Subject – Math AI(Standard Level)
Topic - Calculus
Year - May 2021 – Nov 2024
Paper -1
Answers

Question 1

$\text{volume} = 240\left(\pi \times 8.4^2 - \frac{1}{2} \times 8.4^2 \times 0.872664\ldots\right)$ ***M1M1M1***

: Award ***M1*** $240 \times$ area , award ***M1*** for correctly substituting area sector formula, award ***M1*** for subtraction of their area of the sector from area of circle.

$= 45800\ (= 45811.96071)$ ***A1***

Total [4 marks]

Question 2

(a) $A = \int_0^2 (6-3x)(4+x)\mathrm{d}x$ ***A1A1***

Note: Award ***A1*** for the limits $x = 0,\ x = 2$. Award ***A1*** for an integral of $f(x)$.

[2 marks]

(b) 28 ***A1***

[1 mark]

(c) $28 = 0.5 \times a \times 10$ ***M1***

$5.6\left(\frac{28}{5}\right)$ ***A1***

[2 marks]

Total [5 marks]

Question 3

(a) recognition of need to integrate (*eg* reverse power rule or integral symbol) ***(M1)***

$P(x) = -0.8x^2 + 48x\ (+c)$ ***A1A1***

$260 = -0.8\times(15)^2 + 48\times(15) + c$ ***(M1)***

$c = -280$

$P(x) = -0.8x^2 + 48x - 280$ ***A1***

[5 marks]

(b) profit will decrease (with each new car produced) ***A1***

EITHER

because the profit function is decreasing / the gradient is negative / the rate of change of P is negative ***R1***

OR

$\int_{30}^{50} -1.6x + 48\ (\mathrm{d}x) = -320$ ***R1***

OR

evidence of finding $P(30) = 440$ and $P(50) = 120$ ***R1***

[2 marks]

Total [7 marks]

Question 4

(a) $l'(50) = -0.2\times 50 + 9$ ***(M1)***

$= -1$ ***A1***

the curve is decreasing at $\theta = 50°$. ***A1***

[3 marks]

(b) recognition of need to integrate (e.g. reverse power rule or integral symbol or integrating at least one term correctly) ***(M1)***

$l(\theta) = -0.1\theta^2 + 9\theta\ (+c)$ ***A1A1***

$205.5 = -0.1\times(40)^2 + 9\times(40) + c$ ***(M1)***

$c = 5.5$

$(l(\theta) =)\ -0.1\theta^2 + 9\theta + 5.5$ ***A1***

[5 marks]

Total [8 marks]

Question 5

(a) (i) $A = \frac{1}{2} \times 6 \times q + \frac{1}{2} \times 8 \times p + 48$ **OR** $A = \frac{1}{2}(p+6)(q+8)$ **OR**

$A = 3q + 4p + 48$ ***A1***

(ii) valid attempt to link p and q, using tangents, similar triangles or other method ***(M1)***

eg. $\tan\theta = \frac{8}{p}$ and $\tan\theta = \frac{q}{6}$ **OR** $\tan\theta = \frac{p}{8}$ and $\tan\theta = \frac{6}{q}$ **OR** $\frac{8}{p} = \frac{q}{6}$

correct equation linking p and q ***A1***

eg. $pq = 48$ **OR** $p = \frac{48}{q}$ **OR** $q = \frac{48}{p}$

substitute $p = \frac{48}{q}$ into a correct area expression ***M1***

eg. $(A =)\frac{1}{2} \times 6 \times q + \frac{1}{2} \times 8 \times \frac{48}{q} + 48$ **OR** $(A =)\frac{1}{2}\left(\frac{48}{q} + 6\right)(q+8)$

$A = 3q + \frac{192}{q} + 48$ ***AG***

Note: The ***AG*** line must be seen with no incorrect, intermediate working, for the final ***M1*** to be awarded.

[4 marks]

(b) $\frac{-192}{q^2} + 3$ ***A1A1***

Note: Award ***A1*** for $\frac{-192}{q^2}$, ***A1*** for 3. Award ***A1A0*** if extra terms are seen.

[2 marks]

(c) (i) $\frac{-192}{q^2} + 3 = 0$ ***A1***

(ii) $q = 8$ cm ***A1***

[2 marks]

Total [8 marks]

Question 6

(a) (i) $c = 10$ ***A1***

(ii) $64a + 8b + 10 = 10$ ***A1***

$16a + 4b + 10 = 12$ ***A1***

Note: Award ***A1*** for each equivalent expression or ***A1*** for the use of the axis of symmetry formula to find $4 = \frac{-b}{2a}$ or from use of derivative.
Award ***A0A1*** for $64a + 8b + c = 10$ and $16a + 4b + c = 12$.

