

Subject - Math AI(Standard Level)

Topic - Calculus

Year - May 2021 - Nov 2024

Paper -1

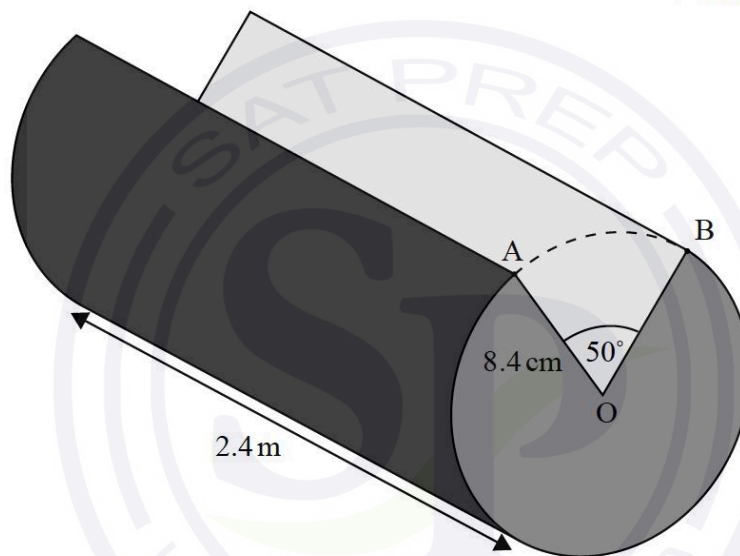
Questions

Question 1

[Maximum mark: 4]

Helen is building a cabin using cylindrical logs of length 2.4m and radius 8.4 cm. A wedge is cut from one log and the cross-section of this log is illustrated in the following diagram.

diagram not to scale

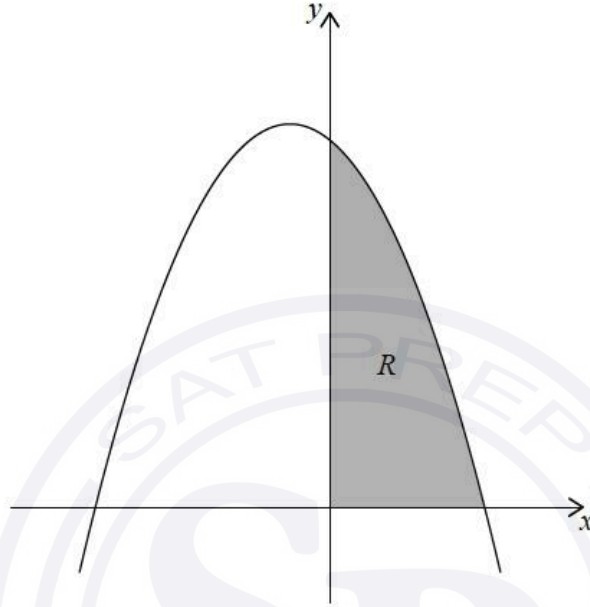


Find the volume of this log.

Question 2

[Maximum mark: 5]

The following diagram shows part of the graph of $f(x) = (6 - 3x)(4 + x)$, $x \in \mathbb{R}$. The shaded region R is bounded by the x -axis, y -axis and the graph of f .



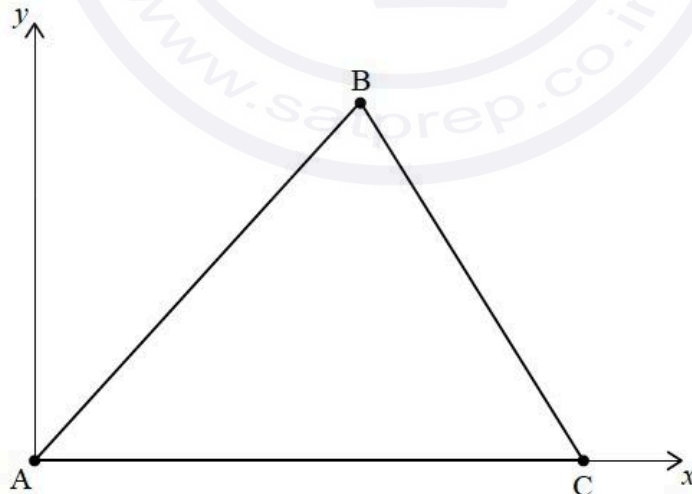
(a) Write down an integral for the area of region R .

