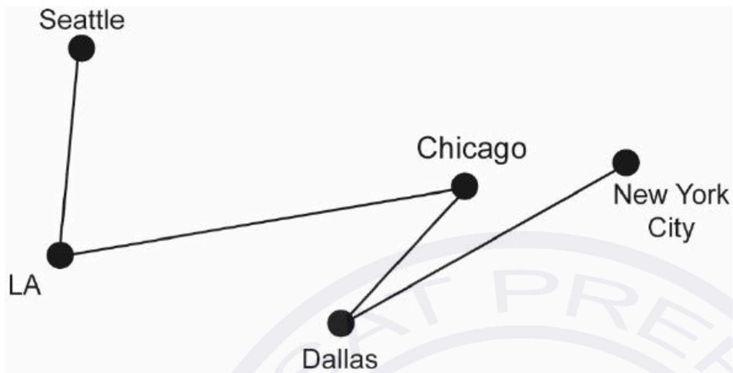
OR
but not every vertex has degree 4 ***R1***

Note: Accept equivalent statements for the cities being connected and the graph not being complete.

[2 marks]

(b) edge CD selected first *M1*
DN,
CL,
LS *A1*

Note:	Award marks if the answers are written as sums in the correct order. ***M1*** if 30 is seen first, ***A1*** for $30 + 39 + 41 + 58$.



A1

Note:	The final ***A1*** can be awarded independently. Award ***M0A0A1*** for a correct MST graph with no other working. Award ***M1A0A1*** if Prim's algorithm is seen to be used correctly with CD first.

[3 marks]

(c) $2 \times \text{MST}$ weight ***(M1)***
$= \$336$ ***A1***

Note:	Allow any integer multiple (>1) of MST weight for ***M1***, and if correctly calculated, award ***M1A1***.

[2 marks]

(d) attempt at nearest neighbour algorithm ***M1***
order is $\text{LA} \to \text{D} \to \text{C} \to \text{NYC} \to \text{S} \to \text{LA}$ ***A1***

Note:	Award ***M1*** for a route that begins with LA and then D, this includes seeing 26 as the first value in a sum. Award ***A1*** if 26+30+68+66+58 seen in order.

Note:	Award ***M1A0*** for an incorrect first nearest neighbour proceeding 'correctly' to the next vertex. For example, LA to C and then C to D.

upper bound is $(26+30+68+66+58=)\ \$248$ ***A1***

Note:	Award ***M1A0*** for correct nearest neighbour algorithm starting from a vertex other than LA. Condone the correct tour written backwards i.e. $58+66+68+30+26=248$

[3 marks]

(e) (i) attempt to find MST of L, N, D and S *(M1)*

by deleting C, Kruskal gives MST for the remainder as LD, DN, LS

weight 123 *(A1)*

(lower bound is therefore $123+(30+41)=$) $194 ***A1***

Note:	Award ***(M1)*** for a graph or list of edges that does not include C. Award ***(A1)*** if 26 + 39 + 58 seen in any order.

(ii) by deleting S, Kruskal gives MST for the remainder as LD, DC, DN

weight 95 *(A1)*

(lower bound is therefore $95+(58+66)=$) $219 ***A1***

Note:	Award ***(A1)*** if 26 + 30 + 39 seen in any order.

[5 marks]

(f) $219 \le C \le 248$ ***A1A1***

Note:	Award ***A1*** for $219 \le C$ and ***A1*** for $C \le 248$. Award at most ***A1A0*** for $219 < C < 248$. ***FT*** for their values from part (e) if higher value from (e)(i) and (e)(ii) used for the lower bound, and part (d) for the upper.

[2 marks]

(g) any valid tour, within their interval from part (f), from any starting point **OR**

any valid tour that starts and finishes at N *(M1)*

valid tour starting point N **AND** within their interval ***A1***

e.g NDCLSN (weight 234)

Note:	If part (f) not correct, **only** award ***A1FT*** if their valid tour begins and ends at N **AND** lies within **BOTH** their interval (including if one-sided) in part (f) **AND** $219 \le C \le 248$. If no response in the form of an interval seen in part (f) then award ***M1A0*** for a valid tour beginning and ending at N **AND** within $219 \le C \le 248$.

