Subject – Math AI(Standard Level)
Topic - Function
Year - May 2021 – Nov 2022
Paper -1
Questions

Question 1

[Maximum mark: 6]

Professor Vinculum investigated the migration season of the Bulbul bird from their natural wetlands to a warmer climate.

He found that during the migration season their population, P could be modelled by $P = 1350 + 400(1.25)^{-t}$, $t \geq 0$, where t is the number of days since the start of the migration season.

(a) Find the population of the Bulbul birds,

(i) at the start of the migration season.

(ii) in the wetlands after 5 days. [3]

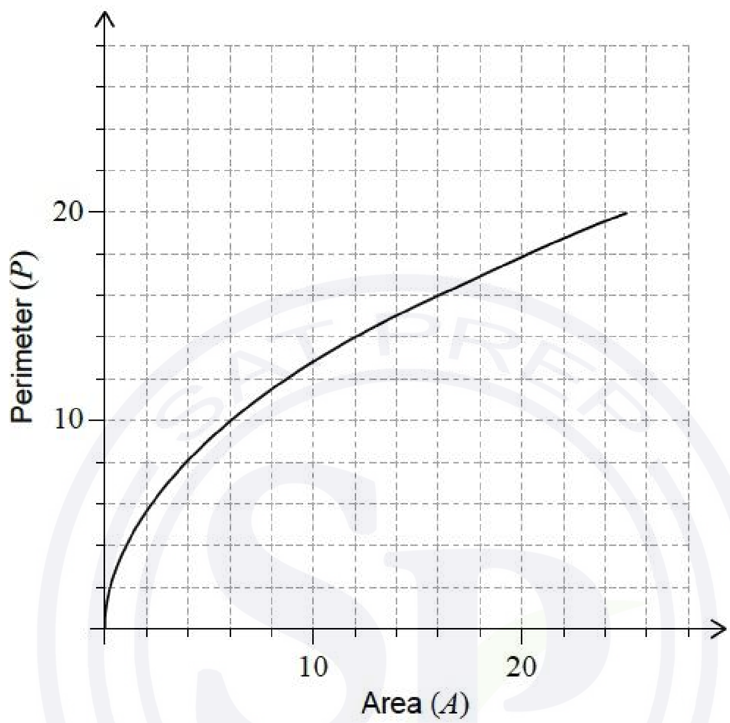
(b) Calculate the time taken for the population to decrease below 1400. [2]

(c) According to this model, find the smallest possible population of Bulbul birds during the migration season. [1]

Question 2

[Maximum mark: 6]

The perimeter of a given square P can be represented by the function $P(A) = 4\sqrt{A}$, $A \geq 0$, where A is the area of the square. The graph of the function P is shown for $0 \leq A \leq 25$.



(a) Write down the value of $P(25)$. [1]

The range of $P(A)$ is $0 \leq P(A) \leq n$.

(b) Hence write down the value of n. [1]

(c) On the axes above, draw the graph of the inverse function, P^{-1}. [3]

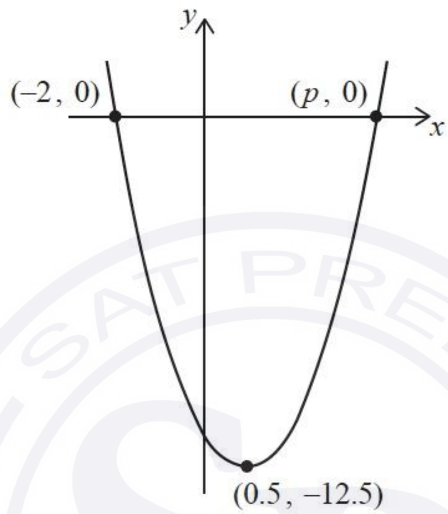
(d) In the context of the question, explain the meaning of $P^{-1}(8) = 4$. [1]

Question 3

[Maximum mark: 7]

Consider the function $f(x) = ax^2 + bx + c$. The graph of $y = f(x)$ is shown in the diagram. The vertex of the graph has coordinates $(0.5, -12.5)$. The graph intersects the x-axis at two points, $(-2, 0)$ and $(p, 0)$.

diagram not to scale



(a) Find the value of p. [1]

(b) Find the value of

(i) a.

(ii) b.

(iii) c. [5]

(c) Write down the equation of the axis of symmetry of the graph. [1]

Question 4

[Maximum mark: 5]

A function is defined by $f(x) = 2 - \frac{12}{x+5}$ for $-7 \le x \le 7$, $x \ne -5$.

(a) Find the range of f. [3]

(b) Find the value of $f^{-1}(0)$. [2]

Question 5

[Maximum mark: 5]

The amount, in milligrams, of a medicinal drug in the body t hours after it was injected is given by $D(t) = 23(0.85)^t$, $t \geq 0$. Before this injection, the amount of the drug in the body was zero.

(a) Write down

(i) the initial dose of the drug.