[2]

(b) Find the area of region R .

[1]

The three points $A(0, 0)$, $B(3, 10)$ and $C(a, 0)$ define the vertices of a triangle.



(c) Find the value of a , the x -coordinate of C , such that the area of the triangle is equal to the area of region R .

[2]

Question 3

[Maximum mark: 7]

A company produces and sells electric cars. The company's profit, P , in thousands of dollars, changes based on the number of cars, x , they produce per month.

The rate of change of their profit from producing x electric cars is modelled by

$$\frac{dP}{dx} = -1.6x + 48, \quad x \geq 0.$$

The company makes a profit of 260 (thousand dollars) when they produce 15 electric cars.

- (a) Find an expression for P in terms of x . [5]

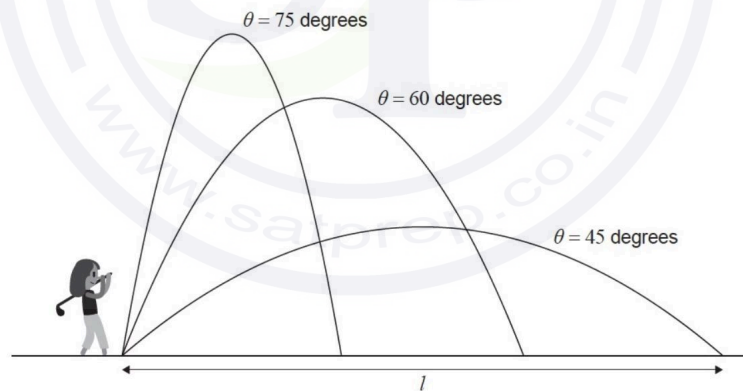
The company regularly increases the number of cars it produces.

- (b) Describe how their profit changes if they increase production to over 30 cars per month and up to 50 cars per month. Justify your answer. [2]

Question 4

[Maximum mark: 8]

Sieun hits golf balls into the air. Each time she hits a ball she records θ , the angle at which the ball is launched into the air, and l , the horizontal distance, in metres, which the ball travels from the point of contact to the first time it lands. The diagram below represents this information.



Sieun analyses her results and concludes:

$$\frac{dl}{d\theta} = -0.2\theta + 9, \quad 35^\circ \leq \theta \leq 75^\circ.$$

- (a) Determine whether the graph of l against θ is increasing or decreasing at $\theta = 50^\circ$. [3]

Sieun observes that when the angle is 40° , the ball will travel a horizontal distance of 205.5 m.

- (b) Find an expression for the function $l(\theta)$. [5]

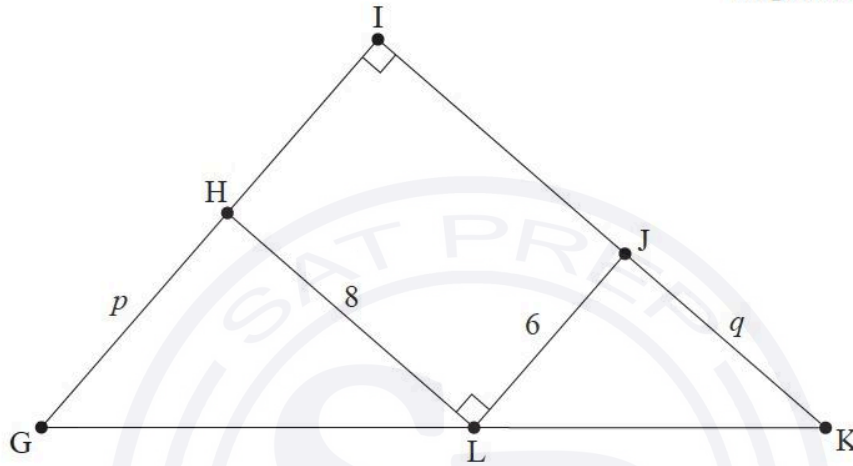
Question 5

[Maximum mark: 8]