[2 marks]

Total [19 marks]

Question 12

(a) attempt to use area of triangle formula *(M1)*

$\frac{1}{2}\times 25.9\times 6.36\times \sin(125^\circ)$ *(A1)*

67.5 m^2 $(67.4700\ldots\text{ m}^2)$ ***A1***

Note: Units are required. The final ***A1*** is only awarded if the correct units are seen in their answer; hence award ***(M1)(A1)A0*** for an unsupported answer of 67.5.

[3 marks]

(b) attempt to use cosine rule *(M1)*

$(\text{BK} =)\sqrt{12^2 + 6.36^2 - 2\times 12\times 6.36\times \cos 45^\circ}$ *(A1)*

8.75 (m) (8.74738…(m)) ***A1***

Note: Award ***(M1)(A1)(A0)*** for radian answer of 10.2 (m) (10.2109…(m)) with or without working shown.

[3 marks]

(c) **METHOD 1**

attempt to use sine rule with measurements from triangle OKX *(M1)*

$\frac{\text{OX}}{\sin 51.1^\circ} = \frac{22.2}{\sin 53.8^\circ}$ *(A1)*

$(\text{OX} =)$ 21.4 (m) (21.4099…)(m) ***A1***

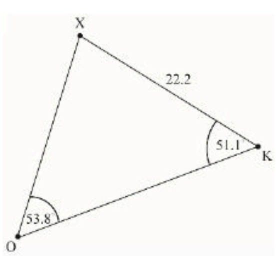
$(21.4\text{ (m)} < 22.2\text{ (m)})$

Odette is closer to the football / Khemil is further from the football ***A1***

Note: For the final ***A1*** to be awarded 21.4 (21.4099…) must be seen. Follow through within question part for final ***A1*** for a consistent comparison with their OX.

METHOD 2

sketch of triangle OXK with vertices, angles and lengths *(A1)*



51.1° is smallest angle in triangle OXK ***R1***

opposite side (OX) is smallest length ***R1***

therefore Odette is closest ***A1***

[4 marks]

(d) attempt to use length of arc formula **(M1)**

$\frac{135}{360} \times 2\pi \times 12$ **(A1)**

28.3(m) $(9\pi,\ 28.2743...)$ (m) **A1**

[3 marks]
Total [13 marks]

Question 13

(a) there are more than two vertices with odd degree **R1**

so it is not possible to travel along each road exactly once **A1**

Note: Do not award ***R0A1***. Award ***R1*** for "There are 4 vertices with odd degree".

[2 marks]

(b) $a = 11$, $b = 18$, $c = 17$, $d = 15$ **A2**

Note: Award ***A1*** for any one correct, ***A2*** for all four correct.

[2 marks]

(c) attempt to use nearest neighbour algorithm **(M1)**

Note: Award ***M1*** for first 3 vertices correct or 11, 4, 3 seen.

$\text{G} - \text{E} - \text{F} - \text{B} - \text{D} - \text{A} - \text{C}(-\text{E}) - \text{G}$ **OR** $11 + 4 + 3 + 5 + 5 + 8 + \text{their } b$ **(A1)**

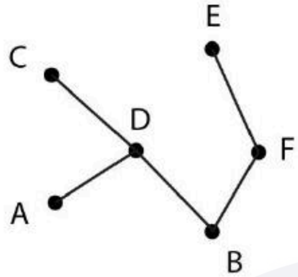
upper bound $= 54$ (km) **A1**

[3 marks]

(d) (i) a diagram of **any** spanning tree of the subgraph ABCDEF ***(A1)***

attempt at Kruskal's algorithm or Prim's algorithm ***(M1)***

e.g. edges BF (3), EF (4) and an edge of length 5 listed or seen in any spanning tree



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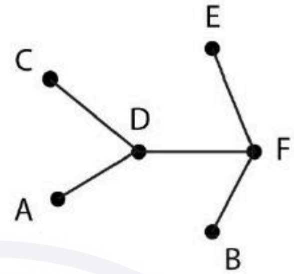

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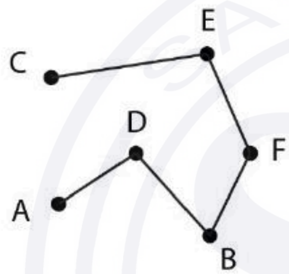

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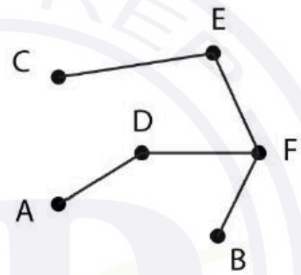


A1

(ii) 24 (km) ***A1***

Note: ***FT*** from their sketch, only if it is a spanning tree. It is not required to see the edge lengths on the sketch, since they are given in the question.