(iii) $y = -\frac{1}{8}x^2 + x + 10$ ***A1A1***

Note: Award ***A1A0*** if one term is incorrect, ***A0A0*** if two or more terms are incorrect.
Award at most ***A1A0*** if correct a, b and c values are seen but answer not expressed as an equation.

[5 marks]

(b) recognizing the need to integrate their expression ***(M1)***

$\int_0^8 -\frac{1}{8}x^2 + x + 10 \, \mathrm{d}x$ ***(A1)***

Note: Award ***(A1)*** for correct integral, including limits. Condone absence of $\mathrm{d}x$.

$90.7 \text{ cm}^2 \left(\frac{272}{3}, 90.6666\ldots\right)$ ***A1***

[3 marks]
Total: [8 marks]

Question 7

(a) $(S(x) =)\ x^2 + 128x^{-1}$ ***(M1)***

Note: Award ***(M1)*** for expressing second term with a negative power. This may be implied by $\frac{1}{x^2}$ seen as part of their answer.

$2x - \frac{128}{x^2}$ **OR** $2x - 128x^{-2}$ ***A1A1***

Note: Award ***A1*** for $2x$ and ***A1*** for $-\frac{128}{x^2}$. The first ***A1*** is for x^2 differentiated correctly and is independent of the ***(M1)***.

[3 marks]

(b) (i) **EITHER**

any correct manipulation of $2x - \frac{128}{x^2} = 0$ e.g. $2x^3 - 128 = 0$ ***(M1)***

OR

sketch of graph of $S'(x)$ with root indicated ***(M1)***

OR

sketch of graph of $S(x)$ with minimum indicated ***(M1)***

THEN

$x = 4$ ***A1***

Note: Value must be positive. Follow through from their part (a) irrespective of working.

(ii) the value of x that will minimize surface area of the box ***A1***

Note: Accept 'optimize' in place of minimize.

[3 marks]

Total: [6 marks]

Question 8

(a) **METHOD 1**

(when $t = 2$)

$\frac{dP}{dt} = -4$ **OR** $\frac{dP}{dt} < 0$ (equivalent in words) **OR** $3(2)^2 - 8(2) = -4$ ***M1***

therefore P is decreasing ***A1***

METHOD 2

sketch with $t = 2$ indicated in 4th quadrant **OR** t-intercepts identified ***M1***

therefore P is decreasing ***A1***

[2 marks]

(b) $(P(t) =)\ t^3 - 4t^2\ (+c)$ ***A1A1***

$4 = 1^3 - 4(1)^2 + c$ ***(M1)***

Question 9

(a) $f'(x) = -2x^{-2} + 6x$ **OR** $f'(x) = -\frac{2}{x^2} + 6x$ ***A1(M1)A1***

[3 marks]

(b) finding gradient at $x = 1$

$\left.\frac{dy}{dx}\right|_{x=1} = 4$ ***A1***

finding the perpendicular gradient ***M1***

$m_{\perp} = -\frac{1}{4}$

$2 = -\frac{1}{4}(1) + c$ **OR** $y - 2 = -\frac{1}{4}(x - 1)$ ***M1***

$x + 4y - 9 = 0$ ***A1***

[4 marks]

[Total 7 marks]

Question 10

(a) $\frac{1}{2}(0.6+0+2(1.2+1.2))$ ***(A1)(M1)***

Note: Award ***A1*** for evidence of $h=1$, ***M1*** for a correct substitution into trapezoidal rule (allow for an incorrect h only). The zero can be omitted in the working.

$2.7\ \text{m}^2$ ***A1***

[3 marks]

(b) $\int_{-1}^{2}\frac{-x^3-3x^2+4x+12}{10}\,\text{d}x$ **OR** $\int_{-1}^{2}f(x)\,\text{d}x$ ***(M1)***

Note: Award ***M1*** for using definite integration with correct limits.

$2.925\ \text{m}^2$ ***A1***

Note: Question requires exact answer, do not award final ***A1*** for 2.93.

[2 marks]

(c) $9-2.925$ ***(M1)***

Note: Award ***M1*** for 9 seen as part of a subtraction.

$=6.08\ \text{m}^2\ \ (6.075)$ ***A1***

[2 marks]
[Total 7 marks]

Question 11

(a) $(f'(x) =)\ 2x + \frac{3}{x^2}$ **A1A1**

Note: Award ***A1*** for $2x$, ***A1*** for $+\frac{3}{x^2}$ **OR** $+3x^{-2}$.

