Subject – Math AI(Higher Level)
Topic - Statistics and Probability
Year - May 2021 – Nov 2024
Paper -2
Answers

Question 1

(a) (i) $\mathrm{P}(Y) = 0.8 \times 0.1 + 0.2 \times 0.3$ ***M1***

$= 0.14$ ***A1***

(ii) $\mathrm{P}(\text{Star} \mid Y) = \dfrac{0.8 \times 0.1}{0.14}$ ***M1***

$= 0.571 \left(\dfrac{4}{7}, 0.571428\ldots\right)$ ***A1***

[4 marks]

(b) the colours of the sweets are distributed according to manufacturer specifications ***A1***

[1 mark]

(c)

Colour	Brown	Red	Green	Orange	Yellow	Purple
Expected Frequency	12	20	16	16	8	8

A2

Note: Award ***A2*** for all 6 correct expected values, ***A1*** for 4 or 5 correct values, ***A0*** otherwise.

[2 marks]

(d) 5 ***A1***

[1 mark]

(e) 0.469 (0.4688117…) ***A2***

[2 marks]

(f) since $0.469 > 0.05$ ***R1***

fail to reject the null hypothesis. There is insufficient evidence to reject the manufacturer's specifications ***A1***

Note: Award ***R1*** for a correct comparison of their correct p-value to the test level, award ***A1*** for the correct result from that comparison. Do not award ***R0A1***.

[2 marks]

Total [12 marks]

Question 2

(a)	(i)	evidence of correct probability e.g. sketch **OR** correct probability statement $\mathrm{P}(X < 6.5)$	***(M1)***
		0.0151	***A1***
	(ii)	0.0228	***A1***

Note: Answers should be given to 4 decimal place.

[3 marks]

(b)	(i)	multiplying **their** probability by 1000 451.7	***(M1)*** ***A1***
	(ii)	510.5	***A1***

[3 marks]

Note: Answers should be given to 4 sf.

(c) H_0 : stopping distances can be modelled by $\mathrm{N}(6.76,\ 0.12^2)$

H_1 : stopping distances cannot be modelled by $\mathrm{N}(6.76,\ 0.12^2)$ ***A1A1***

Note: Award ***A1*** for correct H_0, including reference to the mean and standard deviation.
Award ***A1*** for the negation of their H_0.

[2 marks]

(d) 15.1 or 22.8 seen ***(M1)***

0.0727 $(0.0726542\ldots,\ 7.27\%)$ ***A2***

[3 marks]

(e) $0.05 < 0.0727$ ***R1***

there is insufficient evidence to reject H_0 (or "accept H_0") ***A1***

Note: Do not award ***R0A1***.

[2 marks]

Total [13 marks]

Question 3

(a) $X_1 \sim \text{Po}(3.1)$

$\text{P}(X_1 = 4) = 0.173 \ (0.173349...)$ ***A1***

[1 mark]

(b) (i) $X_2 \sim \text{Po}(3 \times 3.1) = \text{Po}(9.3)$ ***(M1)***

$\text{P}(X_2 = 12) = 0.0799 \ (0.0798950...)$ ***A1***

(ii) $(\text{P}(X_1 > 0))^2 \times \text{P}(X_1 = 0)$ ***(M1)***

$0.95495^2 \times 0.04505$ ***(A1)***

$= 0.0411 \ (0.0410817...)$ ***A1***

[5 marks]

(c) $\text{P}(X_1 = 0) = 0.04505$ ***(A1)***

$X_1 \sim \text{B}(12, 0.04505)$ ***(M1)(A1)***

Note: Award ***M1*** for recognizing binomial probability, and ***A1*** for correct parameters.

$= 0.0133 \ (0.013283....)$ ***A1***

[4 marks]

(d) **METHOD ONE**

n	λ	$\text{P}(X \geq 30)$
...	…	…
10	24.1	0.136705
11	26.2	0.253384

(M1)(A1)(A1)

Note: Award ***M1*** for evidence of a cumulative Poisson with $\lambda = 3.1 + 2.1n$, ***A1*** for 0.136705 and ***A1*** for 0.253384.

so require 12 magpies (including Bill) ***A1***

METHOD TWO

evidence of a cumulative Poisson with $\lambda = 3.1 + 2.1n$ ***(M1)***

sketch of curve and $y = 0.2$ ***(A1)***

(intersect at) $10.5810\ldots$ ***(A1)***

rounding up gives $n = 11$

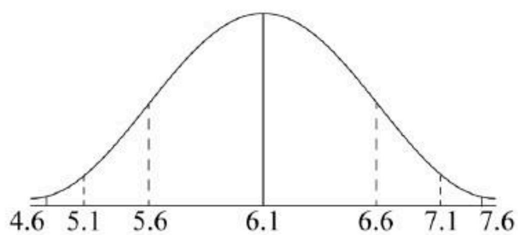
so require 12 magpies (including Bill) ***A1***

[4 marks]

Total [14 marks]

Question 4

(a)



A1A1

Note: Award ***A1*** for a normal curve with mean labelled 6.1 or μ, ***A1*** for indication of SD (0.5): marks on horizontal axis at 5.6 and/or 6.6 **OR** $\mu-0.5$ and/or $\mu+0.5$ on the correct side and approximately correct position.

