Subject – Math AI(Standard Level)
Topic - Geometry and Trigonometry
Year - May 2021 – Nov 2022
Paper -1
Answers

Question 1

(a) $\frac{3-1}{7-3}$ *(M1)*

$= 0.5$ ***A1***

[2 marks]

(b) $y - 2 = -2(x - 5)$ ***(A1)(M1)***

Note: Award ***(A1)*** for their -2 seen, award ***(M1)*** for the correct substitution of $(5, 2)$ and their normal gradient in equation of a line.

$2x + y - 12 = 0$ ***A1***

[3 marks]

(c) every point in the cell is closer to E than any other snow shelter ***A1***

[1 mark]

Total [6 marks]

Question 2

(a) $135^\circ \times \frac{12\pi}{360^\circ}$ ***(M1)(A1)***

14.1 (m) (14.1371…) ***A1***

[3 marks]

(b) evidence of splitting region into two areas ***(M1)***

$135^\circ \times \frac{\pi 6^2}{360^\circ} - \frac{6 \times 6 \times \sin 135^\circ}{2}$ ***(M1)(M1)***

$42.4115\ldots - 12.7279\ldots$

$29.7\ \text{m}^2$ (29.6835…) ***A1***

[4 marks]

Total [7 marks]

Question 3

(a) gradient $AB = \frac{4}{12} \left(\frac{1}{3}\right)$ *(A1)*

midpoint AB: $(8, 22)$ *(A1)*

gradient of bisector $= -\frac{1}{\text{gradient } AB} = -3$ *(M1)*

perpendicular bisector: $22 = -3 \times 8 + b$ **OR** $(y - 22) = -3(x - 8)$ *(M1)*

perpendicular bisector: $y = -3x + 46$ ***A1***

[5 marks]

(b) attempt to solve simultaneous equations *(M1)*

$x + 4 = -3x + 46$

$(10.5, 14.5)$ ***A1***

[2 marks]

Total [7 marks]

Question 4

(a) 25° ***A1***

[1 mark]

(b) $AC = \frac{380}{\tan 25^\circ}$ **OR** $AC = \sqrt{\left(\frac{380}{\sin 25^\circ}\right)^2 - 380^2}$ **OR** $\frac{380}{\sin 25^\circ} = \frac{AC}{\sin 65^\circ}$ *(M1)*

$AC = 815 \text{ m} \ (814.912\ldots)$ ***A1***

[2 marks]

(c) **METHOD 1**

attempt to find AB *(M1)*

$AB = \frac{380}{\tan 40^\circ}$

$= 453 \text{ m} \ (452.866\ldots)$ *(A1)*

$BC = 814.912\ldots - 452.866\ldots$

$= 362 \text{ m} \ (362.046\ldots)$ ***A1***

(d) $362.046\ldots \times 4$

$= 1450 \text{ m h}^{-1} \ (1448.18\ldots)$ ***A1***

[1 mark]

Total [7 marks]

Question 5

$2\times 90\times 34\ (=6120)$ **AND** $2\times 42\times 34\ (=2856)$ ***(A1)***

$90\times 42\ (=3780)$ ***(A1)***

$r=21$ ***(A1)***

$\pi\times 21^2\ (=441\pi,\ 1385.44...)$ ***(M1)***

use of curved surface area formula ***(M1)***

$21\pi\times 90\ (=1890\pi,\ 5937.61...)$ ***(A1)***

$20100\ \text{cm}^2\ (20079.0\ldots)$ ***A1***

Total [7 marks]

Question 6

(a) attempt at substitution into 3D distance formula ***(M1)***

$\text{AB}=\sqrt{(140-20)^2+(15-5)^2+250^2}\quad\left(=\sqrt{77\,000}\right)$

$=277\ \text{m}\ (10\sqrt{770},\ 277.488...)$ ***A1***

[2 marks]

(b) attempt at substitution in the midpoint formula ***(M1)***

$\left(\frac{140+20}{2},\frac{15+5}{2},\frac{0+250}{2}\right)$

$(80,\ 10,\ 125)$ ***A1***

[2 marks]

(c) $125\ \text{m}$ ***A1***

[1 mark]

Total [5 marks]

Question 7

attempt to find any relevant maximum value ***(M1)***

largest sides are 56.5 and 82.5 ***(A1)***

smallest possible angle is 102.5 ***(A1)***

attempt to substitute into area of a triangle formula ***(M1)***

$\frac{1}{2}\times 56.5\times 82.5\times\sin(102.5^\circ)$

$=2280(\text{m}^2)\ (2275.37...)$ ***A1***

Total [5 marks]

Question 8

(a) every point in the shaded region is closer to tower T4 **R1**

Note: Specific reference must be made to the closeness of tower T4.