[2 marks]

(b) attempt to substitute 1 into their part (a) ***(M1)***

$(f'(1) =)\ 2(1) + \frac{3}{1^2}$

5 ***A1***

[2 marks]

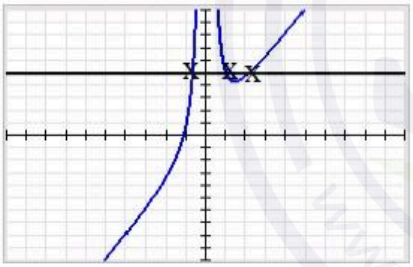
(c) **EITHER**

$5 = 2x + \frac{3}{x^2}$ ***M1***

$x = -0.686,\ 1,\ 2.19\ \ (-0.686140..., 1, 2.18614...)$ ***A1***

OR

sketch of $y = f'(x)$ with line $y = 5$ ***M1***



three points of intersection marked on this graph ***A1***
(and it can be assumed no further intersections occur outside of this window)

THEN

there are two other tangent lines to $f(x)$ that are parallel to L ***A1***

[3 marks]

Total [7 marks]

Question 12

(a) **EITHER**

attempt to substitute 3, 4 and 7 into area of a trapezoid formula *(M1)*

$(A=)\ \frac{1}{2}(7+4)(3)$

OR

given line expressed as an integral *(M1)*

$(A=)\ \int_{-1}^{2}(6-x)\ \mathrm{d}x$

OR

attempt to sum area of rectangle and area of triangle *(M1)*

$(A=)\ 4\times 3+\frac{1}{2}\ (3)(3)$

THEN

16.5 (square units) *A1*

[2 marks]

(b) (i) $(A=)\ \int_{-1}^{2}1.5x^2-2.5x+3\ \mathrm{d}x$ *A1A1*

(ii) 9.75 (square units) *A1*

[3 marks]

(c) $16.5-9.75$ *(M1)*

6.75 (square units) *A1*

[2 marks]

Total [7 marks]

Question 13

(a) $0 = 20 - \frac{980}{t^2}$ **OR** $\frac{dP}{dt} = 0$ ***(M1)***

Note: Accept equivalent information presented in a labelled sketch.

$(h =)$ 7 hours ***A1***

Note: Award ***M1A0*** for an answer of $(7, 280)$.

[2 marks]

(b) recognition of need to integrate (e.g. reverse power rule or integral symbol) ***(M1)***

$P(t) = 20t + \frac{980}{t} \ (+c)$ ***A1A1***

$328 = 20 \times 5 + \frac{980}{5} + c$ ***(M1)***

Note: Award ***(M1)*** for substitution of $P = 328$ and $t = 5$ into their $P(t)$. A constant of integration must be seen (can be implied by a correct answer).

$c = 32$ ***A1***

$P(7) = 20 \times 7 + \frac{980}{7} + 32$ ***M1***

Note: Award ***M1*** for substituting 7 and their 32 into their $P(t)$.
Do not award the final ***M*** mark if their substituted values do not lead to 312.

312 NOK ***AG***

[6 marks]

Total [8 marks]

Question 14

(a) attempt to substitute $h = 10$ and at least two different values of y into the trapezoidal rule ***(M1)***

$$\frac{10}{2}((0+0)+2(3+8+9))$$

$$= 200\ (\text{cm}^2)$$ ***A1***

[2 marks]

(b) (i) $\int_0^{40} 0.04x^2 - 0.001x^3 \, dx$ **OR** $\int_0^{40} y \, dx$ ***A1A1***

Note: Award ***A1*** for a correct integral (including dx), ***A1*** for correct limits in the correct location.

(ii) $213.33\ (\text{cm}^2)$ ***A2***

Note: Answer must be given to 2 decimal places to award ***A2***. Award ***A1A0*** for a correct answer given to an incorrect accuracy of at least 3 significant figures, e.g. $213\ (\text{cm}^2)$.

[4 marks]

(c) attempt to substitute their parts (a) and (b)(ii) into percentage error formula ***(M1)***

$$\left|\frac{213.333... - 200}{213.333...}\right| \times 100$$

$$= 6.25(\%)\ \ (6.24999...(\%))$$ ***A1***

Note: Award ***(M1)A0*** for a final answer of $-6.25(\%)$ or 0.0625.