[2 marks]

(b) $X \sim \mathrm{N}(6.1,\ 0.5^2)$

$\mathrm{P}(5.5 < X < 6.5)$ **OR** labelled sketch of region ***(M1)***

$= 0.673\ \ (0.673074...)$ ***A1***

[2 marks]

(c) $(\mathrm{P}(X < 5.3) =)\ 0.0547992...$ ***(A1)***

$0.0547992... \times 80$ ***(M1)***

$= 4.38\ \ (4.38393...)$ ***A1***

[3 marks]

(d) (i) $Y \sim \mathrm{N}(4.5,\ 0.45^2)$,

$(\mathrm{P}(Y > 4.62) =)\ 0.394862...$ ***(A1)***

use of binomial seen or implied ***(M1)***

using $\mathrm{B}(10,\ 0.394862...)$ ***(M1)***

$0.0430\ \ (0.0429664...)$ ***A1***

(ii) $np(1-p) = 2.39\ \ (2.38946...)$ ***A1***

[5 marks]

(e) $\mathrm{P}(F \cap (W > 4.7)) = 0.5 \times 0.3284\ \ (= 0.1642)$ ***(A1)***

attempt use of tree diagram **OR** use of $\mathrm{P}(F \mid W > 4.7) = \dfrac{\mathrm{P}(F \cap (W > 4.7))}{\mathrm{P}(W > 4.7)}$ ***(M1)***

$$\frac{0.5 \times 0.3284}{0.5 \times 0.9974 + 0.5 \times 0.3284}$$ ***(A1)***

$= 0.248\ \ (0.247669...)$ ***A1***

[4 marks]

Total [16 marks]

Question 5

(a) (i) let X be the random variable "number of patients arriving in a minute", such that $X \sim \mathrm{Po}(m)$.

$\mathrm{H}_0 : m = 1.5$ ***A1***

$\mathrm{H}_1 : m > 1.5$ ***A1***

> **Note:** Allow a value of 270 for m. Award at most ***A0A1*** if it is not clear that it is the population mean being referred to e.g
> H_0 : *The number of patients is equal to 1.5 every minute*
> H_1 : *The number of patients exceeds 1.5 every minute.*
> Referring to the "expected" number of patients or the use of μ or λ is sufficient for ***A1A1***.

(ii) under H_0 let Y be the number of patients in 3 hours

$Y \sim \mathrm{Po}(270)$ ***(A1)***

$\mathrm{P}(Y \geq 320) \; (= 1 - \mathrm{P}(Y \leq 319)) = 0.00166 \;\; (0.00165874)$ ***(M1)A1***

since $0.00166 < 0.05$ ***R1***

(reject H_0)

Loreto should employ more staff ***A1***

[7 marks]

(b) (i) H_0 : The probability of a patient waiting less than 20 minutes is 0.95 ***A1***

H_1 : The probability of a patient waiting less than 20 minutes is less than 0.95 ***A1***

(ii) under H_0 let W be the number of patients waiting more than 20 minutes

$W \sim \mathrm{B}(150,\, 0.05)$ ***(A1)***

$\mathrm{P}(W \geq 11) = 0.132 \;\; (0.132215...)$ ***(M1)A1***

since $0.132 > 0.1$ ***R1***

(fail to reject H_0)

insufficient evidence to suggest they are not meeting their target ***A1***

> **Note:** Do not accept "they are meeting target" for the ***A1***.
> Accept use of $\mathrm{B}(150,\, 0.95)$ and $\mathrm{P}(W \leq 139)$ and any consistent use of a random variable, appropriate p-value and significance level.

[7 marks]

Total: [14 marks]

Question 6

(a) (i) Let X be the random variable "distance from O".

$X \sim \mathrm{N}(10,\ 3^2)$

$\mathrm{P}(X < 13) = 0.841\ \ (0.841344...)$ ***(M1)A1***

(ii) $\left(\mathrm{P}(X > 15) =\right)\ 0.0478\ \ (0.0477903)$ ***A1***

[3 marks]

(b) $\mathrm{P}(X > 15) \times \mathrm{P}(X > 15)$ ***(M1)***

$= 0.00228\ \ (0.00228391...)$ ***A1***

[2 marks]

(c) $1 - (0.8143)^3$ ***(M1)***

$= 0.460\ \ (0.460050...)$ ***A1***

[2 marks]

(d) (i) let Y be the random variable "number of points scored"

evidence of use of binomial distribution ***(M1)***

$Y \sim \mathrm{B}(10,\ 0.539949...)$ ***(A1)***

$\left(\mathrm{E}(Y) =\right) 10 \times 0.539949...$ ***(M1)***

$= 5.40$ ***A1***

(ii) $\left(\mathrm{P}(Y \geq 5) =\right)\ 0.717\ \ (0.716650...)$ ***A1***

(iii) $\mathrm{P}(5 \leq Y < 8)$ ***(M1)***

$= 0.628\ \ (0.627788...)$ ***A1***

> **Note:** Award ***M1*** for a correct probability statement or indication of correct lower and upper bounds, 5 and 7.

(iv) $\dfrac{\mathrm{P}(5 \leq Y < 8)}{\mathrm{P}(Y \geq 5)} \left(= \dfrac{0.627788...}{0.716650...} \right)$ ***(M1)***

$= 0.876\ \ (0.876003...)$ ***A1***

[9 marks]

Total: [16 marks]

Question 7

(a) (let T be the number of passengers who arrive)

$\left(\mathrm{P}(T > 72) =\right)$ $\mathrm{P}(T \geq 73)$ **OR** $1 - \mathrm{P}(T \leq 72)$ ***(A1)***

$T \sim \mathrm{B}(74,\ 0.9)$ **OR** $n = 74$ ***(M1)***

$= 0.00379$ $(0.00379124...)$ ***A1***

Note:	Using the distribution $\mathrm{B}(74,\ 0.1)$, to work with the 10% that do not arrive for the flight, here and throughout this question, is a valid approach.

[3 marks]

(b) (i) 72×0.9 ***(M1)***

64.8 ***A1***

(ii) $n \times 0.9 = 72$ ***(M1)***

80 ***A1***

[4 marks]

(c) **METHOD 1**

EITHER

when selling 74 tickets

	$T \leq 72$	$T = 73$	$T = 74$
Income minus compensation (I)	11100	10800	10500
Probability	0.9962…	0.003380…	0.0004110…

top row ***A1A1***

bottom row ***A1A1***

Note:	Award ***A1A1*** for each row correct. Award ***A1*** for one correct entry and ***A1*** for the remaining entries correct.