Ellis designs a gift box. The top of the gift box is in the shape of a right-angled triangle GIK.

A rectangular section HIJL is inscribed inside this triangle. The lengths of GH, JK, HL, and LJ are p cm, q cm, 8 cm and 6 cm respectively.

diagram not to scale



The area of the top of the gift box is A cm².

(a) (i) Find A in terms of p and q .

(ii) Show that $A = \frac{192}{q} + 3q + 48$.

[4]

(b) Find $\frac{dA}{dq}$.

[2]

Ellis wishes to find the value of q that will minimize the area of the top of the gift box.

(c) (i) Write down an equation Ellis could solve to find this value of q .

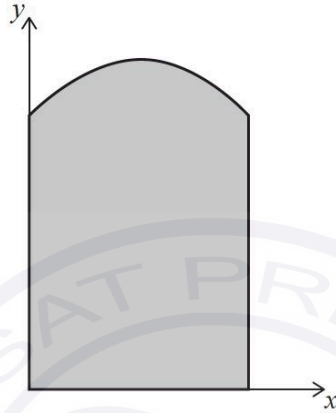
(ii) Hence, or otherwise, find this value of q .

[2]

Question 6

[Maximum mark: 8]

Irina uses a set of coordinate axes to draw her design of a window. The base of the window is on the x -axis, the upper part of the window is in the form of a quadratic curve and the sides are vertical lines, as shown on the diagram. The curve has end points $(0, 10)$ and $(8, 10)$ and its vertex is $(4, 12)$. Distances are measured in centimetres.



The quadratic curve can be expressed in the form $y = ax^2 + bx + c$ for $0 \leq x \leq 8$.

- (a) (i) Write down the value of c .
- (ii) Hence form two equations in terms of a and b .
- (iii) Hence find the equation of the quadratic curve. [5]
- (b) Find the area of the shaded region in Irina's design. [3]

Question 7

[Maximum mark: 6]

The surface area of an open box with a volume of 32 cm^3 and a square base with sides of length $x \text{ cm}$ is given by $S(x) = x^2 + \frac{128}{x}$ where $x > 0$.

- (a) Find $S'(x)$. [3]
- (b) (i) Solve $S'(x) = 0$.
- (ii) Interpret your answer to (b)(i) in context. [3]

Question 8

[Maximum mark: 6]

A company's profit per year was found to be changing at a rate of

$$\frac{dP}{dt} = 3t^2 - 8t$$

where P is the company's profit in thousands of dollars and t is the time since the company was founded, measured in years.

- (a) Determine whether the profit is increasing or decreasing when $t = 2$. [2]

One year after the company was founded, the profit was 4 thousand dollars.

- (b) Find an expression for $P(t)$, when $t \geq 0$. [4]

