

**Subject - Math AI(Standard Level)**  
**Topic - Geometry and Trigonometry**  
**Year - May 2021 - Nov 2022**  
**Paper -1**  
**Questions**

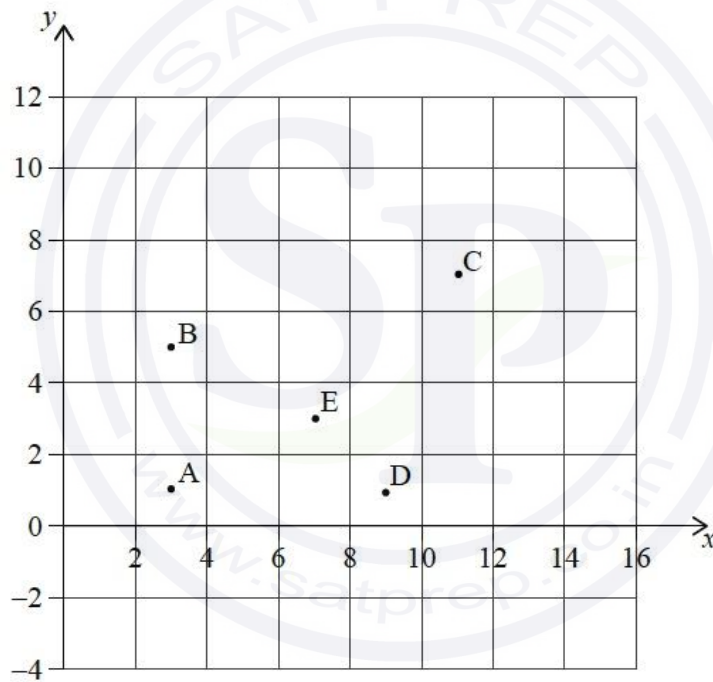
**Question 1**

[Maximum mark: 6]

Points  $A(3, 1)$ ,  $B(3, 5)$ ,  $C(11, 7)$ ,  $D(9, 1)$  and  $E(7, 3)$  represent snow shelters in the Blackburn National Forest. These snow shelters are illustrated in the following coordinate axes.

Horizontal scale: 1 unit represents 1 km.

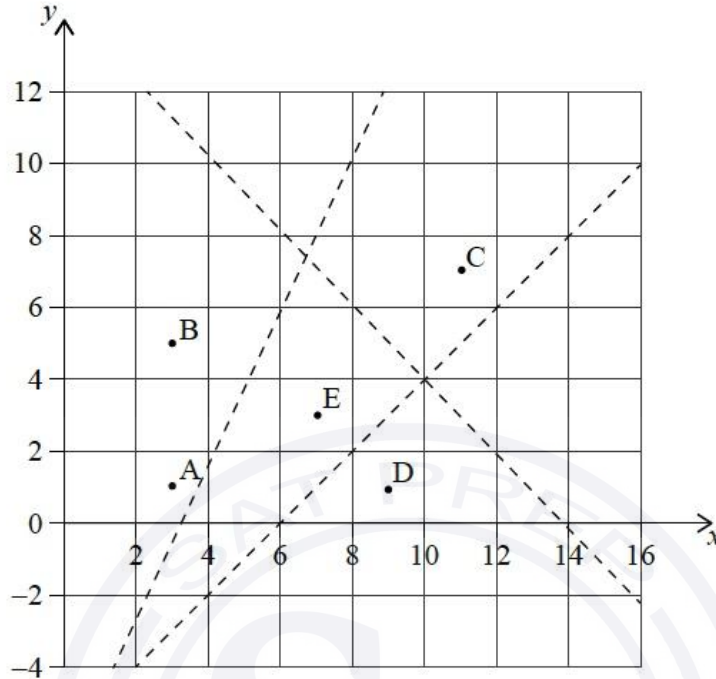
Vertical scale: 1 unit represents 1 km.



(a) Calculate the gradient of the line segment  $AE$ .

[2]

The Park Ranger draws three straight lines to form an incomplete Voronoi diagram.