[1 mark]

(b) $(-9, 1)$ **A1A1**

Note: Award ***A1*** for each correct coordinate. Award at most ***A0A1*** if parentheses are missing.

[2 marks]

(c) correct use of gradient formula ***(M1)***

e.g. $(m=)\ \frac{5-3}{-9--13}\ \left(=\frac{1}{2}\right)$

taking negative reciprocal of **their** m (at any point) ***(M1)***

edge gradient $=-2$ ***A1***

[3 marks]

Total [6 marks]

Question 9

(a) $\frac{1}{2}\times 4\times \pi\times 6^2+\pi\times 6^2$ **OR** $3\times \pi\times 6^2$ ***(M1)(A1)(M1)***

Note: Award ***M1*** for use of surface area of a sphere formula (or curved surface area of a hemisphere), ***A1*** for substituting correct values into hemisphere formula, ***M1*** for adding the area of the circle.

$=339\ \text{mm}^2\ (108\pi,\ 339.292...)$ ***A1***

[4 marks]

(b) $\frac{339.292...}{240}$ ***(M1)***

$=1.41\ \text{(g)}\ \left(\frac{9\pi}{20},\ 0.45\pi,\ 1.41371...\right)$ ***A1***

[2 marks]

Total [6 marks]

Question 10

(a) (i) 13π cm **A1**

Note: Answer must be in terms of π.

(ii) **METHOD 1**

$\frac{\theta}{360} \times 2\pi(18) = 13\pi$ **OR** $\frac{\theta}{360} \times 2\pi(18) = 40.8407...$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into length of an arc formula.

$(\theta =)\ 130°$ ***A1***

METHOD 2

$\frac{\theta}{360} \times \pi \times 18^2 = \pi \times 6.5 \times 18$ ***(M1)***

$(\theta =)\ 130°$ ***A1***

[3 marks]

(b) **EITHER**

$\frac{130}{360} \times \pi(18)^2$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into area of a sector formula.

OR

$\pi(6.5)(18)$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into curved area of a cone formula.

THEN

(Area =) $368\ \text{cm}^2$ $(367.566...,117\pi)$ ***A1***

Note: Allow ***FT*** from their part (a)(ii) even if their angle is not obtuse.

[2 marks]
Total: [5 marks]

Question 11

(a) (the best placement is either point P or point Q)

attempt at using the distance formula *(M1)*

$\text{AP} = \sqrt{(10-6)^2 + (6-2)^2}$ **OR**

$\text{BP} = \sqrt{(10-14)^2 + (6-2)^2}$ **OR**

$\text{DP} = \sqrt{(10-10.8)^2 + (6-11.6)^2}$ **OR**

$\text{BQ} = \sqrt{(13-14)^2 + (7-2)^2}$ **OR**

$\text{CQ} = \sqrt{(13-18)^2 + (7-6)^2}$ **OR**

$\text{DQ} = \sqrt{(13-10.8)^2 + (7-11.6)^2}$

(AP or BP or DP =) $\sqrt{32} = 5.66$ (5.65685...) **AND**

(BQ or CQ or DQ =) $\sqrt{26} = 5.10$ (5.09901...) ***A1***

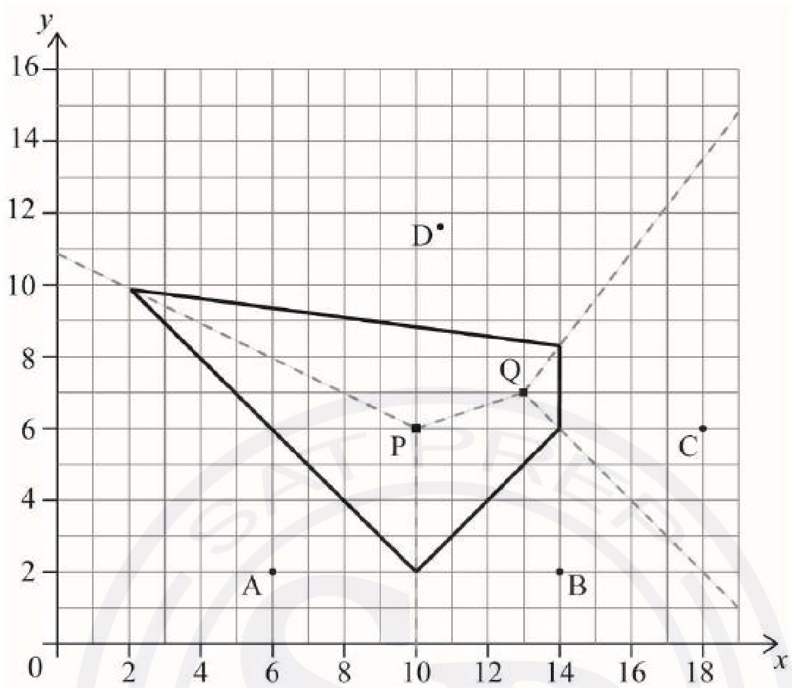
$\sqrt{32} > \sqrt{26}$ **OR** AP (or BP or DP) is greater than BQ (or CQ or DQ) ***A1***

point P is the furthest away ***AG***

[3 marks]

(b) (i) $x = 14$ ***A1***

(ii)



A1A1

Note: Award ***A1*** for each correct straight line. Do not ***FT*** from their part (b)(i).

