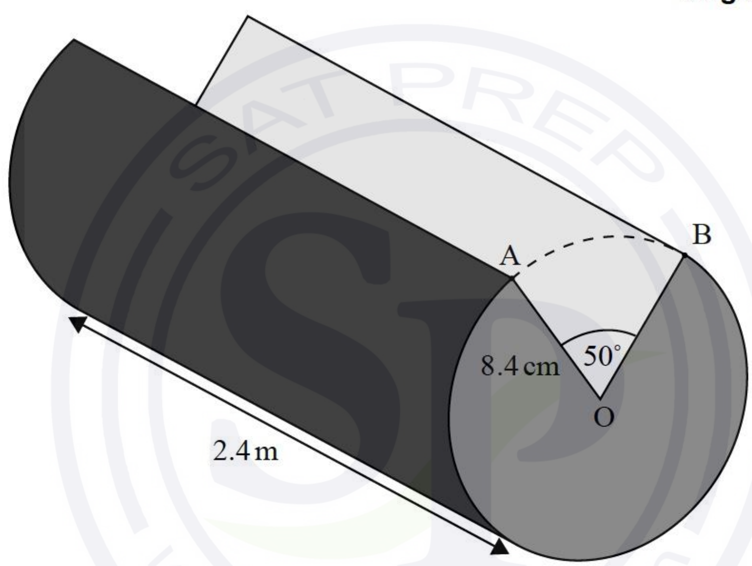
Subject – Math AI(Standard Level)
Topic - Calculus
Year - May 2021 – Nov 2022
Paper -1
Questions

Question 1

[Maximum mark: 4]

Helen is building a cabin using cylindrical logs of length $2.4\,\text{m}$ and radius $8.4\,\text{cm}$. A wedge is cut from one log and the cross-section of this log is illustrated in the following diagram.

diagram not to scale

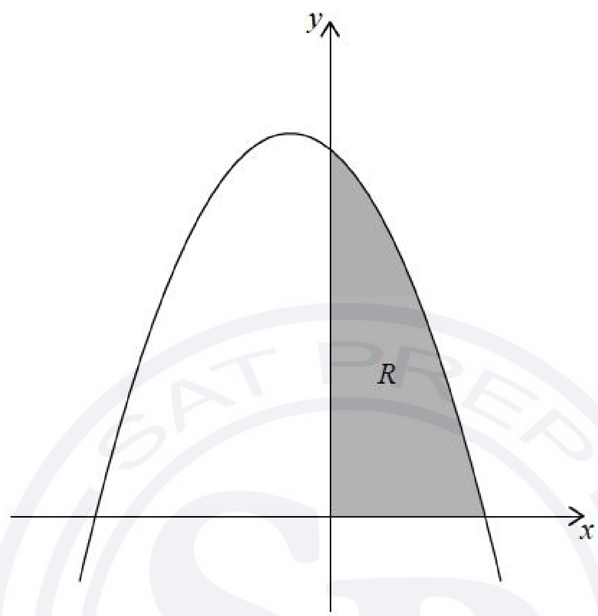


Find the volume of this log.

Question 2

[Maximum mark: 5]

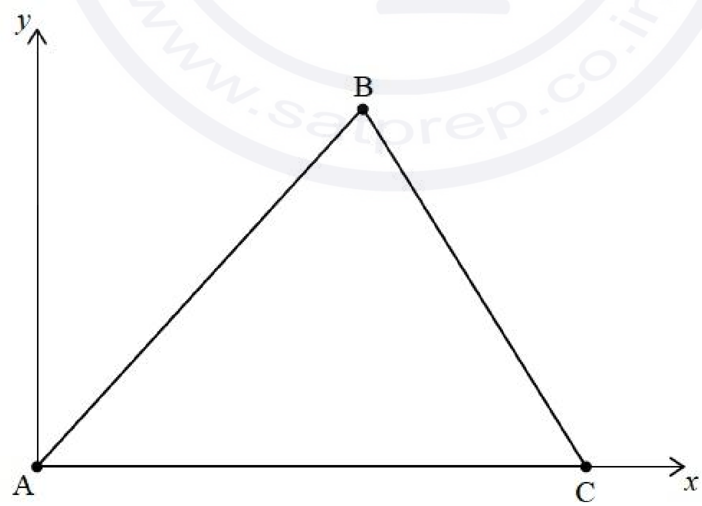
The following diagram shows part of the graph of $f(x) = (6 - 3x)(4 + x)$, $x \in \mathbb{R}$. The shaded region R is bounded by the x-axis, y-axis and the graph of f.