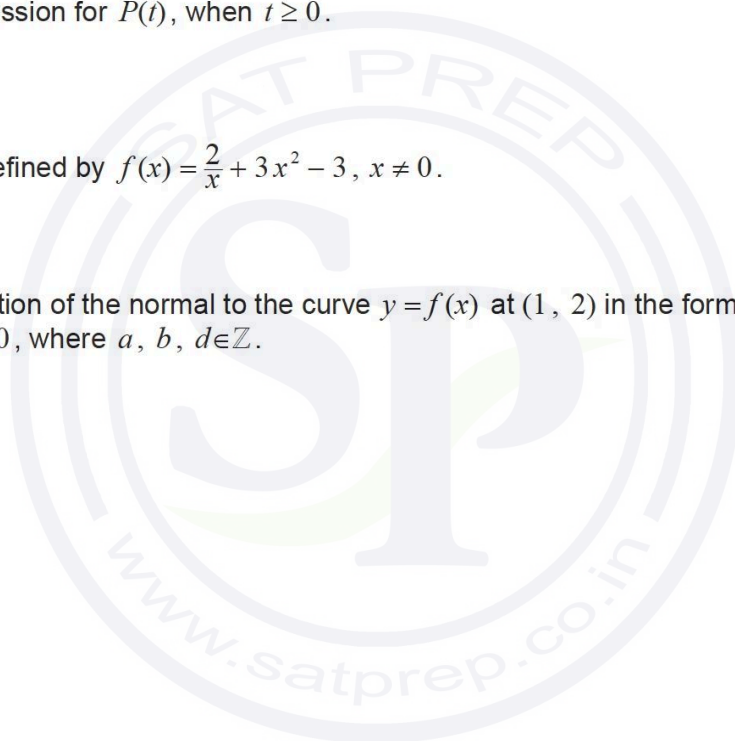
Question 9

[Maximum mark: 7]

The function f is defined by $f(x) = \frac{2}{x} + 3x^2 - 3$, $x \neq 0$.

- (a) Find $f'(x)$. [3]

- (b) Find the equation of the normal to the curve $y = f(x)$ at $(1, 2)$ in the form $ax + by + d = 0$, where $a, b, d \in \mathbb{Z}$. [4]

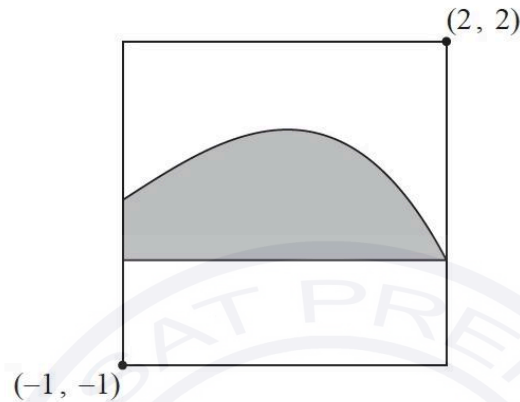


Question 10

[Maximum mark: 7]

A modern art painting is contained in a square frame. The painting has a shaded region bounded by a smooth curve and a horizontal line.

diagram not to scale



When the painting is placed on a coordinate axes such that the bottom left corner of the painting has coordinates $(-1, -1)$ and the top right corner has coordinates $(2, 2)$, the curve can be modelled by $y = f(x)$ and the horizontal line can be modelled by the x -axis. Distances are measured in metres.

- (a) Use the trapezoidal rule, with the values given in the following table, to approximate the area of the shaded region. [3]

x	-1	0	1	2
y	0.6	1.2	1.2	0

The artist used the equation $y = \frac{-x^3 - 3x^2 + 4x + 12}{10}$ to draw the curve.

- (b) Find the exact area of the shaded region in the painting. [2]
- (c) Find the area of the unshaded region in the painting. [2]

Question 11

[Maximum mark: 7]

Consider the function $f(x) = x^2 - \frac{3}{x}$, $x \neq 0$.

(a) Find $f'(x)$. [2]

Line L is a tangent to $f(x)$ at the point $(1, -2)$.