- (b) Find the equation of the line which would complete the Voronoi cell containing site E. Give your answer in the form  $ax + by + d = 0$  where  $a, b, d \in \mathbb{Z}$ . [3]
- (c) In the context of the question, explain the significance of the Voronoi cell containing site E. [1]

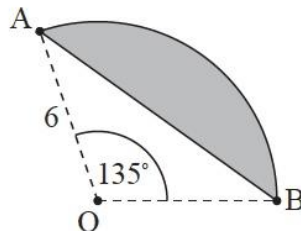
**Question 2**

[Maximum mark: 7]

A garden includes a small lawn. The lawn is enclosed by an arc AB of a circle with centre O and radius 6 m, such that  $\widehat{AOB} = 135^\circ$ . The straight border of the lawn is defined by chord [AB].

The lawn is shown as the shaded region in the following diagram.

diagram not to scale

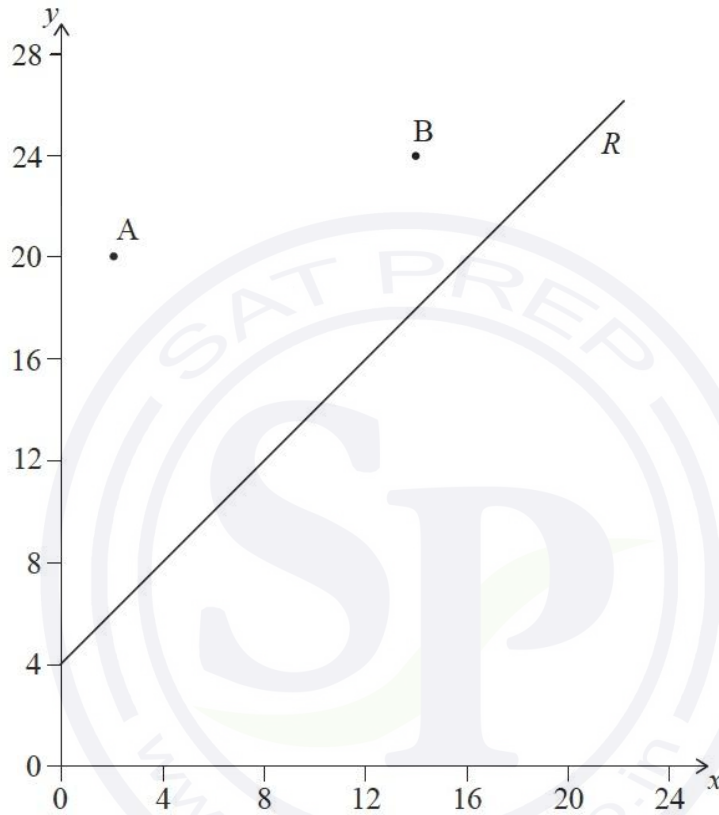


- (a) A footpath is to be laid around the curved side of the lawn. Find the length of the footpath. [3]
- (b) Find the area of the lawn. [4]

### Question 3

[Maximum mark: 7]

Two schools are represented by points  $A(2, 20)$  and  $B(14, 24)$  on the graph below. A road, represented by the line  $R$  with equation  $-x + y = 4$ , passes near the schools. An architect is asked to determine the location of a new bus stop on the road such that it is the same distance from the two schools.



