Subject – Math AI(Standard Level)
Topic - Calculus
Year - May 2021 – Nov 2022
Paper -1
Answers

Question 1

volume $= 240\left(\pi \times 8.4^2 - \frac{1}{2} \times 8.4^2 \times 0.872664\ldots\right)$ ***M1M1M1***

: Award ***M1*** $240 \times$ area, award ***M1*** for correctly substituting area sector formula, award ***M1*** for subtraction of their area of the sector from area of circle.

$= 45800 \ (= 45811.96071)$ ***A1***

Total [4 marks]

Question 2

(a) $A = \int_0^2 (6-3x)(4+x)\,\mathrm{d}x$ ***A1A1***

Note: Award ***A1*** for the limits $x = 0,\ x = 2$. Award ***A1*** for an integral of $f(x)$.

[2 marks]

(b) 28 ***A1***

[1 mark]

(c) $28 = 0.5 \times a \times 10$ ***M1***

$5.6\left(\frac{28}{5}\right)$ ***A1***

[2 marks]

Total [5 marks]

Question 3

(a) recognition of need to integrate (*eg* reverse power rule or integral symbol) ***(M1)***

$P(x) = -0.8x^2 + 48x \ (+c)$ ***A1A1***

$260 = -0.8 \times (15)^2 + 48 \times (15) + c$ ***(M1)***

$c = -280$

$P(x) = -0.8x^2 + 48x - 280$ ***A1***

[5 marks]

(b) profit will decrease (with each new car produced) ***A1***

EITHER

because the profit function is decreasing / the gradient is negative / the rate of change of P is negative ***R1***

OR

$\int_{30}^{50} -1.6x + 48 \ (\mathrm{d}x) = -320$ ***R1***

OR

evidence of finding $P(30) = 440$ and $P(50) = 120$ ***R1***

[2 marks]

Total [7 marks]

Question 4

(a) $l'(50) = -0.2 \times 50 + 9$ ***(M1)***

$= -1$ ***A1***

the curve is decreasing at $\theta = 50°$. ***A1***

[3 marks]

(b) recognition of need to integrate (e.g. reverse power rule or integral symbol or integrating at least one term correctly) ***(M1)***

$l(\theta) = -0.1\theta^2 + 9\theta \ (+c)$ ***A1A1***

$205.5 = -0.1 \times (40)^2 + 9 \times (40) + c$ ***(M1)***

$c = 5.5$

$(l(\theta) =) \ -0.1\theta^2 + 9\theta + 5.5$ ***A1***

[5 marks]

Total [8 marks]

Question 5

(a) (i) $A=\frac{1}{2}\times 6\times q+\frac{1}{2}\times 8\times p+48$ ***OR*** $A=\frac{1}{2}(p+6)(q+8)$ ***OR***

$A=3q+4p+48$ ***A1***

(ii) valid attempt to link p and q, using tangents, similar triangles or other method ***(M1)***

eg. $\tan\theta=\frac{8}{p}$ and $\tan\theta=\frac{q}{6}$ **OR** $\tan\theta=\frac{p}{8}$ and $\tan\theta=\frac{6}{q}$ **OR** $\frac{8}{p}=\frac{q}{6}$

correct equation linking p and q ***A1***

eg. $pq=48$ **OR** $p=\frac{48}{q}$ **OR** $q=\frac{48}{p}$

substitute $p=\frac{48}{q}$ into a correct area expression ***M1***

eg. $(A=)\frac{1}{2}\times 6\times q+\frac{1}{2}\times 8\times\frac{48}{q}+48$ **OR** $(A=)\frac{1}{2}\left(\frac{48}{q}+6\right)(q+8)$

$A=3q+\frac{192}{q}+48$ ***AG***

Note: The ***AG*** line must be seen with no incorrect, intermediate working, for the final ***M1*** to be awarded.

[4 marks]

(b) $\frac{-192}{q^2}+3$ ***A1A1***

Note: Award ***A1*** for $\frac{-192}{q^2}$, ***A1*** for 3. Award ***A1A0*** if extra terms are seen.