(ii) the percentage of the drug that leaves the body each hour. [3]

(b) Calculate the amount of the drug remaining in the body 10 hours after the injection. [2]

Question 6

[Maximum mark: 6]

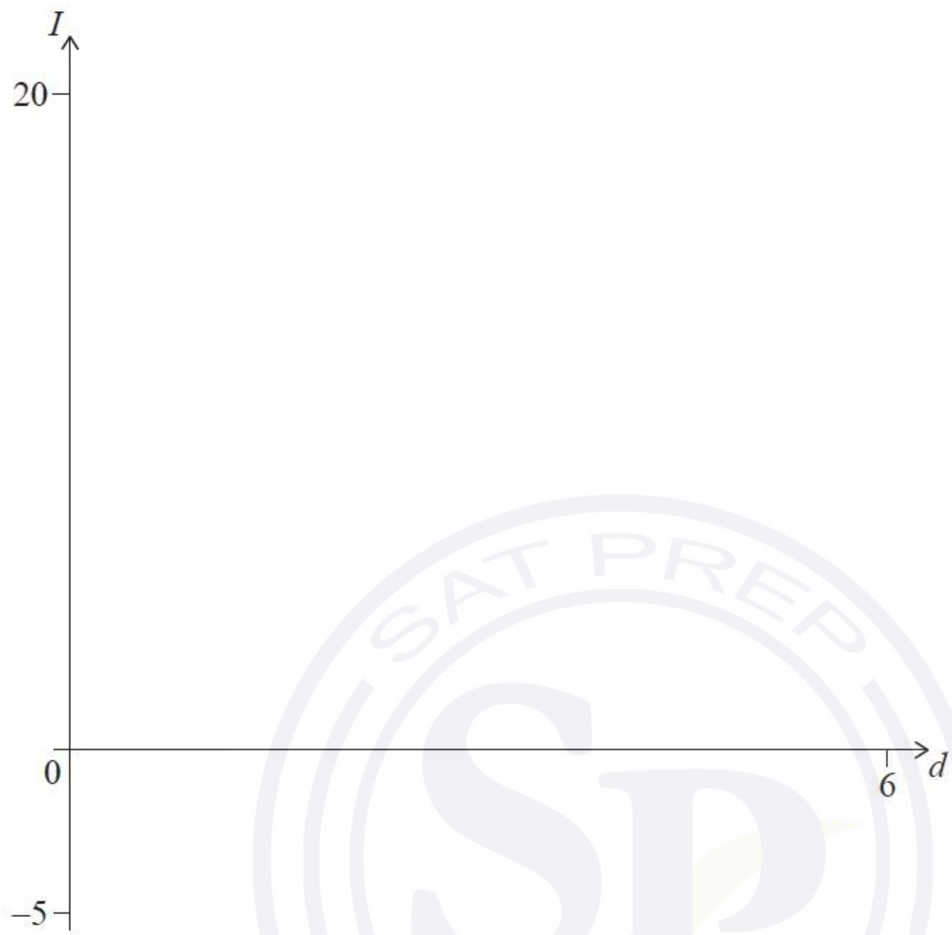
If a shark is spotted near to Brighton beach, a lifeguard will activate a siren to warn swimmers.

The sound intensity, I, of the siren varies inversely with the square of the distance, d, from the siren, where $d > 0$.

It is known that at a distance of 1.5 metres from the siren, the sound intensity is 4 watts per square metre $(\mathrm{W\,m^{-2}})$.

(a) Show that $I = \dfrac{9}{d^2}$. [2]

(b) Sketch the curve of I on the axes below showing clearly the point $(1.5, 4)$. [2]



Whilst swimming, Scarlett can hear the siren only if the sound intensity at her location is greater than $1.5 \times 10^{-6}\,\mathrm{Wm}^{-2}$.

(c) Find the values of d where Scarlett cannot hear the siren. [2]

Question 7

[Maximum mark: 6]

Professor Wei observed that students have difficulty remembering the information presented in his lectures.

He modelled the percentage of information retained, R, by the function $R(t) = 100\,e^{-pt},\ t \geq 0$, where t is the number of days after the lecture.

He found that 1 day after a lecture, students had forgotten 50% of the information presented.

(a) Find the value of p. [2]

(b) Use this model to find the percentage of information retained by his students 36 hours after Professor Wei's lecture. [2]

Based on his model, Professor Wei believes that his students will always retain some information from his lecture.

(c) State a mathematical reason why Professor Wei might believe this. [1]

(d) Write down one possible limitation of the **domain** of the model. [1]

Question 8

[Maximum mark: 7]

The price of gas at Leon's gas station is $\$1.50$ per litre. If a customer buys a minimum of 10 litres, a discount of $\$5$ is applied.

This can be modelled by the following function, L, which gives the total cost when buying a minimum of 10 litres at Leon's gas station.

$$L(x) = 1.50x - 5,\ x \geq 10$$

where x is the number of litres of gas that a customer buys.