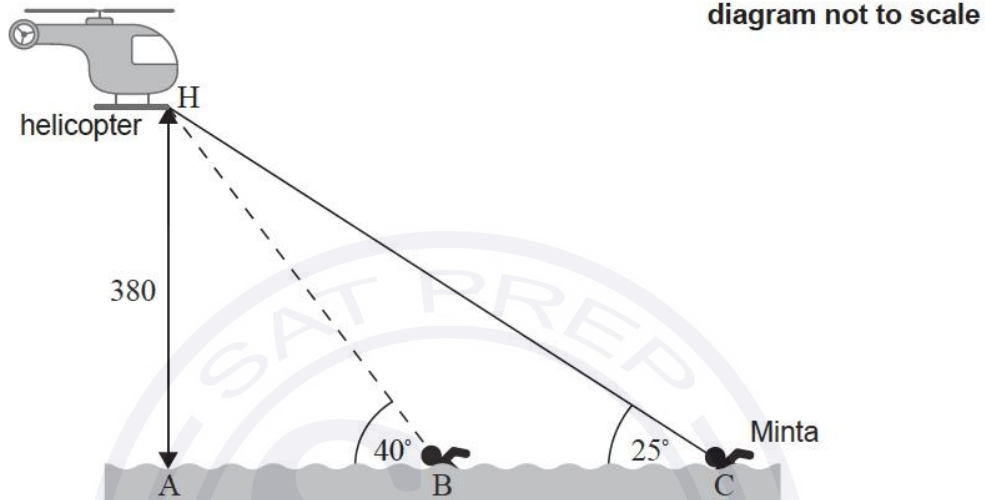
- (a) Find the equation of the perpendicular bisector of  $[AB]$ . Give your equation in the form  $y = mx + c$ . [5]
- (b) Determine the coordinates of the point on  $R$  where the bus stop should be located. [2]

#### Question 4

[Maximum mark: 7]

The diagram below shows a helicopter hovering at point H, 380 m vertically above a lake.

Point A is the point on the surface of the lake, directly below the helicopter.



Minta is swimming at a constant speed in the direction of point A. Minta observes the helicopter from point C as she looks upward at an angle of  $25^\circ$ . After 15 minutes, Minta is at point B and she observes the same helicopter at an angle of  $40^\circ$ .

- Write down the size of the angle of depression from H to C. [1]
- Find the distance from A to C. [2]
- Find the distance from B to C. [3]
- Find Minta's speed, in metres per hour. [1]

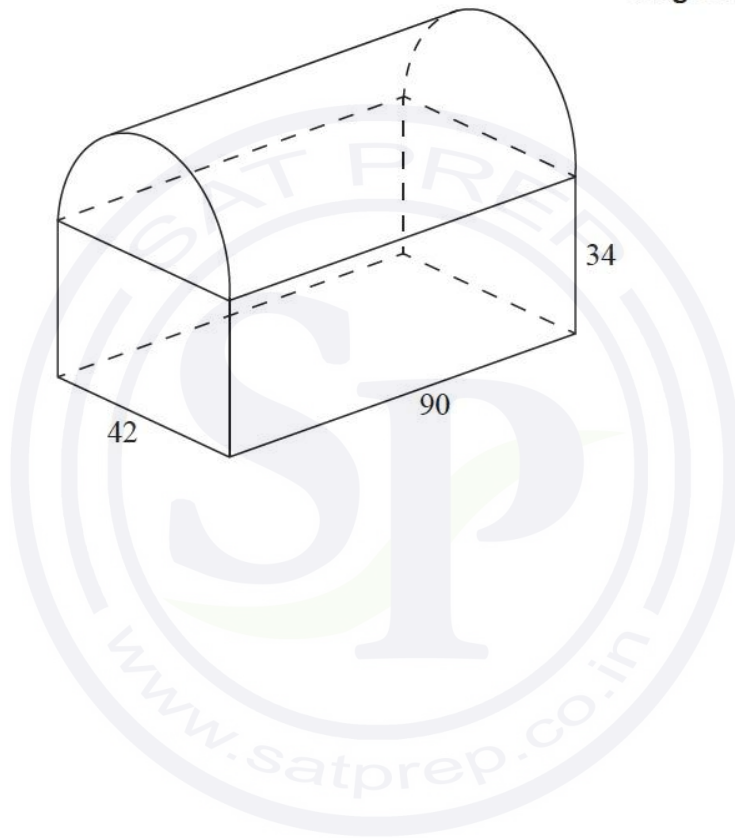
### Question 5

[Maximum mark: 7]

A storage container consists of a box of length 90 cm, width 42 cm and height 34 cm, and a lid in the shape of a half-cylinder, as shown in the diagram. The lid fits the top of the box exactly. The total exterior surface of the storage container is to be painted.