[2 marks]

(c) (i) $\frac{-192}{q^2}+3=0$ ***A1***

(ii) $q=8\text{ cm}$ ***A1***

[2 marks]

Total [8 marks]

Question 6

(a) (i) $c = 10$ **A1**

(ii) $64a + 8b + 10 = 10$ **A1**

$16a + 4b + 10 = 12$ **A1**

Note: Award ***A1*** for each equivalent expression or ***A1*** for the use of the axis of symmetry formula to find $4 = \frac{-b}{2a}$ or from use of derivative.
Award ***A0A1*** for $64a + 8b + c = 10$ and $16a + 4b + c = 12$.

(iii) $y = -\frac{1}{8}x^2 + x + 10$ **A1A1**

Note: Award ***A1A0*** if one term is incorrect, ***A0A0*** if two or more terms are incorrect.
Award at most ***A1A0*** if correct a, b and c values are seen but answer not expressed as an equation.

[5 marks]

(b) recognizing the need to integrate their expression ***(M1)***

$\int_0^8 -\frac{1}{8}x^2 + x + 10 \, dx$ ***(A1)***

Note: Award ***(A1)*** for correct integral, including limits. Condone absence of dx.

$90.7 \text{ cm}^2 \left(\frac{272}{3}, 90.6666\ldots\right)$ **A1**

[3 marks]
Total: [8 marks]

Question 7

(a) $(S(x)=)\ x^2+128x^{-1}$ ***(M1)***

Note: Award ***(M1)*** for expressing second term with a negative power. This may be implied by $\frac{1}{x^2}$ seen as part of their answer.

$2x-\frac{128}{x^2}$ **OR** $2x-128x^{-2}$ ***A1A1***

Note: Award ***A1*** for $2x$ and ***A1*** for $-\frac{128}{x^2}$. The first ***A1*** is for x^2 differentiated correctly and is independent of the ***(M1)***.

[3 marks]

(b) (i) **EITHER**

any correct manipulation of $2x-\frac{128}{x^2}=0$ e.g. $2x^3-128=0$ ***(M1)***

OR

sketch of graph of $S'(x)$ with root indicated ***(M1)***

OR

sketch of graph of $S(x)$ with minimum indicated ***(M1)***

THEN

$x=4$ ***A1***

Note: Value must be positive. Follow through from their part (a) irrespective of working.

(ii) the value of x that will minimize surface area of the box ***A1***

Note: Accept 'optimize' in place of minimize.

[3 marks]
Total: [6 marks]

Question 8

(a) **METHOD 1**

(when $t = 2$)

$\frac{dP}{dt} = -4$ **OR** $\frac{dP}{dt} < 0$ (equivalent in words) **OR** $3(2)^2 - 8(2) = -4$ ***M1***

therefore P is decreasing ***A1***

METHOD 2

sketch with $t = 2$ indicated in 4th quadrant **OR** t-intercepts identified ***M1***

therefore P is decreasing ***A1***

[2 marks]

(b) $(P(t) =)\ t^3 - 4t^2\ (+c)$ ***A1A1***

$4 = 1^3 - 4(1)^2 + c$ ***(M1)***

Question 9

(a) $f'(x) = -2x^{-2} + 6x$ **OR** $f'(x) = -\frac{2}{x^2} + 6x$ ***A1(M1)A1***

[3 marks]

(b) finding gradient at $x = 1$

$\left.\frac{dy}{dx}\right|_{x=1} = 4$ ***A1***

finding the perpendicular gradient ***M1***

$m_\perp = -\frac{1}{4}$

$2 = -\frac{1}{4}(1) + c$ **OR** $y - 2 = -\frac{1}{4}(x - 1)$ ***M1***

$x + 4y - 9 = 0$ ***A1***

[4 marks]

[Total 7 marks]

Question 10

(a) $\frac{1}{2}(0.6+0+2(1.2+1.2))$ ***(A1)(M1)***

Note: Award ***A1*** for evidence of $h=1$, ***M1*** for a correct substitution into trapezoidal rule (allow for an incorrect h only). The zero can be omitted in the working.