(a) Write down an integral for the area of region R. [2]

(b) Find the area of region R. [1]

The three points $\mathrm{A}(0, 0)$, $\mathrm{B}(3, 10)$ and $\mathrm{C}(a, 0)$ define the vertices of a triangle.



(c) Find the value of a, the x-coordinate of C, such that the area of the triangle is equal to the area of region R. [2]

Question 3

[Maximum mark: 7]

A company produces and sells electric cars. The company's profit, P, in thousands of dollars, changes based on the number of cars, x, they produce per month.

The rate of change of their profit from producing x electric cars is modelled by

$$\frac{dP}{dx} = -1.6x + 48, \ x \geq 0.$$

The company makes a profit of 260 (thousand dollars) when they produce 15 electric cars.

(a) Find an expression for P in terms of x. [5]

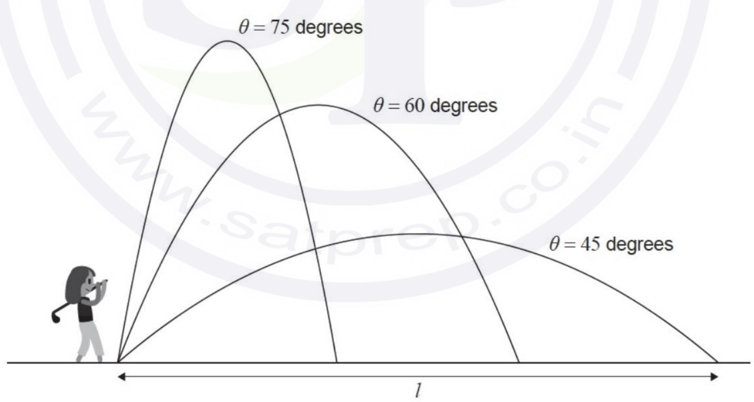
The company regularly increases the number of cars it produces.

(b) Describe how their profit changes if they increase production to over 30 cars per month and up to 50 cars per month. Justify your answer. [2]

Question 4

[Maximum mark: 8]

Sieun hits golf balls into the air. Each time she hits a ball she records θ, the angle at which the ball is launched into the air, and l, the horizontal distance, in metres, which the ball travels from the point of contact to the first time it lands. The diagram below represents this information.



Sieun analyses her results and concludes:

$$\frac{dl}{d\theta} = -0.2\theta + 9, \ 35^\circ \leq \theta \leq 75^\circ.$$

(a) Determine whether the graph of l against θ is increasing or decreasing at $\theta = 50^\circ$. [3]

Sieun observes that when the angle is 40°, the ball will travel a horizontal distance of $205.5\,\text{m}$.

(b) Find an expression for the function $l(\theta)$. [5]

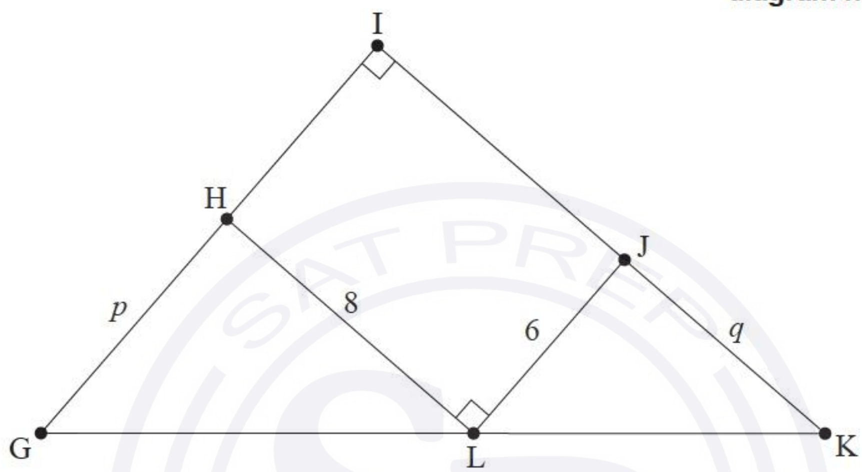
Question 5

[Maximum mark: 8]

Ellis designs a gift box. The top of the gift box is in the shape of a right-angled triangle GIK.