[4 marks]

(e) adding vertex G's two shortest edges to their part (d)(ii) *(M1)*

$24+11+13$

$=48$ ***A1***

[2 marks]

(f) try removing a different vertex ***A1***

[1 mark]

(g) recognize 7 edges in optimum route *(M1)*

Note: Award ***M1*** for a total length of 52 seen.

subtracting $0.5\times$ edges from 52 *(M1)*

$52-7\times 0.5$

$=48.5$ (km) ***A1***

[3 marks]

[Total: 17 marks]

Question 14

(a) $\frac{18-4}{2}$ **(M1)**

$(a=)\ 7$ **A1**

[2 marks]

(b) $\frac{18+4}{2}$ **OR** $18-7$ **OR** $4+7$ **(M1)**

$(d=)\ 11$ **A1**

[2 marks]

(c) (time between high and low tide is) 6h15m **OR** 375 minutes **(A1)**

multiplying by 2 **(M1)**

750 minutes **A1**

[3 marks]

(d) **EITHER**

$\frac{360^\circ}{b}=750$ **(A1)**

OR

$7\cos(b\times 375)+11=4$ **(A1)**

THEN

$(b=)\ 0.48$ **A1**

Note: Award ***A1A0*** for an answer of $\frac{2\pi}{750}\left(=\frac{\pi}{375}=0.00837758...\right)$.

[2 marks]

(e) equating their cos function to 6 **OR** graphing their cos function and 6 **(M1)**

$7\cos(0.48t)+11=6$

$\Rightarrow t=282.468...$ (minutes) **(A1)**

$=4.70780...$ (hr) **OR** 4hr 42 mins (4hr 42.4681… mins) **(A1)**

so the time is $10:42$ **A1**

[4 marks]

(f) next solution is $t = 467.531...$ ***(A1)***

$467.531... - 282.468...$

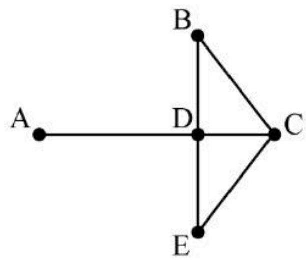
185 (mins) $(185.063...)$ ***A1***

[2 marks]

[Total: 15 marks]

Question 15

(a)



A1

[1 mark]

(b) (i) $$P = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$ *(M1)*

$$P^3 = \begin{pmatrix} 0 & 1 & 2 & 4 & 1 \\ 1 & 2 & 5 & 6 & 2 \\ 2 & 5 & 4 & 6 & 5 \\ 4 & 6 & 6 & 4 & 6 \\ 1 & 2 & 5 & 6 & 2 \end{pmatrix}$$

$a = 6$ **A1**

(ii) 5 (routes) **A1**

[3 marks]

(c) A and C identified as start/finish points (in either order) ***(A1)***
for example : A – D – E – C – D – B – C **A1**

[2 marks]

(d) cost of their Eulerian trail A to C (=180) ***(A1)***
consider edges to get from C to A ***(M1)***

235 (USD) **A1**

[3 marks]

(e)	(i)	A to C (or C to A)	*A1*
	(ii)	best is CBDA 55 (USD)	 *A1*
			[2 marks]
(f)	(i)	$A - D - C - B - E - A$ **OR** 50, 45, 30, 120, 60 summing their 5 edges $50 + 45 + 30 + 120 + 60$ (upper bound =) 305 (km)	*(A1)* *(M1)* *A1*
	(ii)	attempt to find MST without vertex A (MST =) 130 $130 + 50 + 60$ (lower bound =) 240 (km)	*(M1)* *(A1)* *(M1)* *A1*
			[7 marks]
			[Total 18 marks]

Question 16

(a) 25 (m) **A1**

[1 mark]