$2.7\ \text{m}^2$ ***A1***

[3 marks]

(b) $\int_{-1}^{2}\frac{-x^3-3x^2+4x+12}{10}\,\mathrm{d}x$ **OR** $\int_{-1}^{2} f(x)\,\mathrm{d}x$ ***(M1)***

Note: Award ***M1*** for using definite integration with correct limits.

$2.925\ \text{m}^2$ ***A1***

Note: Question requires exact answer, do not award final ***A1*** for 2.93.

[2 marks]

(c) $9-2.925$ ***(M1)***

Note: Award ***M1*** for 9 seen as part of a subtraction.

$=6.08\ \text{m}^2\ \ (6.075)$ ***A1***

[2 marks]

[Total 7 marks]

Question 11

(a) $\left(f'(x)=\right)\ 2x+\dfrac{3}{x^2}$ ***A1A1***

> **Note:** Award ***A1*** for $2x$, ***A1*** for $+\dfrac{3}{x^2}$ **OR** $+3x^{-2}$.

[2 marks]

(b) attempt to substitute 1 into their part (a) ***(M1)***

$\left(f'(1)=\right)\ 2(1)+\dfrac{3}{1^2}$

5 ***A1***

[2 marks]

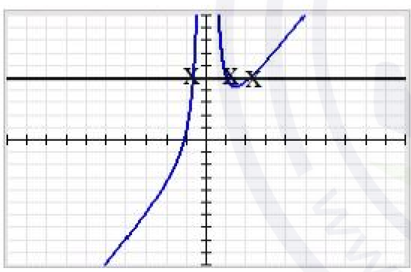
(c) **EITHER**

$5=2x+\dfrac{3}{x^2}$ ***M1***

$x=-0.686,\ 1,\ 2.19\ \ (-0.686140...,\ 1,\ 2.18614\ldots)$ ***A1***

OR

sketch of $y=f'(x)$ with line $y=5$ ***M1***



three points of intersection marked on this graph ***A1***
(and it can be assumed no further intersections occur outside of this window)

THEN

there are two other tangent lines to $f(x)$ that are parallel to L ***A1***

[3 marks]
Total [7 marks]

Question 12

(a) **EITHER**

attempt to substitute 3, 4 and 7 into area of a trapezoid formula *(M1)*

$(A =) \frac{1}{2}(7+4)(3)$

OR

given line expressed as an integral *(M1)*

$(A =) \int_{-1}^{2}(6-x)\ \mathrm{d}x$

OR

attempt to sum area of rectangle and area of triangle *(M1)*

$(A =) 4 \times 3 + \frac{1}{2}\ (3)(3)$

THEN

16.5 (square units) *A1*

[2 marks]

(b) (i) $(A =) \int_{-1}^{2} 1.5x^2 - 2.5x + 3\ \mathrm{d}x$ *A1A1*

(ii) 9.75 (square units) *A1*

[3 marks]

(c) $16.5 - 9.75$ *(M1)*

6.75 (square units) *A1*

[2 marks]

Total [7 marks]

Question 13

(a) $0 = 20 - \frac{980}{t^2}$ **OR** $\frac{dP}{dt} = 0$ ***(M1)***

Note: Accept equivalent information presented in a labelled sketch.

$(h =)$ 7 hours ***A1***

Note: Award ***M1A0*** for an answer of $(7, 280)$.

[2 marks]

(b) recognition of need to integrate (e.g. reverse power rule or integral symbol) ***(M1)***

$P(t) = 20t + \frac{980}{t}$ $(+c)$ ***A1A1***

$328 = 20 \times 5 + \frac{980}{5} + c$ ***(M1)***

Note: Award ***(M1)*** for substitution of $P = 328$ and $t = 5$ into their $P(t)$. A constant of integration must be seen (can be implied by a correct answer).

$c = 32$ ***A1***

$P(7) = 20 \times 7 + \frac{980}{7} + 32$ ***M1***

Note: Award ***M1*** for substituting 7 and their 32 into their $P(t)$.
Do not award the final ***M*** mark if their substituted values do not lead to 312.

312 NOK ***AG***

[6 marks]

Total [8 marks]