[3 marks]
Total: [6 marks]

Question 12

(a) $m = \frac{6-0}{4-2} = 3$ **(M1)A1**

[2 marks]

(b) $(m =)\ -\frac{1}{3}\ (-0.333,\ -0.333333...)$ **A1**

[1 mark]

(c) an equation of line with a correct intercept and either of their gradients from (a) or (b) **(M1)**

e.g. $y = -\frac{1}{3}x + 4$ **OR** $y - 4 = -\frac{1}{3}(x - 0)$

Note: Award ***(M1)*** for substituting either of their gradients from parts (a) or (b) and point B or $(3, 3)$ into equation of a line.

$x + 3y - 12 = 0$ or any integer multiple **A1**

[2 marks]

(d) $(x =)\ 12$ **A1**

[1 mark]
Total: [6 marks]

Question 13

(a) attempt at substituting the cosine rule formula **(M1)**

$$\cos\theta = \frac{1660^2 + 1550^2 - 1670^2}{2(1660)(1550)}$$ **(A1)**

$(\theta =)\ 62.6^\circ\ \ (62.5873\ldots)$ (accept 1.09 rad $(1.09235\ldots)$) **A1**

[3 marks]

(b) correctly substituted area of triangle formula **(M1)**

$A = \frac{1}{2}(1660)(1550)\sin(62.5873\ldots)$

$(A =)\ 1140000\ \ (1.14 \times 10^6,\ 1142043.327\ldots)\ \text{km}^2$ **A1**

Note: Accept $1150000\ \ (1.15 \times 10^6,\ 1146279.893\ldots)\ \text{km}^2$ from use of 63°. Other angles and their corresponding sides may be used.

[2 marks]
Total: [5 marks]

Question 14

(a) midpoint $(1, 2.5)$ **A1**

$m_{AB} = \frac{6-(-1)}{8-(-6)} = \frac{1}{2}$ ***(M1)A1***

Note: Accept equivalent gradient statements including using midpoint.

$m_{\perp} = -2$ ***M1***

Note: Award ***M1*** for finding the negative reciprocal of their gradient.

$y - 2.5 = -2(x-1)$ **OR** $y = -2x + \frac{9}{2}$ **OR** $4x + 2y - 9 = 0$ ***A1***

[5 marks]

(b) substituting $x = -6$ into their equation from part (a) ***(M1)***

$y = -2(-6) + \frac{9}{2}$

$y = 16.5$ ***A1***

Note: Award ***M1A0*** for $(-6, 16.5)$ as their final answer.

[2 marks]
[Total 7 marks]

Question 15

(a) $\sqrt{3.2^2 + 4.5^2 + 5.8^2}$ ***(M1)***

$= 8.01\ (8.00812...)\ \text{m}$ ***A1***

[2 marks]

(b) $\text{F}\hat{\text{A}}\text{O} = \sin^{-1}\left(\frac{5.8}{8.00812...}\right)$ **OR** $\cos^{-1}\left(\frac{5.52177...}{8.00812...}\right)$ **OR** $\tan^{-1}\left(\frac{5.8}{5.52177...}\right)$ ***(M1)***

$46.4^\circ\ (46.4077...^\circ)$ ***A1***

[2 marks]
[Total 4 marks]

Question 16

height of triangle at roof $= 1.35 - 0.9 = 0.45$ **(A1)**

slant height $= \sqrt{0.45^2 + 0.45^2}$ **OR** $\sin(45^\circ) = \dfrac{0.45}{\text{slant height}}$ **(M1)**

$= \sqrt{0.405}$ $(0.636396..., 0.45\sqrt{2})$ **A1**

area of one rectangle on roof $= \sqrt{0.405} \times 0.9$ $(= 0.572756...)$ **M1**

area painted $= (2 \times \sqrt{0.405} \times 0.9 = 2 \times 0.572756...)$

1.15 m^2 $(1.14551... \text{ m}^2, 0.81\sqrt{2} \text{ m}^2)$ **A1**

[Total 5 marks]