(b) (i) recognition of need to use Pythagoras theorem **(M1)**

$BF^2 = 20^2 + 25^2$

$(BF =)$ 32.0 (32.0156…, $\sqrt{1025}$, $5\sqrt{41}$) (m) **A1**

(ii) correct use of trig ratio for $B\hat{F}M$ **(M1)**

$(B\hat{F}M =)\ \tan^{-1}\left(\frac{25}{20}\right)$ or equivalent

$(B\hat{F}M =)$ 51.3 (51.3401...) **A1**

Note: Accept radian answer of 0.896 (0.896055…) Accept an answer of 51.4 from use of 3sf answer to part (b)(i) and then either cosine rule or inverse sine.

[4 marks]

(c) attempt to use arc length formula **(M1)**

(arc length =) $\frac{2 \times 51.3401...}{360} \times 2\pi(32.0156\ldots)$ **(A1)**

(arc length =) 57.4 (57.3755…) (m) **A1**

Note: Accept 57.3 from use of 3 sf. values of their answers from parts (b)(i) and (b)(ii).

[3 marks]

(d) 34.0156… (seen anywhere) **(A1)**

use of area of sector formula **(M1)**
recognition of subtracting areas of two sectors **(M1)**

(area =) $\frac{102.680\ldots}{360} \times \pi\left((34.0156\ldots)^2 - (32.0156\ldots)^2\right)$

(area =) 118 (m^2) (118.335…) **A1**

[4 marks]

(e) multiplying their area from part (d) by 0.12 or 12 **(M1)**

0.12 (m) seen **OR** 1183350 (cm^2) seen **(A1)**

$118.335\ldots \times 0.12$ **OR** 1183350×12

14.2 (14.2002…) m^3 **OR** 14200000 (14200236) cm^3 **A1**

[3 marks]
[Total 15 marks]

Question 17

(a) AEDCFBA **A1A1**

> **Note:** Award ***A1*** for AE at start, ***A1*** for correct completed route.

attempt to find the length of their route
length $22+21+19+24+25+31$ ***(M1)***
$=142$ (km) ***A1***

> **Note:** Award ***A1A0M1A0*** for omitted final edge and their sum.

[4 marks]

(b) attempt to form MST without vertex A ***(M1)***

> **Note:** Exactly 4 edges that form a spanning tree are required.

BD DC DE DF **OR** 20, 19, 21, 22 seen in that order ***A1***

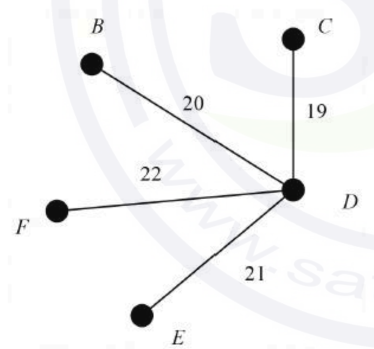
> **Note:** Award ***M1A0*** for diagram of MST.

attempt to reconnect vertex A (one edge is sufficient) ***(M1)***
reconnecting A: AE (22) and AF (23) ***(A1)***
lower bound: $20+19+21+22+22+23$
$=127$ ***A1***

> **Note:** If 127 seen, unsupported or without the explicit evidence of Prim's algorithm, award ***M1A0M1A1A1***.

[5 marks]

(c)



(A1)

> **Note:** Condone the omission of the weights from their diagram.
> The diagram may include A with its two edges.

correct reasoning based on lack of cycle (once A is reattached) ***R1***
e.g. edges BD and CD would be repeated
this lower bound is not achievable (in this way) ***A1***

> **Note:** Do not award ***R0A1***.

[3 marks]
[Total: 12 marks]

Question 18

(a) $BC = 20$ (m) ***A1***

[1 mark]

(b) use of Pythagoras ***(M1)***

$AB = \sqrt{12^2 + 4^2}$

$= 12.6$ (m) $(12.6491\ldots, \sqrt{160})$ ***A1***

[2 marks]

(c) **METHOD 1 – finding angle ABC**

correct use of a trig ratio to find $A\hat{B}C$ (or finding the bearing of B from A) ***(A1)***

e.g. $\tan\left(A\hat{B}C\right) = \frac{12}{4}$, $\cos A\hat{B}C = \frac{20^2 + 12.649^2 - 20^2}{2 \times 20 \times 12.649}$, $\cos A\hat{B}C = \frac{6.3245}{20}$

$A\hat{B}C = 71.6$ $(71.5650\ldots)$ ***(A1)***

Note:	Angle $A\hat{B}C$ can be 71.5 or 72.2 depending on their working out. Bearings should be given in degrees.