$\mathrm{E}(I) = 11100 \times 0.9962... + 10800 \times 0.00338... + 10500 \times 0.000411 \approx 11099$ ***(M1)A1***

OR

income is $74 \times 150 = 11100$ *(A1)*

expected compensation is

$0.003380... \times 300 + 0.0004110... \times 600 \ (= 1.26070...)$ *(M1)A1A1*

> **Note:** The *(M1)* is for an attempt to work out expected compensation by multiplying a probability for tickets sold by either 300 or 600.

expected income when selling 74 tickets is $11100 - 1.26070\ldots$ *(M1)*

> **Note:** Award *(M1)* for subtracting their expected compensation from 11100.

$= 11098.73.. \ (= \$11099)$ *A1*

THEN

income for 72 tickets $= 72 \times 150 = 10800$ *(A1)*

so expected gain $\approx 11099 - 10800 = \$299$ *A1*

METHOD 2

for 74 tickets sold, let C be the compensation paid out

$\mathrm{P}(T = 73) = 0.00338014\ldots, \ \mathrm{P}(T = 74) = 0.000411098\ldots$ *A1A1*

$\mathrm{E}(C) = 0.003380... \times 300 + 0.0004110... \times 600 \ (= 1.26070...)$ *(M1)A1A1*

extra expected revenue $= 300 - 1.01404... - 0.246658\ldots \ (300 - 1.26070\ldots)$

(A1)(M1)

> **Note:** Award *A1* for the 300 and *M1* for the subtraction.

$= \$299$ (to the nearest dollar) *A1*

METHOD 3

let D be the change in income when selling 74 tickets.

	$T \le 72$	$T = 73$	$T = 74$
Change in income	300	0	−300

(A1)(A1)

> **Note:** Award *A1* for one error, however award *A1A1* if there is no explicit mention that $T = 73$ would result in $D = 0$ and the other two are correct.

$\mathrm{P}(T \le 73) = 0.9962..., \ \mathrm{P}(T = 74) = 0.000411098\ldots$ *A1A1*

$\mathrm{E}(D) = 300 \times 0.9962... + 0 \times 0.003380... - 300 \times 0.0004110$ *(M1)A1A1*

$= \$299$ *A1*

[8 marks]

[Total 15 marks]

Question 8

(a) quota **A1**

[1 mark]

(b) (i) $27.125 \approx 27.1$ ***(M1)A1***

(ii) $8.29815\ldots \approx 8.30$ ***A1***

[3 marks]

(c) (let μ be the national mean)

H$_0$: $\mu = 25.2$
H$_1$: $\mu > 25.2$ ***A1***

Note: Accept hypotheses in words if they are clearly expressed and '**population** mean' or 'school mean' is referred to. Do not accept H$_0$: $\mu = \mu_0$ unless μ_0 is explicitly defined as "national standard mark" or given as 25.2.

recognizing t-test ***(M1)***
p-value $= 0.279391\ldots$ ***A1***

$0.279391\ldots > 0.05$ ***R1***

Note: The ***R1*** mark is for the comparison of their p-value with 0.05.

insufficient evidence to reject the null hypothesis (that the mean for the school is 25.2) ***A1***

Note: Award the final ***A1*** only if the **null** hypothesis is also correct (e.g. $\mu_0 = 25.2$ or (population) mean $= 25.2$) and the conclusion is consistent with both the direction of the inequality and the alternative hypothesis.

[5 marks]

(d) **EITHER**
the sampling process is not random ***R1***

For example:
the school asked for volunteers
the students were selected from a single class

OR
the quota might not be representative of the student population ***R1***

For example:
the school may have only 4 boys and 400 girls.

Note: Do not accept 'the sample is too small'.

[1 mark]

(e) (i) $(28.1 \times 2 + 20 =)\ 76.2$ ***A1***

(ii) 8.4×2 ***(A1)***
$= 16.8$ ***A1***

[3 marks]
[Total 13 marks]

Question 9

(a) wood layer, $W \sim \mathrm{N}(7, 0.3^2)$; plastic, $P \sim \mathrm{N}(3, 0.16^2)$

door: $X = W + P$

$\mathrm{E}(X) = 10$ (mm) **(A1)**

$\mathrm{Var}(X) = \mathrm{Var}(W) + \mathrm{Var}(P) = 0.1156$ (mm^2) **(M1)(A1)**

recognizing the distribution is Normal, with their mean and variance **(M1)**

$X \sim \mathrm{N}(10, 0.34^2)$

$\mathrm{P}(X < 9.5) = 0.0707$ $(0.07070125...)$ **A1**

[5 marks]

(b) $\mathrm{E}(T) = 80$ **(A1)**

$\mathrm{Var}(T)(= 0.1156 \times 8) = 0.9248$ **(M1)(A1)**

$T \sim \mathrm{N}(80, 0.9248)$

$\mathrm{P}(T > 82) = 0.0188$ $(0.0187753\ldots)$ **A1**

[4 marks]

(c) (i) 6.93 mm $(6.93428\ldots)$ **A1**

(ii) $(s_{n-1} =)$ 0.404 **(A1)**

$(s_{n-1}^2 =)$ 0.163 mm^2 $(0.162928\ldots)$ **A1**

[3 marks]

(d) $\mathrm{H}_0: \mu_A = \mu_B$ and $\mathrm{H}_1: \mu_A > \mu_B$ **A1A1**

> **Note:** Award ***A1*** for use of μ or in words "population mean", and ***A1*** for both correct equality in null hypothesis and correct inequality in alternative hypothesis. Accept an equivalent statement in words, must include mean and reference to "**population** mean" / "mean for **all** Machine B layers" for the first ***A1*** to be awarded.

use a two-sample t-test **(M1)**

p-value $= 0.406975...$ **A1**

since $0.406975... > 0.05$ **OR** $p\text{-value} > 0.05$ **R1**

Do not reject H_0 (Insufficient evidence to support the employee's claim) **A1**

> **Note:** Accept a p-value of $0.415861\ldots$ from use of 3sf values from part (c). Follow through within the question for the final ***R1*** and ***A1*** for their p-value provided $0 \le p \le 1$. Do not award ***R0A1***.