(b) Use your answer to part (a) to find the gradient of L . [2]

(c) Determine the number of lines parallel to L that are tangent to $f(x)$. Justify your answer. [3]



Question 12

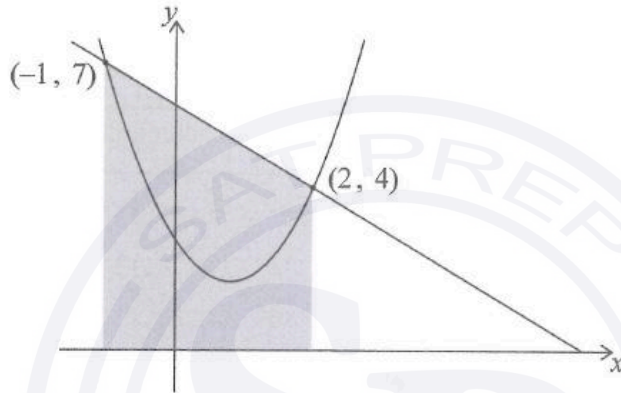
[Maximum mark: 7]

The graphs of $y = 6 - x$ and $y = 1.5x^2 - 2.5x + 3$ intersect at $(2, 4)$ and $(-1, 7)$, as shown in the following diagrams.

In **diagram 1**, the region enclosed by the lines $y = 6 - x$, $x = -1$, $x = 2$ and the x -axis has been shaded.

diagram not to scale

Diagram 1



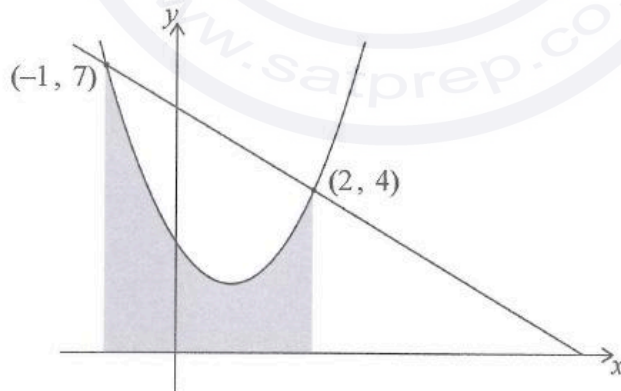
- (a) Calculate the area of the shaded region in **diagram 1**.

[2]

In **diagram 2**, the region enclosed by the curve $y = 1.5x^2 - 2.5x + 3$, and the lines $x = -1$, $x = 2$ and the x -axis has been shaded.

diagram not to scale

Diagram 2



- (b) (i) Write down an integral for the area of the shaded region in **diagram 2**.

- (ii) Calculate the area of this region.

[3]

- (c) Hence, determine the area enclosed between $y = 6 - x$ and $y = 1.5x^2 - 2.5x + 3$.

[2]

Question 13

[Maximum mark: 8]

Giles charges a customer per hour to hire his boat. It is known that

$$\frac{dP}{dt} = 20 - \frac{980}{t^2}, \quad 0 < t \leq 12$$

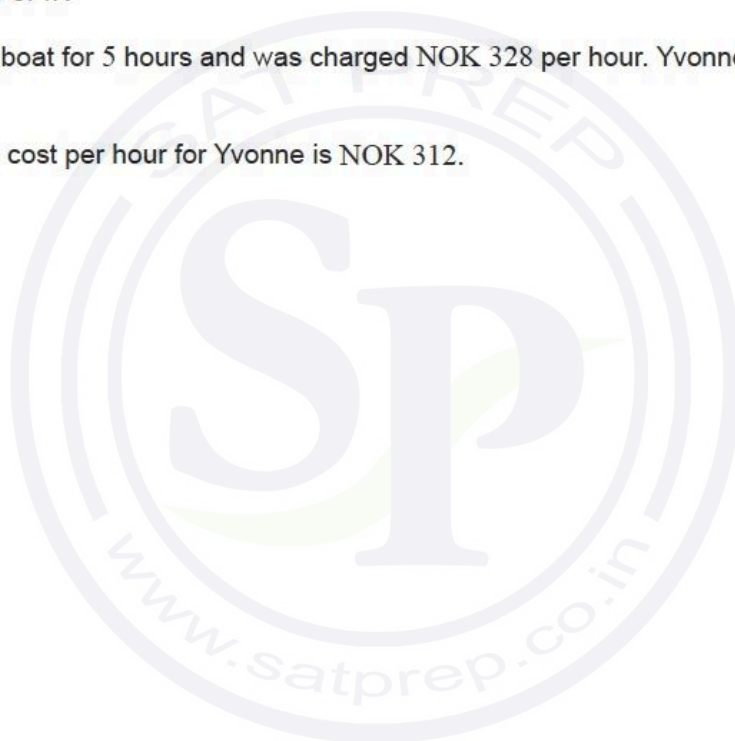
where P is the cost per hour, in Norwegian krone (NOK), that the customer is charged and t is the time, in hours, spent on the boat.

