

**Subject - Math AI(Standard Level)**  
**Topic - Function**  
**Year - May 2021 - Nov 2022**  
**Paper -1**  
**Answers**

**Question 1**

(a) (i) 1750

**A1**

(ii)  $1350 + 400(1.25)^{-5}$

**(M1)**

= 1480

**A1**

**Note:** Accept 1481.

**[3 marks]**

(b)  $1400 = 1350 + 400(1.25)^{-t}$   
9.32 (days (9.31885...)) (days)

**(M1)**

**A1**

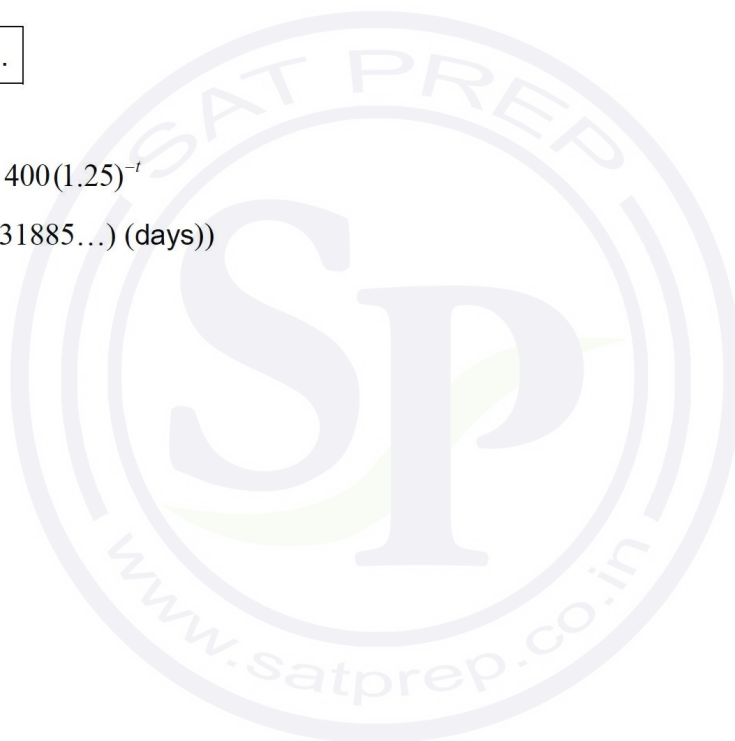
**[2 marks]**

(c) 1350

**A1**

**[1 mark]**

**Total [6 marks]**



## Question 2

(a) 20

**A1**  
**[1 mark]**

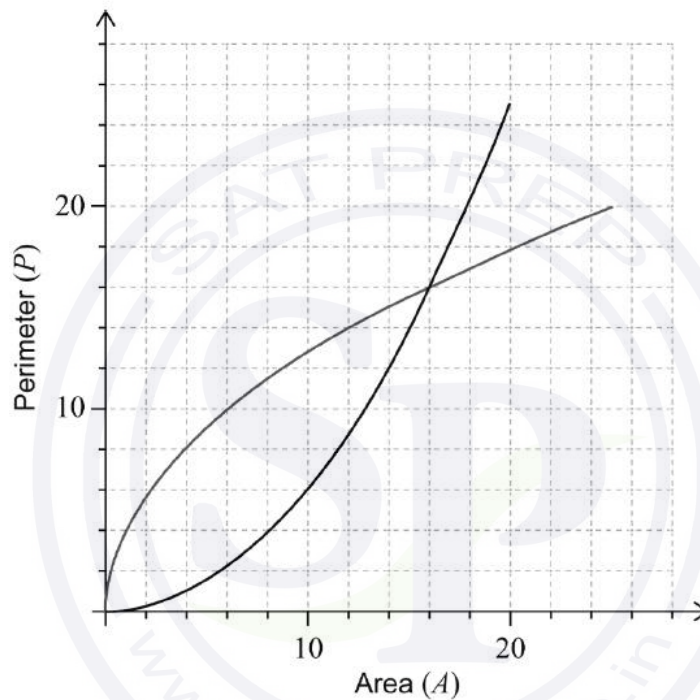
(b)  $n = 20$

**A1**

**Note:** Follow through from part (a).

**[1 mark]**

(c)



**(M1)A1A1**

**Note:** Award **(M1)** for reflection in the line  $P = A$ , award **A1** for endpoint at  $(20, 25)$ , award **A1** for passing through  $(16, 16)$ .

**[3 marks]**

(d) when the perimeter is 8, the area is 4

**A1**  
**[1 mark]**

**Total [6 marks]**

### Question 3

(a) 3

A1

Note: Accept (3, 0) seen.

[1 mark]

(b) METHOD 1

$$0 = 4a - 2b + c, 0 = 9a + 3b + c, -\frac{25}{2} = \frac{1}{4}a + \frac{1}{2}b + c$$

(M1)(A1)

(i) 2

A1

(ii) -2

A1

(iii) -12

A1

Note: Award the (M1)(A1) if at least one correct value is seen.  
Do not apply FT from part (a) if workings are not shown.