(a) Find the total cost of buying 40 litres of gas at Leon's gas station. [2]

(b) Find $L^{-1}(70)$. [2]

The price of gas at Erica's gas station is $\$1.30$ per litre. A customer must buy a minimum of 10 litres of gas. The total cost at Erica's gas station is cheaper than Leon's gas station when $x > k$.

(c) Find the minimum value of k. [3]

Question 9

[Maximum mark: 7]

Let the function $h(x)$ represent the height in centimetres of a cylindrical tin can with diameter x cm.

$$h(x) = \frac{640}{x^2} + 0.5 \text{ for } 4 \le x \le 14.$$

(a) Find the range of h. [3]

The function h^{-1} is the inverse function of h.

(b) (i) Find $h^{-1}(10)$.

(ii) In the context of the question, interpret your answer to part (b)(i).

(iii) Write down the range of h^{-1}. [4]

Question 10

[Maximum mark: 4]

Natasha carries out an experiment on the growth of mould. She believes that the growth can be modelled by an exponential function

$$P(t) = Ae^{kt},$$

where P is the area covered by mould in mm^2, t is the time in days since the start of the experiment and A and k are constants.

The area covered by mould is $112\,\text{mm}^2$ at the start of the experiment and $360\,\text{mm}^2$ after 5 days.

(a) Write down the value of A. [1]

(b) Find the value of k. [3]

Question 11

[Maximum mark: 5]

The height of a baseball after it is hit by a bat is modelled by the function

$$h(t) = -4.8t^2 + 21t + 1.2$$

where $h(t)$ is the height in metres above the ground and t is the time in seconds after the ball was hit.

(a) Write down the height of the ball above the ground at the instant it is hit by the bat. [1]

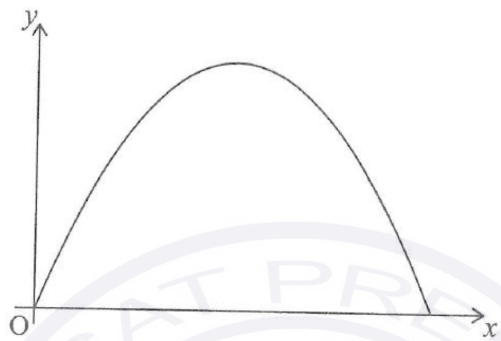
(b) Find the value of t when the ball hits the ground. [2]

(c) State an appropriate domain for t in this model. [2]

Question 12

[Maximum mark: 5]

The cross-section of an arched entrance into the ballroom of a hotel is in the shape of a parabola. This cross-section can be modelled by part of the graph $y = -1.6x^2 + 4.48x$, where y is the height of the archway, in metres, at a horizontal distance, x metres, from the point O, in the bottom corner of the archway.



(a) Determine an equation for the axis of symmetry of the parabola that models the archway. [2]

To prepare for an event, a square-based crate that is $1.6\,\text{m}$ wide and $2.0\,\text{m}$ high is to be moved through the archway into the ballroom. The crate must remain upright while it is being moved.

(b) Determine whether the crate will fit through the archway. Justify your answer. [3]

Question 13

[Maximum mark: 6]

Celeste heated a cup of coffee and then let it cool to room temperature. Celeste found the coffee's temperature, T, measured in $^\circ\text{C}$, could be modelled by the following function,

$$T(t) = 71\,\text{e}^{-0.0514t} + 23,\ t \geq 0,$$

where t is the time, in minutes, after the coffee started to cool.

(a) Find the coffee's temperature 16 minutes after it started to cool. [2]

The graph of T has a horizontal asymptote.

(b) Write down the equation of the horizontal asymptote. [1]

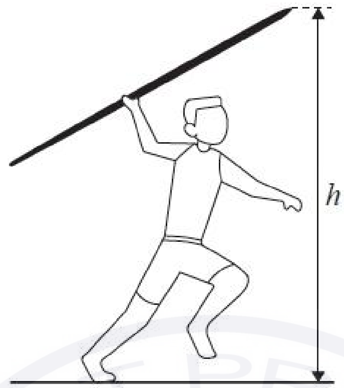
(c) Write down the room temperature. [1]

(d) Given that $T^{-1}(50) = k$, find the value of k. [2]

Question 14

[Maximum mark: 5]

DeVaughn throws a javelin in a school track and field competition.



The height, h, of the front tip of the javelin above the ground, in metres, is modelled by the following quadratic function,

$$h(t) = -3.6t^2 + 10.8t + 1.8,\ t \geq 0$$

where t is the time in seconds after the javelin is thrown.

(a) Write down the height of the front tip of the javelin at the time it is thrown. [1]

(b) Find the value of t when the front tip of the javelin reaches its maximum height. [2]

(c) Find the value of t when the front tip of the javelin strikes the ground. [2]