[6 marks]

Total [18 marks]

Question 10

(a) (i) $\frac{370+472}{2}$ *(M1)*

Note: This ***(M1)*** can also be awarded for either a correct Q_3 or a correct Q_1 in part (a)(ii).	

$Q_3 = 421$ ***A1***

(ii) their part (a)(i) – their Q_1 (clearly stated) ***(M1)***

$\text{IQR} = (421-318=)\ 103$ ***A1***

[4 marks]

(b) $(Q_3 + 1.5(\text{IQR}) =)\ 421 + (1.5 \times 103)$ ***(M1)***

$= 575.5$

since $498 < 575.5$ ***R1***

Netherlands is not an outlier ***A1***

Note: The ***R1*** is dependent on the ***(M1)***. Do not award ***R0A1***.	

[3 marks]

(c) not appropriate ("no" is sufficient) ***A1***

as r is too close to zero / too weak a correlation ***R1***

[2 marks]

(d) (i) 6 ***A1***

(ii) 4.5 ***A1***

(iii) 4.5 ***A1***

[3 marks]

(e) (i) $r_s = 0.683\ (0.682646\ldots)$ ***A2***

(ii) **EITHER**

there is a (positive) association between the population size and the score ***A1***

OR

there is a (positive) linear correlation between the ranks of the population size and the ranks of the scores (when compared with the PMCC of 0.249) ***A1***

[3 marks]

(f) lowering the top score by 20 does not change its rank so r_s is unchanged ***R1***

Note: Accept "this would not alter the rank" or "Netherlands still top rank" or similar. Condone any statement that clearly implies the ranks have not changed, for example: "The Netherlands still has the highest score."	

[1 mark]

[Total 16 marks]

Question 11

(a) (i) $(m =)\ 54\,(\%)$ **A1**

(ii) $(n =)\ 14\,(\%)$ **A1**

(iii) $(p =)\ 22\,(\%)$ **A1**

(iv) $(q =)\ 10\,(\%)$ **A1**

Note: Based on their n, follow through for parts (i) and (iii), but only if it does not contradict the given information. Follow through for part (iv) but only if the total is 100%.

[4 marks]

(b) (i) $0.54\ \left(\frac{54}{100}, \frac{27}{50}, 54\%\right)$ **A1**

(ii) $\frac{54}{64}\ \left(0.844, \frac{27}{32}, 84.4\%, 0.84375\right)$ **A1A1**

Note: Award ***A1*** for a correct denominator (0.64 or 64 seen), ***A1*** for the correct final answer.

[3 marks]

(c) (i) recognizing Binomial distribution with correct parameters ***(M1)***

$X \sim \mathrm{B}(10, 0.68)$

$(\mathrm{P}(X = 5) =)\ 0.123\ \ (0.122940\ldots, 12.3\%)$ **A1**

(ii) $1 - \mathrm{P}(X \le 3)$ **OR** $\mathrm{P}(X \ge 4)$ **OR** $\mathrm{P}(4 \le X \le 10)$ ***(M1)***

$0.984\ (0.984497\ldots, 98.4\%)$ **A1**

(iii) $(0.68)^9 \times 0.32$ ***(M1)***

recognition of two possible cases ***(M1)***

$2 \times \left((0.68)^9 \times 0.32\right)$

$0.0199\ \ (0.0198957\ldots, 1.99\%)$ **A1**

[7 marks]

(d) **EITHER**

the probability is not constant **A1**

OR

the events are not independent **A1**

OR

the events should be modelled by the hypergeometric distribution instead **A1**

[1 mark]

Total [15 marks]

Question 12

(a) slugs appear discretely / independently / randomly / at a constant (average) rate / mean is (approximately) equal to variance **R1R1**

[2 marks]

(b) new $(m =)\ 0.2 \times 12\ (= 2.4)$ (so $X \sim \text{Po}(2.4)$) ***(A1)***

attempt to use a pdf (e.g $\text{P}(X = 4)$) ***(M1)***

$0.125\ (0.125408...)$ ***A1***

[3 marks]

(c) $\text{P}(X < 3)$ **OR** $\text{P}(X \leq 2)$ ***(A1)***

$0.570\ (0.569708...)$ ***A1***

[2 marks]

(d) $\text{P}(X \geq 1) = 0.909282...$ ***(A1)***

raising a probability to a power of 3 ***(M1)***

$0.909282...^3$

$= 0.752\ (0.751788...)$ ***A1***

Note: Award at most ***(A1)(M1)(A0)*** for a final answer of 0.751. Working may not be seen.

[3 marks]

(e) $\text{H}_0 : m = 2.4$, ***A1***

$\text{H}_1 : m > 2.4$ ***A1***

Note: The hypotheses may be written in words but must include reference to the mean (e.g. "number of snails" is not sufficient to award ***A1***), and state clearly for H_1 that the mean increases.