A rectangular section HIJL is inscribed inside this triangle. The lengths of GH, JK, HL, and LJ are p cm, q cm, 8 cm and 6 cm respectively.

diagram not to scale



The area of the top of the gift box is $A\,\text{cm}^2$.

(a) (i) Find A in terms of p and q.

(ii) Show that $A = \frac{192}{q} + 3q + 48$. [4]

(b) Find $\frac{dA}{dq}$. [2]

Ellis wishes to find the value of q that will minimize the area of the top of the gift box.

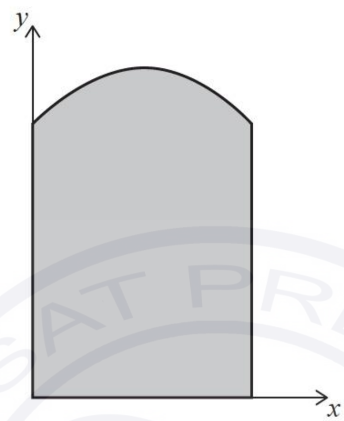
(c) (i) Write down an equation Ellis could solve to find this value of q.

(ii) Hence, or otherwise, find this value of q. [2]

Question 6

[Maximum mark: 8]

Irina uses a set of coordinate axes to draw her design of a window. The base of the window is on the x-axis, the upper part of the window is in the form of a quadratic curve and the sides are vertical lines, as shown on the diagram. The curve has end points $(0,\ 10)$ and $(8,\ 10)$ and its vertex is $(4,\ 12)$. Distances are measured in centimetres.



The quadratic curve can be expressed in the form $y = ax^2 + bx + c$ for $0 \le x \le 8$.

(a) (i) Write down the value of c.

(ii) Hence form two equations in terms of a and b.

(iii) Hence find the equation of the quadratic curve. [5]

(b) Find the area of the shaded region in Irina's design. [3]

Question 7

[Maximum mark: 6]

The surface area of an open box with a volume of $32\,\text{cm}^3$ and a square base with sides of length $x\,\text{cm}$ is given by $S(x) = x^2 + \frac{128}{x}$ where $x > 0$.

(a) Find $S'(x)$. [3]

(b) (i) Solve $S'(x) = 0$.

(ii) Interpret your answer to (b)(i) in context. [3]

Question 8

[Maximum mark: 6]

A company's profit per year was found to be changing at a rate of

$$\frac{dP}{dt} = 3t^2 - 8t$$

where P is the company's profit in thousands of dollars and t is the time since the company was founded, measured in years.

(a) Determine whether the profit is increasing or decreasing when $t = 2$. [2]

One year after the company was founded, the profit was 4 thousand dollars.

(b) Find an expression for $P(t)$, when $t \geq 0$. [4]

Question 9

[Maximum mark: 7]

The function f is defined by $f(x) = \frac{2}{x} + 3x^2 - 3$, $x \neq 0$.

(a) Find $f'(x)$. [3]

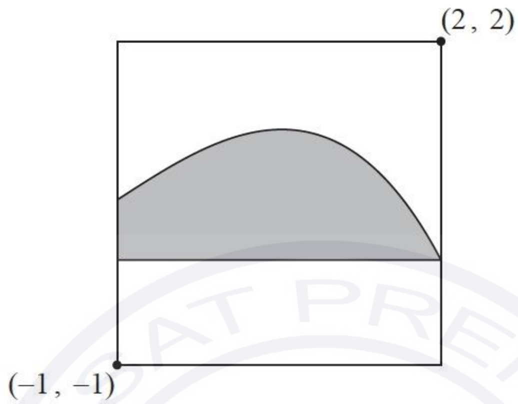
(b) Find the equation of the normal to the curve $y = f(x)$ at $(1, 2)$ in the form $ax + by + d = 0$, where $a, b, d \in \mathbb{Z}$. [4]

Question 10

[Maximum mark: 7]

A modern art painting is contained in a square frame. The painting has a shaded region bounded by a smooth curve and a horizontal line.

diagram not to scale



When the painting is placed on a coordinate axes such that the bottom left corner of the painting has coordinates $(-1\,,\,-1)$ and the top right corner has coordinates $(2\,,\,2)$, the curve can be modelled by $y = f(x)$ and the horizontal line can be modelled by the x-axis. Distances are measured in metres.