The cost per hour has a local minimum when the boat is hired for h hours.

(a) Find the value of h . [2]

Sandra hired Giles' boat for 5 hours and was charged NOK 328 per hour. Yvonne hires Giles' boat for 7 hours.

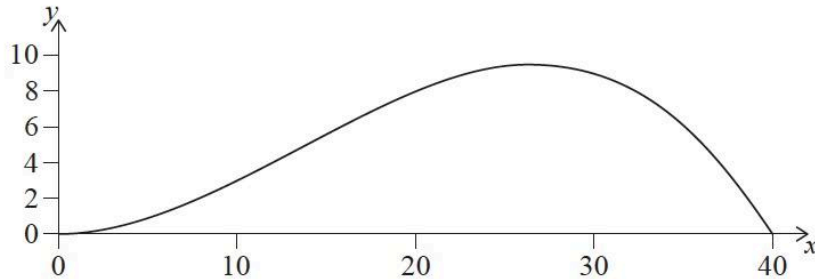
(b) Show that the cost per hour for Yvonne is NOK 312. [6]



Question 14

[Maximum mark: 8]

The cross section of a scale model of a hill is modelled by the following graph.



The heights of the model are measured at horizontal intervals and are given in the table.

Horizontal distance, x cm	0	10	20	30	40
Vertical distance, y cm	0	3	8	9	0

- (a) Use the trapezoidal rule with $h = 10$ to find an approximation for the cross-sectional area of the model. [2]

It is given that the equation of the curve is $y = 0.04x^2 - 0.001x^3$, $0 \leq x \leq 40$.

- (b) (i) Write down an integral to find the exact cross-sectional area. [4]
(ii) Calculate the value of the cross-sectional area to two decimal places. [4]
- (c) Find the percentage error in the area found using the trapezoidal rule. [2]

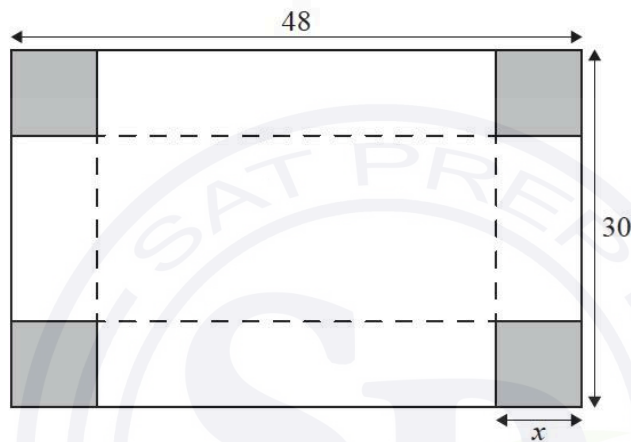
Question 15

[Maximum mark: 6]

A rectangular box, with an open top, is to be constructed from a piece of cardboard that measures 48 cm by 30 cm.

Squares of equal size will be cut from the corners of the cardboard, as indicated by the shading in the diagram. The sides will then be folded along the dotted lines to form the box.

diagram not to scale



The volume of the box, in cubic centimetres, can be modelled by the function $V(x) = (48 - 2x)(30 - 2x)(x)$, for $0 < x < k$, where x is the length of the sides of the squares removed in centimetres.