METHOD 2

$$-12.5 = a(0.5 + 2)(0.5 - 3)$$

(M1)

(i)  $a = 2$

A1

$$0 = 2 \times (3)^2 + 3b + c$$

$$0 = 2 \times (-2)^2 + (-2)b + c$$

(M1)

(ii)  $b = -2$

A1

(iii)  $c = -12$

A1

[5 marks]

(c)  $x = 0.5$

A1

Note: Do not FT from their part (b), this is a contradiction with the diagram.

[1 mark]

Total [7 marks]

### Question 4

(a)  $(f(-7) =) 8$  and  $(f(7) =) 1$

(A1)

range is  $f(x) \leq 1, f(x) \geq 8$

A1A1

**Note:** Award at most **A1A1A0** if strict inequalities are used.

[3 marks]

(b) **EITHER**

sketch of  $f$  and  $y = 0$  or sketch of  $f^{-1}$  and  $x = 0$

(M1)

**OR**

finding the correct expression of  $f^{-1}(x) = \frac{-2-5x}{x-2}$

(M1)

**OR**

$$f^{-1}(0) = \frac{-2-5(0)}{0-2}$$

(M1)

**OR**

$$f(x) = 0$$

(M1)

**THEN**

$$f^{-1}(0) = 1$$

A1

[2 marks]

Total [5 marks]

### Question 5

(a) (i) 23 mg

A1

(ii)  $1 - 0.85$  OR  $\frac{23 - 19.55}{23}$  OR  $0.15$   
15 (%)

(M1)

A1

[3 marks]

(b)  $23(0.85)^{10}$

(M1)

4.53 mg (4.52811...)

A1

[2 marks]

Total [5 marks]

**Question 6**

(a)  $I = \frac{k}{d^2}$

(M1)

$$4 = \frac{k}{1.5^2}$$

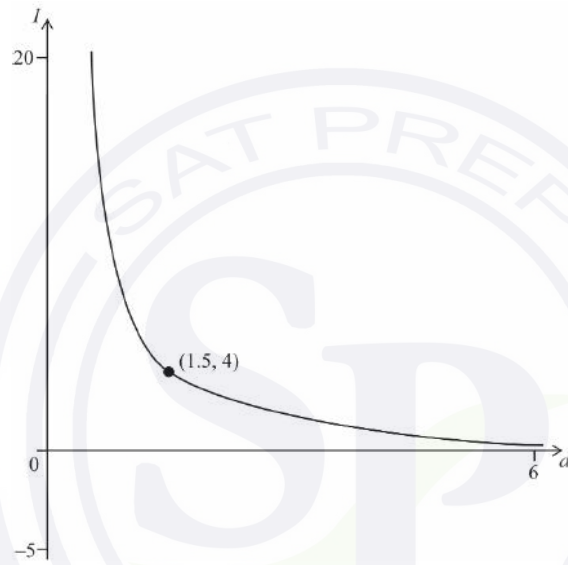
M1

$$I = \frac{9}{d^2}$$

AG

[2 marks]

(b)



A1A1

[2 marks]

(c)  $1.5 \times 10^{-6} \geq \frac{9}{d^2}$

(M1)

**Note:** Award (M1) for a correct inequality.

$$d \geq 2450 \text{ (m) (2449.48...)}$$

A1

**Note:** Award A0 for  $d = 2450$ .

[2 marks]

Total [6 marks]

### Question 7

(a) EITHER

$$50 = 100e^{-1 \times p} \quad \text{OR} \quad 0.5 = e^{-1 \times p}$$

(M1)

OR



(M1)

THEN

$$0.693 \quad (0.693147\dots, \ln 2)$$

A1

[2 marks]

(b)  $R(1.5) = 100e^{-0.693147\dots \times 1.5}$

(M1)

$$35.4(\%) \quad (35.3553\dots)$$

A1

[2 marks]

(c)  $R(t) > 0$  OR  $R(t)$  has a horizontal asymptote

R1

[1 mark]

(d) Award **A1** for **one** reasonable limitation of the domain:

A1

small values of  $t$  produce unrealistic results

$$R(0) = 100\%$$

large values of  $t$  are not possible

people do not live forever

model is not valid at small or large values of  $t$

*The reason should focus on the domain  $t \geq 0$ . Do not accept answers such as:*

recollection varies for different people

memories are discrete not continuous

the nature of the information will change how easily it is recalled

emotional/physical stress can affect recollection/concentration

**Note:** Do not accept  $t \geq 0$  as this is a limitation that has been given in the question.

[1 mark]

**Total [6 marks]**



### Question 8

(a)  $L(40) = 1.50 \times 40 - 5$

(M1)

$= \$ 55$

A1

[2 marks]

(b)  $70 = 1.50x - 5$

(M1)

$(x =) 50$  litres

A1

[2 marks]

(c)  $1.30x$

(A1)

$1.30x < 1.50x - 5$

(M1)

**Note:** Award **M1** for a graph showing two intersecting linear functions, provided one function has a  $y$ -intercept of 0 and the other function has a negative  $y$ -intercept.