[2 marks]

[Total: 8 marks]

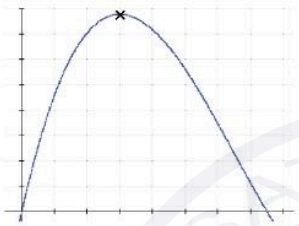
Question 15

. (a) $(k =)\ 15$ ***A1***

[1 mark]

(b) **EITHER**

attempt to sketch the function $V(x)$ with indication of maximum ***(M1)***



OR

recognition of setting the derivative to 0 ***(M1)***

e.g. $V'(x) = 0$

THEN

$(x =)\ 6\ (\text{cm})$ ***A1***

Note: Award ***(M1)A0*** for the maximum given as a coordinate pair.

[2 marks]

(c) 44 or 26 seen ***(A1)***

attempting to adjust the constant(s) in the given volume formula ***(M1)***

volume of second box $= (44-2x)(26-2x)(x)$

(New maximum volume $=$) $2730\ \text{cm}^3$ $(2726.13...\ \text{cm}^3)$ ***A1***

Note: Units must be seen to award the final ***A1***. Award ***(A1)(M1)A0*** for the maximum given as a coordinate pair.

[3 marks]

[Total: 6 marks]

Question 16

(a) attempt at using the trapezoidal rule ***(M1)***

$\text{area} = \frac{1}{2}(3 + 2(8 + 19) + 42)$

$= 49.5\ (\text{m}^2)$ ***A1***

[2 marks]

(b) recognition of need to integrate (*e.g.* reverse power rule or integral symbol) ***(M1)***

$\int 3x^2 + 4\,\text{d}x = x^3 + 4x + c$ ***(A1)(A1)***

Note: Award ***A1*** for each correct term.

$f(x) = x^3 + 4x + 3$ ***A1***

Note: Award ***A1*** for simplified correct answer including the value of c. Accept a value of c of 3.005 or 3.025 or 2.975 for using the non-integer x-values and their corresponding y-values.

[4 marks]

(c) **METHOD 1**

forming expression for sum of integral and deconstructing the trapezoid into a rectangle and triangle ***(M1)***

$\int_0^3 x^3 + 4x + 3\ \text{d}x (= 47.25) + 42 \times 1 + \frac{1}{2} \times 2 \times 42\ (= 84)$ ***(A1)***

$= 131\ (\text{m}^2)\ \ (131.25)$ ***A1***

METHOD 2

forming expression for sum of integral and trapezoid ***(M1)***

$\int_0^3 x^3 + 4x + 3\ \text{d}x (= 47.25)\ + \frac{1}{2} \times 4 \times 42\ (= 84)$ ***(A1)***

$= 131\ (\text{m}^2)\ \ (131.25)$ ***A1***

Note: Award ***(A1)*** for their integral with the correct limits added to 84 or their 47.25 added to 84.

[3 marks]

Total [9 marks]

Question 17

(a) $x = 0$ ***A1***

Note: Answer must be an equation; an answer of "0" or "the y-axis" is awarded ***A0***.

[1 mark]

(b) $(g'(x) =) -8x^{-2} + x$ ***A1A1A1***

Note: Award ***A1*** for -8 seen, ***A1*** for x^{-2} $\left(\text{or } \frac{1}{x^2}\right)$ and ***A1*** for second term being x.
Award at most ***A1A1A0*** if additional terms are seen.

(c) $x > 2$ **OR** $(2, \infty)$ **OR** $2 < x < \infty$ ***A1A1***

Note: Award ***A1*** for 2 seen and award ***A1*** for **correct** inequality.

[2 marks]
[Total 6 marks]

Question 18

. (a) $\pi x + 2y \ (= 20)$ ***A1***

Note: Award ***A0*** for an unsimplified answer.

[1 mark]

(b) attempt to combine area formulas to express A in terms of x and y ***M1***

$A = xy + \frac{\pi x^2}{4}$

$y = \frac{20 - \pi x}{2} \ (= 10 - \frac{\pi x}{2})$ ***A1***

correct substitution for y and expansion of brackets ***A1***

$A = x\left(\frac{20 - \pi x}{2}\right) + \frac{\pi x^2}{4} = \frac{20x - \pi x^2}{2} + \frac{\pi x^2}{4}$

$= 10x - \frac{\pi x^2}{4}$ ***AG***

Note: The ***AG*** line must be stated for the final ***A1*** to be awarded.