Question 17

(a) $\sin\theta = \dfrac{2.1}{2.8}$ **OR** $\tan\theta = \dfrac{2.1}{1.85202\ldots}$ ***(M1)***

$(\theta =)$ 48.6° $(48.5903\ldots^\circ)$ ***A1***

[2 marks]

(b) **METHOD 1**

$\sqrt{2.8^2 - 2.1^2}$ **OR** $2.8\cos(48.5903...)$ **OR** $\dfrac{2.1}{\tan(48.5903\ldots)}$ ***(M1)***

1.85 (m) $(1.85202\ldots)$ ***(A1)***

$(6.4 - 1.85202\ldots)$

4.55 m $(4.54797\ldots)$ ***(A1)***

$\sqrt{(4.54797\ldots)^2 + 2.1^2}$

5.01 m $(5.00939\ldots\text{m})$ ***A1***

METHOD 2

attempt to use cosine rule ***(M1)***

$(c^2 =)$ $2.8^2 + 6.4^2 - 2(2.8)(6.4)\cos(48.5903\ldots)$ ***(A1)(A1)***

$(c =)$ 5.01 m $(5.00939\ldots\text{m})$ ***A1***

[4 marks]

(c) camera 1 is closer to the cash register (than camera 2 and both cameras are at the same height on the wall) ***R1***

the larger angle of depression is from camera 1 ***A1***

[2 marks]

Total [8 marks]

Question 18

(a) attempt to substitute into length of arc formula *(M1)*

$\frac{140°}{360°}\times 2\pi\times 56$

137 cm $\left(136.833\ldots,\ \frac{392\pi}{9}\text{ cm}\right)$ ***A1***

[2 marks]

(b) subtracting two substituted area of sectors formulae *(M1)*

$\left(\frac{140°}{360°}\times\pi\times 56^2\right)-\left(\frac{140°}{360°}\times\pi\times 10^2\right)$ **OR** $\frac{140°}{360°}\times\pi\times\left(56^2-10^2\right)$ *(A1)*

3710 cm^2 (3709.17… cm^2) ***A1***

[3 marks]

Total [5 marks]

Question 19

(a) 78 *A1*

[1 mark]

(b) (i) 65 *A1*

(ii) **EITHER**

(period $=$) 16 (could be seen on sketch) *(M1)*

$b=\frac{2\pi}{16}$ **OR** $b=\frac{360^\circ}{16}$

$(b=)$ 0.393 $\left(0.392699\ldots, \frac{\pi}{8}\right)$ **OR** $(b=)$ 22.5° *A1*

OR

$143=65\sin(4b)+78$ *(M1)*

$(\sin(4b)=1)$

$\left(4b=\frac{\pi}{2}\right.$ **OR** $\left.4b=90^\circ\right)$

$(b=)$ 0.393 $\left(0.392699\ldots, \frac{\pi}{8}\right)$ **OR** $(b=)$ 22.5° *A1*

[3 marks]

(c) 13 *A1*

Note: Apply follow through marking only if their final answer is positive.

[1 mark]

(d) $(b=)$ 0.196 $\left(0.196349\ldots, \frac{\pi}{16}\right)$ **OR** $(b=)\,11.3^\circ$ (11.25°) *A1*

[1 mark]
Total [6 marks]

Question 20

(a) $\sin(\text{B}\hat{\text{S}}\text{K}) = \frac{218}{1200}$ **OR** $\frac{\sin(\text{B}\hat{\text{S}}\text{K})}{218} = \frac{\sin(90^\circ)}{1200}$ ***(M1)***

Note: Award ***M1*** for a correct trig formula. Accept other variables representing $\text{B}\hat{\text{S}}\text{K}$.

$(\text{B}\hat{\text{S}}\text{K} =)\ 10.5^\circ\ (10.4668...)$ ***A1***

Note: Award ***A1*** for the radian answer, $0.182681...$. Award ***M1A0*** if the candidate finds the correct angle of elevation but then uses it to find a complementary angle as their final answer.

[2 marks]

(b) $\text{SB}^2 + 218^2 = 1200^2$ **OR** $\cos(10.4668...) = \frac{\text{SB}}{1200}$ **OR** $\tan(10.4668...) = \frac{218}{\text{SB}}$ **OR**

$\frac{\text{BS}}{\sin(79.5331...^\circ)} = \frac{1200}{\sin(90^\circ)}$ ***(M1)***

$1180\ (\text{m})\ \left(\sqrt{1392476},\ 1180.03...\right)$ ***A1***

[2 marks]

(c) 1.18×10^3 ***A1A1***

Note: Award ***A1*** for 1.18 Award ***A1*** for 10^3 Accept their rounded answer to part (b). Award ***A0A0*** for answers of the type: 11.8×10^2.

[2 marks]
Total [6 marks]