(minimum value of  $k =$ ) 25

A1

**Note:** Accept  $x > 25$ .

[3 marks]

Total [7 marks]

### Question 9

(a)  $h(4) = \frac{640}{4^2} + 0.5$  OR  $h(14) = \frac{640}{14^2} + 0.5$

(M1)

**Note:** Award **(M1)** for substituting 4 or 14 into  $h$ . This can be implicit from seeing 3.77 (3.76530...) or 40.5.

$3.77 \leq h(x) \leq 40.5$  (3.76530...  $\leq h(x) \leq 40.5$ )

A1A1

**Note:** Award **A1** for both correct endpoints seen, **A1** for the endpoints in a correct interval.

[3 marks]

(b) (i)  $h(x) = 10$  OR  $h^{-1}(x) = \sqrt{\frac{640}{x-0.5}}$  OR  $h^{-1}(10) = \sqrt{\frac{640}{10-0.5}}$

(M1)

$(x =) 8.21$  cm (8.20782...)

A1

(ii) a tin that is 10 cm high will have a diameter of 8.21 cm (8.20782...)

A1

**Note:** Condone a correct answer expressed as the converse.

(iii)  $4 \leq h^{-1} \leq 14$

A1

**Note:** Accept  $4 \leq y \leq 14$ . Do not **FT** in this part.

[4 marks]

Total: [7 marks]

### Question 10

(a)  $(A=)$  112

**A1**

[1 mark]

(b)  $112e^{5k} = 360$

**(M1)**

**Note:** Award **(M1)** for their correct equation.

**EITHER**

graph of  $y = 112e^{5x}$  and  $y = 360$  with indication of point of intersection

**(M1)**

**OR**

$$(k =) \frac{1}{5} \ln\left(\frac{360}{112}\right)$$

**(M1)**

**Note:** Award **(M1)** for correct rearranging and use of log.

**THEN**

$$(k =) 0.234 \text{ (0.233521...)}$$

**A1**

**Note:** Award **(M1)(M1)(A0)** for 0.233.

[3 marks]  
Total: [4 marks]

### Question 11

(a) 1.2 metres

**A1**

[1 mark]

(b)  $-4.8t^2 + 21t + 1.2 = 0$

$(t =) 4.43 \text{ s (4.431415... s)}$

**(M1)**

**A1**

**Note:** If both values for  $t$  are seen do not award the **A1** mark unless the negative is explicitly excluded.

[2 marks]

(c)  $0 \leq t \leq 4.43$  **OR**  $[0, 4.43]$

**A1A1**

**Note:** Award **A1** for correct endpoints and **A1** for expressing answer with correct notation. Award at most **A1A0** for use of  $x$  instead of  $t$ .

[2 marks]  
[Total 5 marks]



### Question 12

(a)  $(x =) -\frac{4.48}{2(-1.6)}$  OR coordinates of maximum point (1.4, 3.136)

(M1)

$x = 1.4$

A1

[2 marks]

(b) **METHOD 1**

the cart is centred in the archway when it is between

$x = 0.6$  and  $x = 2.2$ ,

where  $y \geq 2.112$  (m) (which is greater than 2)

A1

R1

the archway is tall enough for the crate

A1

**Note:** Do not award R0A1.

**METHOD 2**

the height of the archway is greater or equal to 2.0 between

$x = 0.557385\dots$  and  $x = 2.24261\dots$

width of this section of archway =

$(2.24261\dots - 0.557385\dots) = 1.68522\dots$  (m) (which is greater than 1.6)

A1

R1

the archway is wide enough for the crate

A1

**Note:** Do not award R0A1.

[3 marks]

Total [5 marks]

### Question 13

(a)  $71e^{-0.0514(16)} + 23$

(M1)

$54.2$  °C (54.1956...)

A1

[2 marks]

(b)  $T = 23$

A1

**Note:** Condone  $y = 23$ .

[1 mark]

(c)  $23$  °C

A1

[1 mark]

(d)  $50 = 71e^{-0.0514(k)} + 23$

(M1)

$k = 18.8 \left( \frac{-5000}{257} \ln \left( \frac{27}{71} \right), 18.8101\dots \right)$

A1

[2 marks]

Total [6 marks]

**Question 14**

(a) 1.8 (m)

**A1**  
**[1 mark]**

(b) **EITHER**

$$\frac{-10.8}{2(-3.6)}$$

**(M1)**

**OR**

$$-7.2(t) + 10.8 = 0$$

**(M1)**

**OR**

sketch indicating maximum

**(M1)**

**THEN**

(t =) 1.5 seconds

**A1**  
**[2 marks]**

(c) **EITHER**

$$0 = -3.6t^2 + 10.8t + 1.8$$

**(M1)**

**OR**

sketch indicating a root

**(M1)**

**THEN**

(t =) 3.16 seconds (3.15831...)

**A1**  
**[2 marks]**  
**Total [5 marks]**