- (a) Write down the maximum possible value of k in this context. [1]
- (b) Find the value of x that maximizes the volume of the box. [2]

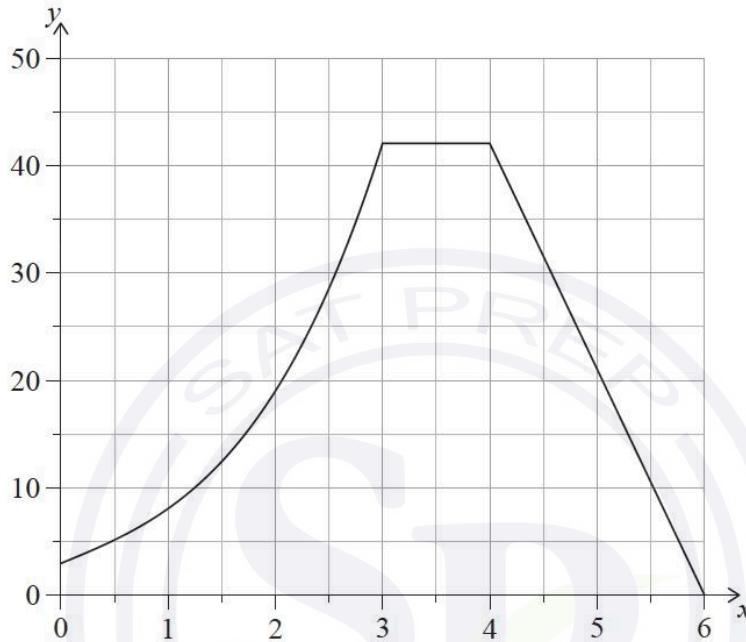
A second piece of 48 cm by 30 cm cardboard is damaged and a strip 2 cm wide must be removed from all four sides. A box will then be constructed in a similar manner from the remaining cardboard.

- (c) Calculate the maximum possible volume of the box made from the second piece of cardboard. [3]

Question 16

[Maximum mark: 9]

An engineer wants to calculate the cross-sectional area of a dam. The cross-section of the dam can be modelled by a curve and two straight lines as shown in the following diagram, where distances are measured in metres.



The curve is modelled by a function $f(x)$. The following table gives values of $f(x)$ for different values of x in the interval $0 \leq x \leq 3$.

x	0	0.5	1	1.5	2	2.5	3
$y = f(x)$	3	5.13	8	12.4	19	28.6	42

- (a) Calculate an estimate for the area in the interval $0 \leq x \leq 3$ by using the trapezoidal rule with three equal intervals. [2]

It is known that $f'(x) = 3x^2 + 4$ in the domain $0 < x < 3$.

- (b) Find an expression for $f(x)$, in the domain $0 < x < 3$. [4]
- (c) **Hence** find the actual area of the **entire** cross-section. [3]

Question 17

[Maximum mark: 6]

Consider the graph of the following function:

$$g(x) = \frac{8}{x} + \frac{x^2}{2}, \text{ for } x \neq 0.$$

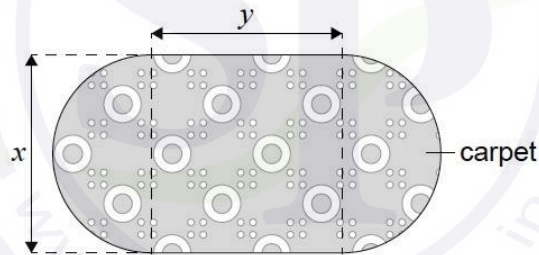
- (a) Write down the equation of the vertical asymptote of $g(x)$. [1]
- (b) Find $g'(x)$. [3]
- (c) Write down the interval in which $g(x)$ is increasing. [2]

Question 18

[Maximum mark: 8]

A company is designing a new carpet. The intended design of the carpet is in the shape of a rectangle with a semi-circle at each end.

The width of the rectangle is y metres and the diameter of each semi-circle is x metres, with $x > 0$ and $y \geq 0$.