$180 + 71.5650\ldots = 252°$ $(251.565\ldots)$ ***A1***

Note:	The final ***A1*** can be awarded for 180 plus their 71.6. If radians used, award ***A1A1*** for 1.24904… or 4.39063… seen, and then ***A0*** for the radian answer.

METHOD 2 – finding angle that AB makes with the horizontal (angle *H*)

correct use of a trig ratio to find *H*, the angle AB makes with horizontal ***(A1)***

e.g. $\tan \hat{H} = \frac{4}{12}$, $\cos \hat{H} = \frac{12^2 + 12.649^2 - 4^2}{2 \times 12 \times 12.649}$

$\hat{H} = 18.4$ $(18.4349...)$ ***(A1)***

Note:	Accept 18.5 (18.5078…) from use of 3sf answer from part (b). Bearings should be given in degrees.

$270 - 18.4348... = 252°$ $(251.565\ldots)$ ***A1***

Note:	The final ***A1*** can be awarded for 270 minus their 18.4. If radians used, award ***A1A1*** for 0.321750… or 4.39063… seen, and then ***A0*** for the radian answer.

[3 marks]

(d) (i) $-\frac{4}{3}$ $\left(-\frac{16}{12}\right)$ ***A1***

(ii) $(6, 8)$ ***A1A1***

Note: Award ***A1A0*** if parentheses are missing.

(iii) gradient of (their) perp line = $\frac{3}{4}$ ***(M1)***

equation of perpendicular bisector of AC ***(A1)***

e.g. $(y-8)=\frac{3}{4}(x-6)$ **OR** $y=\frac{3}{4}x+3.5$

EITHER

equation of perpendicular bisector of BC is $y=10$ ***(A1)***

OR

equation of perpendicular bisector of AB is $y=-3x+36$ ***(A1)***

Note: The ***A1*** is for either equation of perpendicular bisector of BC or AB.

point of intersection $\left(8\frac{2}{3}, 10\right)$ **OR** $(8.67, 10)$ $\left[(8.666\ldots, 10)\right]$ ***(M1)A1***

Note: Award ***M1*** for an attempt to equate their perpendicular bisectors
Award the final ***A1*** for the correct coordinate pair – parentheses omitted or not.

[8 marks]

[Total: 14 marks]

Question 19

(a) (i) $\begin{pmatrix} 7.2 \\ 5.1 \\ 2.4 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 2.8 \end{pmatrix} = \begin{pmatrix} 7.2 \\ 5.1 \\ -0.4 \end{pmatrix}$ ***A1***

e: Accept alternate vector notation, e.g. $(7.2, 5.1, -0.4)$ or $\langle 7.2, 5.1, -0.4 \rangle$

(ii) use of correct formula to find $\left|\overrightarrow{AB}\right|$ ***(M1)***

$\sqrt{7.2^2 + 5.1^2 + (-0.4)^2}$

8.83 (km) $(8.83232\ldots)$ ***A1***

[3 marks]

(b) magnitude of $\begin{pmatrix} 1.1 \\ 8.4 \\ 0.2 \end{pmatrix}$ is

$\sqrt{1.1^2 + 8.4^2 + 0.2^2}$ $(= 8.47407\ldots)$ ***(A1)***

EITHER

$\begin{pmatrix} 1.1 \\ 8.4 \\ 0.2 \end{pmatrix} \bullet \begin{pmatrix} 7.2 \\ 5.1 \\ -0.4 \end{pmatrix}$ ***(M1)***

$1.1 \times 7.2 + 8.4 \times 5.1 - 0.2 \times 0.4$ $(= 50.68)$ ***(A1)***

Note: The ***M*** mark can be implied by a partially correct ***A1*** line.

angle = $\arccos\left(\frac{50.68}{8.83232\ldots \times 8.47407\ldots}\right)$ ***(M1)***