[2 marks]

(f) **EITHER**

finding either $\text{P}(X \geq 7)$ or $\text{P}(X \geq 8)$ ***(M1)***

$(\text{P}(X \geq 7) =)\ 0.01160...$ **AND** $(\text{P}(X \geq 8) =)\ 0.00334...$ ***A1***

OR

finding either $\text{P}(X \leq 7)$ or $\text{P}(X \leq 6)$ ***(M1)***

$(\text{P}(X \leq 7) =)\ 0.996661...$ **AND** $(\text{P}(X \leq 6) =)\ 0.988405...$ ***A1***

THEN

so critical region is $X \geq 8$ **OR** $X > 7$ ***A1***

Note: ***(M1)A0A1*** can be awarded for a correct answer that is unsupported.

[3 marks]

(g) $(0.75 \times 12 =)\ 9$ ***(A1)***

$\text{P}(X \leq 7 \mid m = 9)$ ***(M1)***

$= 0.324$ ***A1***

[3 marks]

Total [18 marks]

Question 13

(a) $(s_{n-1} =) 1.30243...$ *(M1)(A1)*

$1.70\ (1.69632)$ ***A1***

Note: Award ***(M1)A0A0*** for a value of $(s_n =)\ 1.28934...$ or $(s_n{}^2 =)\ 1.6624$ seen.

[3 marks]

(b) the variance and the mean are similar ***R1***

Note: Do not accept a general statement "the variance and the mean are equal" unless their answer in part (a) is 1.76.

[1 mark]

(c) (i) attempt to find $\mathrm{P}(X = 4)$ under the null hypothesis $(= 0.0687830...)$ ***(M1)***

multiplying by 50 ***(M1)***

$j = 3.44\ (3.43915...)$ ***A1***

(ii) **EITHER**

attempt to find $\mathrm{P}(X \geq 5)$ under the null hypothesis and multiply by 50 ***(M1)***

OR

$50 - (8.60 + 15.14 + 13.32 + 7.82 + 3.44) \quad (= 5.12 - 3.44)$ ***(M1)***

THEN

$k = 1.68\ (1.67925...)$ ***A1***

[5 marks]

(d) there are expected frequencies less than 5 ***A1***

[1 mark]

(e) 3 ***A1***

[1 mark]

(f) 0.991 (0.991187) ***(M1)A1***

Note: Award ***M1*** for a table of observed and expected frequencies with columns for 4 and 5 or more combined.

[2 marks]

(g) $99\% > 5\%$ ***R1***

EITHER

so there is insufficient evidence to reject H_0. ***A1***

OR

we accept that the number of sightings follows a Poisson distribution ***A1***

Note: Do not award ***R0A1***.

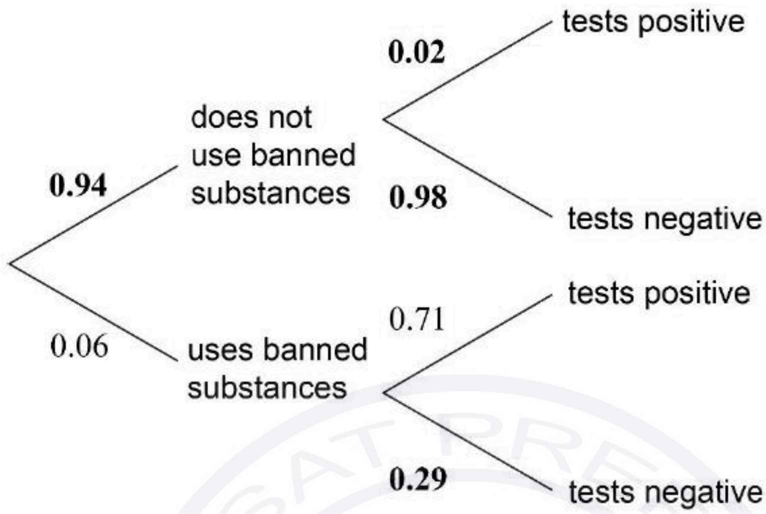
A p-value must be seen in part (f) to award ***FT***.

[2 marks]

[Total: 15 marks]

Question 14

(a)



A1A1

Note:	Award ***A1*** for any one value correct, ***A1*** for other three values correct. Accept percentage responses as equivalent forms on **all** branches.

[2 marks]

(b) (i) multiplication of two probabilities along the tree diagram ***(M1)***

0.94×0.98

$= 0.921 \ (0.9212, 92.1\%, 92.12\%)$ ***A1***

(ii) $(0.9212)^2$ ***(A1)***

$= 0.849 \ (0.848609\ldots, 84.9\%, 84.8609\ldots\%)$ ***A1***

[4 marks]

(c) (i) $0.94 \times 0.02 + 0.06 \times 0.29$ ***(A1)(M1)***

Note: Award ***A1*** for two correct products from their tree diagram seen, ***M1*** for the addition of their two products.

0.0362 (3.62%) ***A1***

(ii) multiplying their part (c)(i) by 1300

0.0362×1300 ***(M1)***

47.1 (47.06) ***A1***

[5 marks]

(d) $p = 0.02$ **OR** $p = 0.98$ ***(A1)***

recognition of binomial probability with $n = 20$ ***(M1)***

$\mathrm{P}(X = 0)$ **OR** $\mathrm{P}(X = 20)$ ***(M1)***

0.668 $(0.667607...)$ ***A1***

Note: Award ***(A1)(M1)(M1)A0*** for an answer of 0.667. $0.98^{20} = 0.668$ $(0.667607\ldots)$ is awarded full marks.

[4 marks]

(e) $\mathrm{P}(X \geq 3)$ **OR** $\mathrm{P}(X \leq 17)$ ***(M1)***

0.00707 $(0.00706869...)$ ***A1***

Note: Award ***(M1)A0*** for an answer of 0.00706. Award ***(M1)A0*** for an answer of 0.0599 $(0.0598989\ldots)$, obtained from the use of $\mathrm{P}(X \geq 2)$. ***FT*** from their value of p in part (d)

[2 marks]

[Total: 17 marks]

Question 15

(a) $\frac{9.45-8.73}{1958-1708}$ ***(M1)***

$=0.00288 \left(\frac{9}{3125}\right)$ ***A1***

[2 marks]

(b) (i) the (mean) yearly change in (mean annual) temperature ***A1***

Note: Accept equivalent statements, e.g. "rate of change of temperature".