(a) Use the trapezoidal rule, with the values given in the following table, to approximate the area of the shaded region. [3]

x	-1	0	1	2
y	0.6	1.2	1.2	0

The artist used the equation $y = \dfrac{-x^3 - 3x^2 + 4x + 12}{10}$ to draw the curve.

(b) Find the exact area of the shaded region in the painting. [2]

(c) Find the area of the unshaded region in the painting. [2]

Question 11

[Maximum mark: 7]

Consider the function $f(x) = x^2 - \frac{3}{x}$, $x \neq 0$.

(a) Find $f'(x)$. [2]

Line L is a tangent to $f(x)$ at the point $(1, -2)$.

(b) Use your answer to part (a) to find the gradient of L. [2]

(c) Determine the number of lines parallel to L that are tangent to $f(x)$. Justify your answer. [3]

Question 12

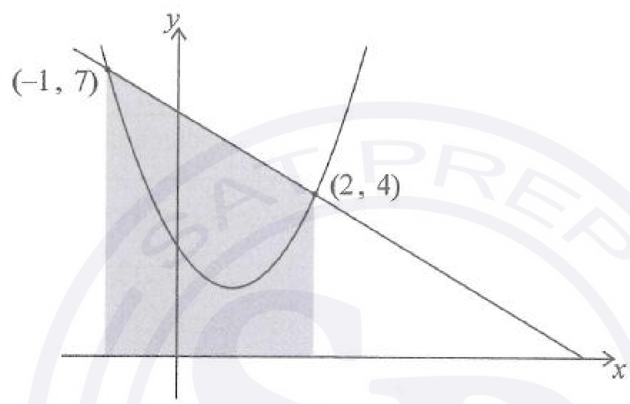
[Maximum mark: 7]

The graphs of $y = 6 - x$ and $y = 1.5x^2 - 2.5x + 3$ intersect at (2 , 4) and (−1 , 7), as shown in the following diagrams.

In **diagram 1**, the region enclosed by the lines $y = 6 - x$, $x = -1$, $x = 2$ and the x-axis has been shaded.

diagram not to scale

Diagram 1

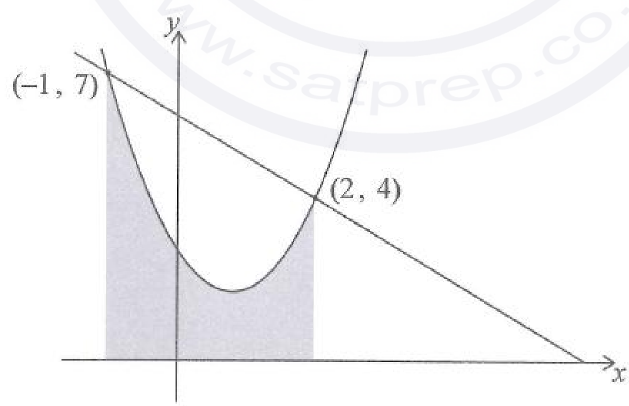


(a) Calculate the area of the shaded region in **diagram 1**. [2]

In **diagram 2**, the region enclosed by the curve $y = 1.5x^2 - 2.5x + 3$, and the lines $x = -1$, $x = 2$ and the x-axis has been shaded.

diagram not to scale

Diagram 2



(b) (i) Write down an integral for the area of the shaded region in **diagram 2**.

(ii) Calculate the area of this region. [3]

(c) Hence, determine the area enclosed between $y = 6 - x$ and $y = 1.5x^2 - 2.5x + 3$. [2]

Question 13

[Maximum mark: 8]

Giles charges a customer per hour to hire his boat. It is known that

$$\frac{dP}{dt} = 20 - \frac{980}{t^2}, \ 0 < t \leq 12$$

where P is the cost per hour, in Norwegian krone (NOK), that the customer is charged and t is the time, in hours, spent on the boat.

The cost per hour has a local minimum when the boat is hired for h hours.

(a) Find the value of h. [2]

Sandra hired Giles' boat for 5 hours and was charged NOK 328 per hour. Yvonne hires Giles' boat for 7 hours.

(b) Show that the cost per hour for Yvonne is NOK 312. [6]