The company has decided that the perimeter of the carpet will be 20 metres and would like to maximize its area.

- (a) Find an expression for the perimeter in terms of x and y . [1]
- (b) Show that the area, $A \text{ m}^2$, of the carpet can be expressed as $A = 10x - \frac{\pi x^2}{4}$. [3]
- (c) Find $\frac{dA}{dx}$. [2]
- (d) Hence find the **exact** value of x for which the area is a maximum. [2]

Question 19

[Maximum mark: 7]

The gradient of the **normal** to the curve $y = ax^2 + bx - 10$ at the point T(2, 4) is $-\frac{1}{3}$.

Calculate the value of a and the value of b .

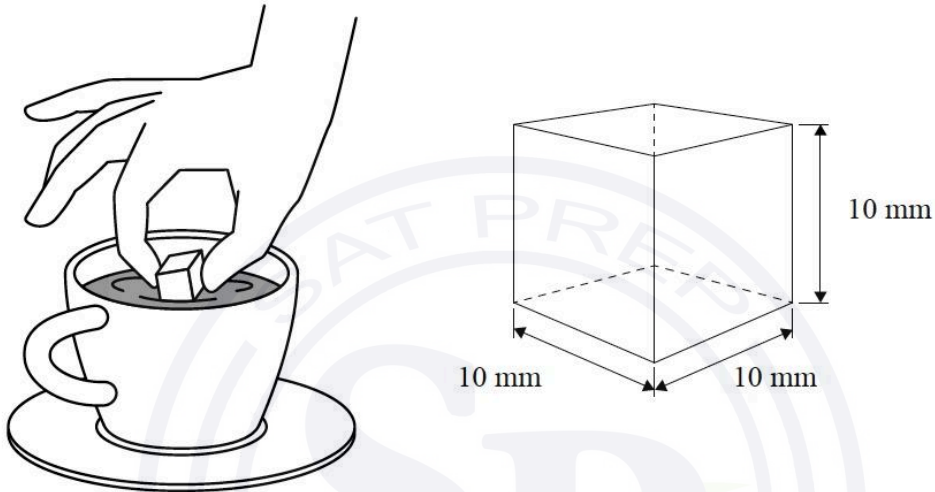


Question 20

[Maximum mark: 8]

Kyungyoon investigates the rate at which a cubical block of sugar dissolves in hot coffee. Initially, the cube has side lengths of 10 mm. This information is illustrated in the following diagrams.

diagram not to scale



Kyungyoon predicts that, as the block of sugar dissolves, each side length will decrease at a constant rate of 0.2 mm per second.

- (a) According to this model, find
- (i) the length of one side of a block of sugar, 20 seconds after it is placed in hot coffee.
 - (ii) the volume of a block of sugar, 20 seconds after it is placed in hot coffee. [3]

Let the function $V(t)$ represent the volume of the block of sugar, mm^3 , t seconds after it is placed in hot coffee. $V(t)$ is given by

$$V(t) = 1000 - 60t + 1.2t^2 - 0.008t^3, \text{ for } 0 \leq t \leq 50.$$

- (b) Find $V'(t)$. [2]
- (c) Find the rate of change of the volume of the block of sugar at $t = 20$. [2]
- (d) State one reason why the side length of the cube may not always decrease at a constant rate. [1]

Question 21

[Maximum mark: 7]

Consider the curve $y = 4x^3 - \frac{2}{x^2}$.

- (a) Find $\frac{dy}{dx}$. [3]
- (b) Write down the gradient of the curve at $x = 1$. [1]
- (c) Hence, find the equation of the normal to the curve at $x = 1$. [3]

Question 22

[Maximum mark: 7]

Consider the curve $y = 5x^2 - \frac{3}{x^3}$.

- (a) Find $\frac{dy}{dx}$. [3]
- (b) Write down the gradient of the curve at $x = 1$. [1]
- (c) Hence, find the equation of the normal to the curve at $x = 1$. [3]