(ii) $^{\circ}\mathrm{C}$ / year **OR** degrees C per year ***A1***

Note: Do not follow through from part (b)(i) into (b)(ii).

[2 marks]

(c) attempt to substitute point and gradient into appropriate formula ***(M1)***

$8.73 = 0.00288 \times 1708 + c \Rightarrow c = 3.81096...$

or

$9.45 = 0.00288 \times 1958 + c \Rightarrow c = 3.81096$.

equation is $y = 0.00288x + 3.81$ ***A1***

[2 marks]

(d) attempt to substitute 2000 into their part (c) ***(M1)***

$0.00288 \times 2000 + 3.81096...$

$= 9.57\ (^{\circ}\mathrm{C})\ \ (9.57096...)$ ***A1***

[2 marks]

(e) (i) $y = 0.00256x + 4.46$ $(0.00255714...x + 4.46454...)$ ***(M1)A1***

Note: Award ***(M1)A0*** for answers that show the correct method, but are presented incorrectly (e.g. no "$y =$" or truncated values etc.). Accept 4.465 as the correct answer to 4 sf.

(ii) 0.861 $(0.861333...)$ ***A1***

[3 marks]

(f) attempt to substitute 2000 into their part (e)(i) ***(M1)***

$0.00255714... \times 2000 + 4.46454...$

$= 9.58(^\circ\text{C})$ $(9.57882...(^\circ\text{C}))$ ***A1***

Note: Award ***A1*** for 9.57 from $0.00255714 \times 2000 + 4.46$.

[2 marks]

[Total: 13 marks]

Question 16

(a) (let X be the random variable the weight of an individual in the city of Melba)

$X \sim \mathrm{N}(72, 10^2)$

recognizing need to find $\mathrm{P}(X > 85)$ (condone "86" for the ***M1***) ***(M1)***

e.g. correct sketch of normal curve **OR** 0.0968 $(= 0.0968005...)$ seen

let Y be the random variable the number of people more than 85 kg

attempt to use a binomial distribution ***(M1)***

$Y \sim \mathrm{B}(10, 0.0968005...)$ ***(A1)***

Note: This ***(A1)*** can be implied by the value $0.988580\ldots$

$(\mathrm{P}(Y \geq 4) =)$ 0.0114 $(= 0.0114196...)$ ***A1***

[4 marks]

(b) let W be the random variable the total weight of a sample of eight people

$W \sim \mathrm{N}(576,\ 8\times 10^2)$ ***A1A1A1***

Note: Award ***A1*** for normal distribution; ***A1*** correct mean; ***A1*** correct variance or SD ($\mathrm{SD} = 28.2842\ldots$).

[3 marks]

(c) attempt to use inverse normal (or equivalent) ***(M1)***

$\mathrm{P}(W > w) = 0.01$

$(w =)\ 642\ (\text{kg})\quad (641.799\ldots)$ ***A1***

[2 marks]

(d) (i) *Any two correct assumptions identified,* ***A1A1***

e.g.

That Laetitia's clients are a random sample of the city's population

That people take only one holiday a year

That the choice of individual holidays is independent

That Laetitia is her clients' only agent

Note: Accept "assumes the proportion that takes a holiday abroad is 42%".

(ii) $\mathrm{H}_0 : p = 0.42$ ***A1***

$\mathrm{H}_1 : p < 0.42$ ***A1***

(iii) let Q be the random variable the number who go holiday abroad

$Q \sim \mathrm{B}(200,\ 0.42)$ ***(A1)***

$(\mathrm{P}(Q \le 67) =)\ 0.00850\quad (= 0.00849906...)$ ***A1***

$0.00850 < 0.05$ ***R1***

EITHER

there is evidence that Laetitia's claim is reasonable ***A1***

OR

there is insufficient evidence to accept the newspaper's claim ***A1***

Note: Follow through within this part, for correctly comparing and concluding with their **probability**, e.g. it is possible to award ***A0A0R1A1***. The conclusion to part (e)(iii) MUST follow through from their hypotheses seen in part (e)(ii); if hypotheses are incorrect/reversed etc., the answer to part (e)(iii) must reflect this in order for the ***A1*** to be credited.

[8 marks]

[Total 17 marks]

Question 17

(a) (i) 150 (cm) ***A1***

(ii) attempt to substitute values in the mean formula with at least one mid-interval value multiplied by a corresponding frequency ***(M1)***

(mean $=$) 176 (176.3) (cm) ***A1***

[3 marks]

(b) 183 **OR** 168 seen ***(A1)***

Note: These values may be seen in the working for part (c).

$(\text{IQR} = 183 - 168 =)$ 15 (cm) ***A1***

[2 marks]

(c) (upper bound =) $183 + 1.5 \times 15$ **OR** 205.5 seen ***A1***
$205.5 > 204$ **OR** $204 - 183 < 22.5$ **OR** $204 - 22.5 < 183$) ***R1***
Laszlo's height is not an outlier ***A1***

Note: Do not award ***R0A1***.

[3 marks]

(d) H_0: The heights of the students can be modelled by $N(176, 13.5^2)$
H_1: The heights of the students cannot be modelled by $N(176, 13.5^2)$ ***A1A1***

Note: Award ***A1*** for each correct hypothesis that includes a reference to normal distribution with a mean of 176 and a standard deviation of 13.5 (or variance of 13.5^2). "Correlation", "independence", "association", and "relationship" are incorrect.