[3 marks]

(c) attempt at power rule e.g. one correct term ***(M1)***

$10 - \frac{\pi x}{2}$ ***A1***

[2 marks]

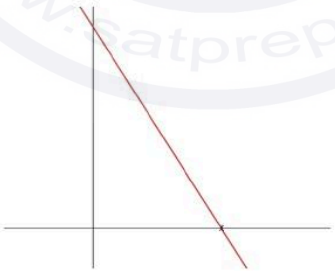
(d) **EITHER**

setting their derivative to zero ***M1***

$10 - \frac{\pi x}{2} = 0$ **OR** $\frac{dA}{dx} = 0$

OR

attempt at finding the root of the graph of their derivative ***M1***



THEN

$x = \frac{20}{\pi}$ (m) ***A1***

(given that function is a negative quadratic, only stationary point is a maximum)

Note: Do not award the M mark if their derivative is not used to find the x-value.
Award at most ***M1A0*** if their answer is not given in exact form.

[2 marks]

[Total: 8 marks]

Question 19

3 (seen at any stage of their work) ***(A1)***

$\frac{dy}{dx} = 2ax + b$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to find $\frac{dy}{dx}$.

substituting $x = 2$ and their gradient into their derivative ***(M1)***

$3 = 2a(2) + b$

substituting $(2, 4)$ into original equation ***(M1)***

$4 = a(2)^2 + b(2) - 10$

solving equations $a = -2$

$b = 11$ ***A1A1***

Note: Using $m = -\frac{1}{3}$, gives $a = -\frac{11}{3}, b = \frac{43}{3}$, award ***A0M1A1M1M1A0A1***.
For the final ***A*** marks, award at most ***A1A0*** if correct values are unlabelled or incorrectly labelled.

[Total: 7 marks]

Question 20

(a) (i) recognizing that 0.2 is removed 20 times ***(M1)***

$10 - 0.2 \times 20$

6 (mm) ***A1***

(ii) $(V =) 216 \text{ mm}^3$ ***A1***

Note: Units are required for the ***A1*** to be awarded.

[3 marks]

(b) attempt at power rule ***(M1)***

$(V'(t) =) -60 + 2.4t - 0.024t^2$ ***A1***

[2 marks]

(c) recognizing need to find $V'(20)$ ***(M1)***

$(V'(20) =) -60 + 2.4(20) - 0.024(20)^2$

$= -21.6 \text{ (mm}^3 \text{ s}^{-1})$ ***A1***

[2 marks]

(d) decreasing/change in coffee temperature ***R1***
decreasing/change in surface area of the cube
the cube breaks apart into smaller pieces
dissolution rate related to volume of the cube

Note: Award ***R1*** for a reasonable explanation of a change in rate.

[1 mark]
[Total: 8 marks]

Question 21

(a) $\left(\frac{dy}{dx} =\right)\ 12x^2 + 4x^{-3}$ $\left(\left(\frac{dy}{dx} =\right)\ 12x^2 + \frac{4}{x^3}\right)$ ***A1(M1)A1***

Note: Award ***A1*** for $12x^2$, ***M1*** for expressing $\frac{1}{x^2}$ as x^{-2}, ***A1*** for $+4x^{-3}$.

[3 marks]

(b) 16 ***A1***

[1 mark]

(c) $m_{\perp} = -\frac{1}{16}$ (seen anywhere) ***(A1)***

$y = 2$ (seen anywhere) ***(A1)***

$y - 2 = -\frac{1}{16}(x - 1)$ **OR** $y = -\frac{1}{16}x + \frac{33}{16}$ $(y = -0.0625x + 2.0625)$ **OR** $y = -0.0625x + 2.06$

A1

Note: Follow through within the question for the final ***A1***, which can be awarded for correctly substituting **their** gradient (seen) and **their** point (seen) into the equation for a line.

[3 marks]

[Total: 7 marks]

Question 22

(a) $\left(\frac{dy}{dx} =\right) 10x + 9x^{-4}$ $\left(\left(\frac{dy}{dx} =\right) 10x + \frac{9}{x^4}\right)$ ***A1(M1)A1***

Note: Award ***A1*** for $10x$, ***M1*** for expressing $\frac{1}{x^3}$ as x^{-3}, ***A1*** for $+9x^{-4}$.

[3 marks]

(b) 19 ***A1***

[1 mark]

(c) $m_{\perp} = -\frac{1}{19}$ (seen anywhere) ***(A1)***

$y = 2$ (seen anywhere) ***(A1)***

$y - 2 = -\frac{1}{19}(x - 1)$ **OR** $y = -\frac{1}{19}x + \frac{39}{19}$ $(y = -0.0526x + 2.05,\ y = -0.0526315...x + 2.05263...)$

A1

Note: Follow through within the question for the final ***A1***, which can be awarded for correctly substituting **their** gradient (seen) and **their** point (seen) into the equation for a line.

[3 marks]

[Total: 7 marks]