OR

Attempt to find $\begin{pmatrix} 1.1 \\ 8.4 \\ 0.2 \end{pmatrix} \times \begin{pmatrix} 7.2 \\ 5.1 \\ -0.4 \end{pmatrix}$ ***(M1)***

$\left|\begin{pmatrix} 1.1 \\ 8.4 \\ 0.2 \end{pmatrix} \times \begin{pmatrix} 7.2 \\ 5.1 \\ -0.4 \end{pmatrix}\right| = \sqrt{4.38^2 + 1.88^2 + 54.87^2}$ $(= 55.0766\ldots)$ ***(A1)***

angle = $\arcsin\left(\frac{55.0766\ldots}{8.83232\ldots \times 8.47407\ldots}\right)$ ***(M1)***

THEN

47.4° $(47.3805\ldots)$ **OR** 0.827 $(0.826947\ldots)$ ***A1***

[5 marks]

(c) using sum of angles in a triangle equals 180 *(M1)*

$\hat{ACB} = 180 - 47.3805 - 55.2 \quad (= 77.4194\ldots^\circ)$ *(A1)*

$$\frac{AC}{\sin(55.2)} = \frac{8.83232...}{\sin(77.4194...)}$$ *(A1)*

7.43 (km) (7.43107…) *A1*

[4 marks]

[Total 12 marks]

Question 20

(a) recognition that speed is the magnitude of $\begin{pmatrix} 50 \\ -33 \\ 0 \end{pmatrix}$ *(M1)*

$\sqrt{50^2 + (-33)^2}$ **OR** $\left|\begin{pmatrix} 50 \\ -33 \\ 0 \end{pmatrix}\right|$

$= 59.9$ (km h^{-1}). (59.9082…) *A1*

[2 marks]

(b) $\begin{pmatrix} 50 \\ -33 \\ 0 \end{pmatrix} \bullet \begin{pmatrix} -15 \\ -20 \\ 0 \end{pmatrix} = -90$ *(A1)*

$$\cos\theta = \frac{-90}{59.9082... \times 25}$$ *(M1)*

$\theta = 93.4^\circ \left(93.4450\ldots^\circ, 1.63092\ldots\right)$ *A1*

[3 marks]

(c) $\vec{OD} = \begin{pmatrix} 200 \\ -100 \\ 0.02 \end{pmatrix} + t\begin{pmatrix} -15 \\ -20 \\ 0 \end{pmatrix}$ *A1*

[1 mark]

(d) (i) $\begin{pmatrix} 200 \\ -100 \\ 0.02 \end{pmatrix} + t\begin{pmatrix} -15 \\ -20 \\ 0 \end{pmatrix} = \begin{pmatrix} 152 \\ p \\ 0.02 \end{pmatrix}$ **OR** $200 = -15t + 152$ *(M1)*

$t_1 = 3.2 \quad \left(\frac{16}{5}\right)$ *A1*

(ii) $p = -164$ *A1*

[3 marks]

(e) (i) attempt to find difference between the two position vectors ***(M1)***

$$\begin{pmatrix} 190-65t \\ -95+13t \\ 0.02 \end{pmatrix}$$ ***A1***

> **Note:** Award ***A1M1A0*** for $\begin{pmatrix} -190+65t \\ 95-13t \\ -0.02 \end{pmatrix}$.

(ii) attempt to find $\left|\begin{pmatrix} 190-65t \\ -95+13t \\ 0.02 \end{pmatrix}\right|$ ***(M1)***

$\sqrt{(190-65t)^2+(13t-95)^2+0.02^2}$ ***(A1)***

attempt to find minimum. (e.g. $t = 3.09$ hours) ***(M1)***

closest distance = $55.9 \quad (55.8931\ldots) \quad (\text{km})$ ***A1***

[6 marks]

[Total: 15 marks]

Question 21

(a) attempt to use Pythagoras' theorem **(M1)**

$\sqrt{3.4^2 - 2^2}$

$= 2.75$ $(2.74954\ldots)$ (m) **A1**

[2 marks]

(b) (i) **METHOD 1** (Use of $\frac{1}{2} \times a \times b \times \sin(\theta)$)

$60°$ **(A1)**

attempt to find area of one triangle using $\frac{1}{2} \times a \times b \times \sin(\theta)$ **(M1)**

$\frac{1}{2} \times 2 \times 2 \times \sin(60°)$

$\left(6 \times \frac{1}{2} \times 2 \times 2 \times \sin(60°)\right) = 10.4$ $(10.3923\ldots,\ 6\sqrt{3})$ (m^2) **A1**

Note: Award ***A0M0A0*** for $\frac{1}{2} \times 2 \times 2$ or equivalent.