Award at most ***A0A1*** for correctly worded hypotheses that include a reference to a normal distribution but omit the distribution's parameters in one or both hypotheses. Award ***A0A1*** for correct hypotheses that are reversed.

[2 marks]

(e) (i) $h \sim \mathrm{N}(176, 13.5^2)$

attempt to find normal probability in either correct range **(M1)**

$\mathrm{P}(170 \le h < 180)$ **OR** $\mathrm{P}(h \ge 190)$

recognition of multiplying either of their probabilities by 200 ***(M1)***

$0.288137... \times 200$ **OR** $0.149859... \times 200$

$a = 57.6 \ (57.6274...)$, $b = 30.0 \ (29.9718...)$ ***A1A1***

(ii) $\mathrm{df} = 4$ ***(A1)***

$(p =) \ 0.0166 \ (= 0.0166282\ldots)$ ***A1***

comparing their p-value to 0.05 ***R1***

$0.0166 < 0.05$

Note: Accept p value of $0.0165 \ (= 0.0164693...)$ from using a and b to 3 sf.

(Reject H_0 There is sufficient evidence to say that) the data has not been drawn from the ($\mathrm{N}(176, 13.5^2)$) distribution. ***A1***

Note: Do not award ***R0A1***.

The conclusion to part (e)(ii) **MUST** follow through from their hypotheses seen in part (d); if hypotheses are incorrect/reversed etc., the answer to part (e)(ii) must reflect this in order for the ***A1*** to be credited.

[8 marks]

[Total 18 marks]

Question18

(a) $\mathrm{P}(X < 340)$ **OR** labelled sketch of region **OR** calc syntax with correct bounds ***(M1)***

$= 0.159 \quad (0.158656\ldots)$ ***A1***

[2 marks]

(b) recognizing endpoint occurs at either 0.975 or 0.025 ***(M1)***

$\mathrm{P}(X < k) = 0.975$ **OR** $\mathrm{P}(X < m) = 0.025$

$330 < X < 370 \quad (330.400... < X < 369.599...)$ ***A1A1***

[3 marks]

(c) (i) recognizing mean of W is sum of individual means within wall ***(M1)***

$W = C_1 + C_2 + L$ may be seen

$\mathrm{E}(W) = 2\mathrm{E}(C) + \mathrm{E}(L)$

$= 800$ ***A1***

recognizing **variance** of W is sum of individual **variances** within wall ***(M1)***

$\mathrm{Var}(W) = 2\mathrm{Var}(C) + \mathrm{Var}(L)$ **OR** 225 seen ***(A1)***

$(\mathrm{SD}(W) =)\ 15$ ***A1***

Note: Award ***M1A0A0*** for an answer of 20.6 from using $\mathrm{Var}(2C)$ in place of $2\mathrm{Var}(C)$.

(ii) recognizing that W is modelled by a normal distribution ***(M1)***

$(\mathrm{P}(780 < W < 810) =)\ 0.656 \quad (0.656296...)$ ***A1***

Note: The answer is 0.521 (0.520) from using $\mathrm{SD} = 20.6 \quad (5\sqrt{17})$. Follow through from part (c)(i) without working seen.

[7 marks]

(d) $810 = 350 + 350 + \mathrm{E}(L)$ (or equivalent) ***(A1)***

$(\mathrm{E}(L) =)\ 110$ ***A1***

$\mathrm{Var}(W) = 2\mathrm{Var}(C) + \mathrm{Var}(L)$ **OR** $256 = 2(100) + \mathrm{Var}(L)$ ***(A1)***

$(\mathrm{SD}(L) =)\ 7.48 \quad (7.48331...,\ \sqrt{56})$ ***A1***

[4 marks]

(e) $116 \quad (116.298)$ ***A1***

Note: Do not follow through from either a negative variance or a negative SD.

[1 mark]

[Total: 17 marks]

Question 19

(a) (i) 15 **A1**

(ii) **EITHER**

attempt to use arithmetic series formula ***(M1)***

OR

attempt to set up simultaneous equations ***(M1)***

OR

attempt to use quadratic regression ***(M1)***

$(T_k =)\frac{1}{2}k^2 + \frac{1}{2}k$ ***A1A1***

> **Note:** Condone variable change (eg in quadratic regression).
> Accept $a = \frac{1}{2},\ b = \frac{1}{2}$.

[4 marks]

(b) (i) $(15 + 10 =)\ 25$ ***A1***

(ii) $\frac{k(k+1)}{2} + \frac{(k-1)((k-1)+1)}{2}$ **OR** $\frac{1}{2}k^2 + \frac{1}{2}k + \frac{1}{2}(k-1)^2 + \frac{1}{2}(k-1)$ ***(A1)***

$= k^2$ ***A1***

[3 marks]

(c) one correct product of probabilities seen: $\frac{15}{25}\times\frac{10}{24}$ **OR** $\frac{10}{25}\times\frac{15}{24}$ ***(A1)***

adding their products ***(M1)***

$\frac{15}{25}\times\frac{10}{24} + \frac{10}{25}\times\frac{15}{24}$

$= \frac{1}{2}$ ***A1***

[3 marks]

(d) attempt to add two products of probabilities involving k only ***M1***
(these may be incorrect or in terms of T_k)

$\frac{\frac{k}{2}(k+1)}{k^2}\times\frac{\frac{k}{2}(k-1)}{k^2-1} + \frac{\frac{k}{2}(k-1)}{k^2}\times\frac{\frac{k}{2}(k+1)}{k^2-1}$ ***A1***

further simplification consistent with given answer ***A1***

$= \frac{1}{2}$ ***A1***

hence independent of k ***AG***

[4 marks]
[Total: 14 marks]

Question 20

(a) let S be the number of spaceships hit and B the number of battleships

(i) mean = 8.4 ***(A1)***

$P(S \le 10) = 0.774 \ \ (0.774301...)$ ***A1***

(ii) attempt to add two means ***(M1)***

$4.2 + 2.3 = 6.5$

$P(S + B > 10) = P(S + B \ge 11)$ ***(M1)***

$0.0668 \ \ (0.0668387\ldots)$ ***A1***

[5 marks]

(b) (i) $E(T) = 3 \times 4.2 + 5 \times 2.3 = 24.1$ ***A1***

(ii) $\text{Var}(T) = 3^2 \times 4.2 + 5^2 \times 2.3 = 95.3$ ***(M1)A1***

[3 marks]

(c) any valid reason ***R1***

for example:

mean is not equal to variance **OR** T cannot take all integer values

[1 mark]

(d) distribution of mean score is $N\left(24.1, \frac{95.3}{60}\right) \ \ (N(24.1, 1.58833...))$ ***(A1)(A1)***

e: Award ***A1*** for normal distribution with mean 24.1, and ***A1*** for variance $\frac{95.3}{60}$.

$P(\bar{T} > 25) = 0.238 \ \ (0.237576...)$ ***A2***

[4 marks]

[Total 13 marks]

Question 21

(a) (i) 0.9 (ii) 0.3 (iii) 0.7 **A2**

e:Award ***A1A0*** if one of the values is incorrect, ***A0A0*** otherwise.

[2 marks]

(b) $(0.1\times 0.3 =)$ 0.03 **A1**

[1 mark]

(c) $\text{P}(\text{no fail}) = 0.63$ ***(A1)***

$\text{P}(\text{one fails}) = 0.34$ ***(A1)***

$\text{P}(\text{two fail}) = 0.03$ ***(A1)***

e: The three ***A1's*** can be awarded independently

multiplying by 200 ***(M1)***

No switch fails	One switch fails	Two switches fail
126	68	6

(A1)

degrees of freedom $= 2$ ***(A1)***

e: Award ***A1*** for df = 2 seen anywhere and may be awarded independent of the ***M1*** mark.
The df=2 cannot be implied from chi squared statistic $= 3.40989$

p-value $= 0.182$ $(0.181781\ldots)$ ***A1***

$0.182 > 0.05$ ***R1***

hence insufficient evidence to reject H_0 (that the manufacturers claims are correct) ***A1***

[9 marks]
[Total 12 marks]

Question 22

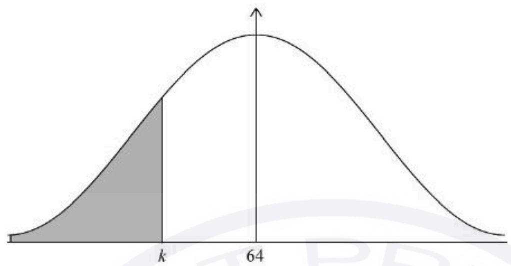
(a) 0.5 **A1**

[1 mark]

(b) 0.452 (0.452209…) **A2**

[2 marks]

(c) (i)



A1A1

: Award ***A1*** for a normal curve (with symmetry and some evidence of change of curvature towards the extreme values).
Award ***A1*** for a shaded region $x < k$, where k<mean.

(ii) $\mathrm{P}(T < k) = 0.3$

solving a cumulative distribution function **OR**
use of inverse function on GDC ***(M1)***
$k = 57.7$ (57.7071…) ***A1***

[4 marks]

(d) recognizing binomial distribution ***(M1)***
$\mathrm{B}(5,\ 0.3)$ $(\mathrm{P}(X = 2))$ ***(A1)***

0.309 (0.3087) ***A1***

[3 marks]

(e) $2(x-1)+4.5$ **OR** $2x+2.5$ ***A1A1***

Award ***A1*** for a linear expression with a gradient of 2,
A1 for a completely correct expression in x.

[2 marks]

(f) ($)13.10 (accept 13.1) ***A1***

[1 mark]

(g) attempt to solve $2(x-1)+4.5 = 7.2$ **OR** $2x+2.5 = 7.2$ ***(M1)***
2.35 (kg) ***A1***

Award ***M1A1FT*** for an answer of 1.35 (kg) from $2x + 4.5$ seen in (e).

[2 marks]
[Total 15 marks]

Question 23

(a) (i) $(Q_3 =)\ 7.2$ **A2**

Note: Award ***A1A0*** if the lower quartile, 6.5, is given as the answer.
Award ***A1A0*** for a correct ordered list of happiness scores, when the correct Q_3 is not seen.

(ii) $Q_1 = 6.5$ ***(A1)***

$IQR = 7.2 - 6.5$

$= 0.7$ ***A1***

[4 marks]

(b) $Q_3 + 1.5 \times IQR$ ***(A1)***

$(7.2 + 1.5 \times 0.7 =)\ 8.25$ ***A1***

since $7.5 < 8.25$ ***R1***

Switzerland is not an outlier ***AG***

Note: Do not award ***A0A0R1***.

[3 marks]

(c) (i) $a = 3.5$ ***A1***

(ii) $b = 8$ ***A1***

(iii) $c = 3.5$ ***A1***

[3 marks]

(d) (i) $(r_s =)\ 0.164\ \ (0.164134\ldots)$ ***A2***

(ii) France rank (of sixth) is unchanged (so the r_s is unchanged) ***R1***

[3 marks]

(e) Because r_s is too close to zero and hence Jose's conclusion is not appropriate ***R1***

Note: Award ***R1*** for a comment on the value r_s and "not appropriate" as a conclusion.
Accept "r_s indicates a weak correlation of the ranks".
Do not accept "r_s indicates a weak correlation of happiness score and country rank".

[1 mark]

[Total 14 marks]