METHOD 2 (Use of altitude)

(altitude is) $\sqrt{3}$ **(A1)**

attempt to find the area of one triangle using $\frac{1}{2} \times b \times h$ with their altitude. **(M1)**

$\frac{1}{2} \times 2 \times \sqrt{3}$

$\left(6 \times \frac{1}{2} \times 2 \times \sqrt{3}\right) = 10.4$ $(10.3923\ldots,\ 6\sqrt{3})$ (m^2) **A1**

Note: Award ***A0M0A0*** for $\frac{1}{2} \times 2 \times 2$ or equivalent.

METHOD 3 (Finding the area of a trapezoid)

(altitude of one trapezoid is) $\sqrt{3}$ **(A1)**

attempt to find area of one trapezoid using $\frac{1}{2} \times (a+b)h$ **(M1)**

$\frac{1}{2} \times (2+4)\sqrt{3}$ $\left(3\sqrt{3}\right)$

$\left(2 \times \frac{1}{2} \times (2+4)\sqrt{3}\right) = 10.4$ $(10.3923\ldots,\ 6\sqrt{3})$ (m^2) **A1**

(ii) $\frac{1}{3}\times 10.3923...\times 2.74954...$ ***(A1)***

$= 9.52\text{ m}^3$ $(9.52470\ldots)$ ***A1***

Note: Units must be seen.

[5 marks]

(c) $\cos(\mathrm{M\hat{A}T}) = \frac{2}{3.4}$ or correct equivalent ***(A1)***

$(\mathrm{MAT} =)\ 54.0^\circ$ $(53.9681\ldots, 0.941921\ldots$ radians$)$ ***A1***

[2 marks]

(d) Angle $\mathrm{YAX} = 180 - 53.9681... = 126.031\ldots^\circ$ ***(A1)***

Angle $\mathrm{YXA} = 180 - 35 - 126.031... = 18.9681\ldots^\circ$ ***(A1)***

Note: These angles may be seen in the sine rule.

Attempt to substitute into sine rule ***(M1)***

$$\frac{\mathrm{AY}}{\sin(18.9681...)} = \frac{2.6}{\sin(126.031)}$$

$\mathrm{AY} = 1.05$ $(1.04503\ldots)$ (m) ***A1***

[4 marks]

(e) **METHOD 1 COSINE RULE**

attempt to substitute into cosine rule to form a quadratic for YZ ***(M1)***

$0.9^2 = \mathrm{YZ}^2 + 1.04503...^2 - 2\times 1.04503...\times \mathrm{YZ}\times\cos(35)$ ***(A1)***

$\mathrm{YZ} = 0.185$ $(0.184692\ldots)$ (m), 1.53 $(1.52739\ldots)$ (m) ***A1A1***

Note: Accept 0.191 $(0.191313\ldots)$ from use of 3 s.f. values.

METHOD 2 SINE RULE

attempt to substitute into sine rule to find $\mathrm{Y\hat{Z}A}$

$$\frac{\sin(\mathrm{Y\hat{Z}A})}{1.04503\ldots} = \frac{\sin(35^\circ)}{0.9}$$ ***(M1)***

$\mathrm{Y\hat{Z}A} = 41.7597\ldots$ or $\mathrm{Y\hat{Z}A} = 138.240\ldots$

$\mathrm{Z\hat{A}Y} = 103.240\ldots$ or $\mathrm{Z\hat{A}Y} = 6.75972\ldots$ ***(A1)***

Note: Award ***A1*** for any of these angles seen.

$$\frac{\mathrm{YZ}}{\sin(\mathrm{Z\hat{A}Y})} = \frac{0.9}{\sin(35^\circ)}$$

$\mathrm{YZ} = 0.185$ $(0.184692\ldots)$ (m), 1.53 $(1.52739\ldots)$ (m) ***A1A1***

[4 marks]

[Total: 17 marks]