Find the area to be painted.

diagram not to scale

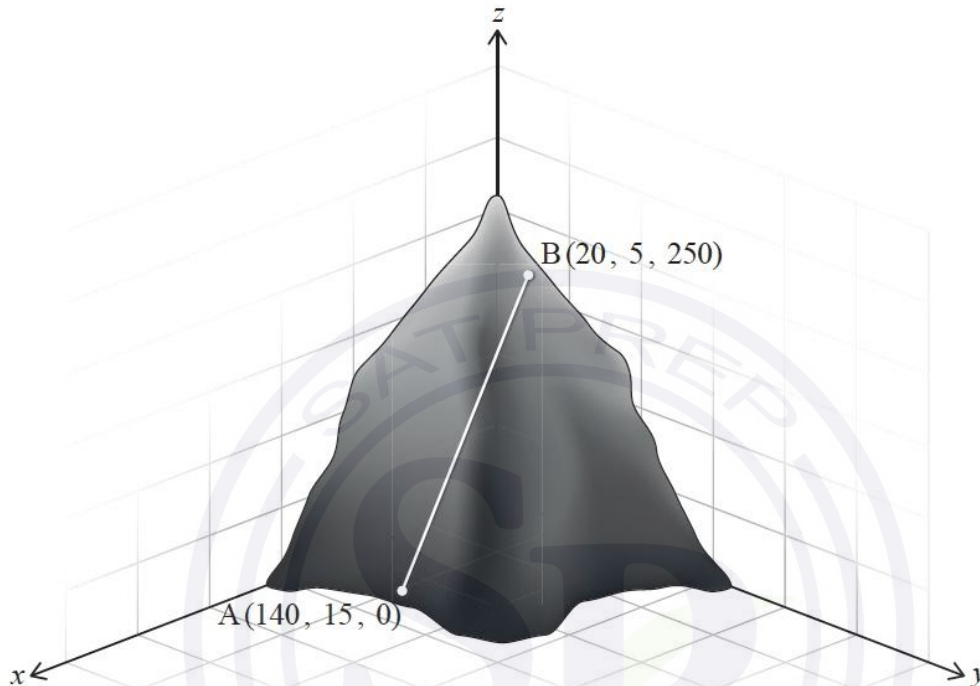


### Question 6

[Maximum mark: 5]

An inclined railway travels along a straight track on a steep hill, as shown in the diagram.

diagram not to scale



The locations of the stations on the railway can be described by coordinates in reference to  $x$ ,  $y$ , and  $z$ -axes, where the  $x$  and  $y$  axes are in the horizontal plane and the  $z$ -axis is vertical.

The ground level station A has coordinates  $(140, 15, 0)$  and station B, located near the top of the hill, has coordinates  $(20, 5, 250)$ . All coordinates are given in metres.

(a) Find the distance between stations A and B. [2]

Station M is to be built halfway between stations A and B.

(b) Find the coordinates of station M. [2]

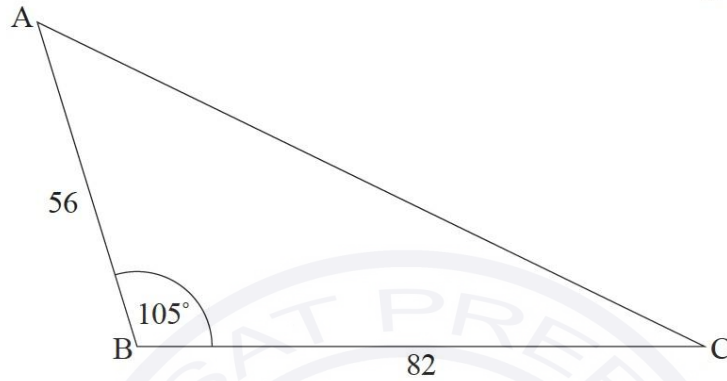
(c) Write down the height of station M, in metres, above the ground. [1]

### Question 7

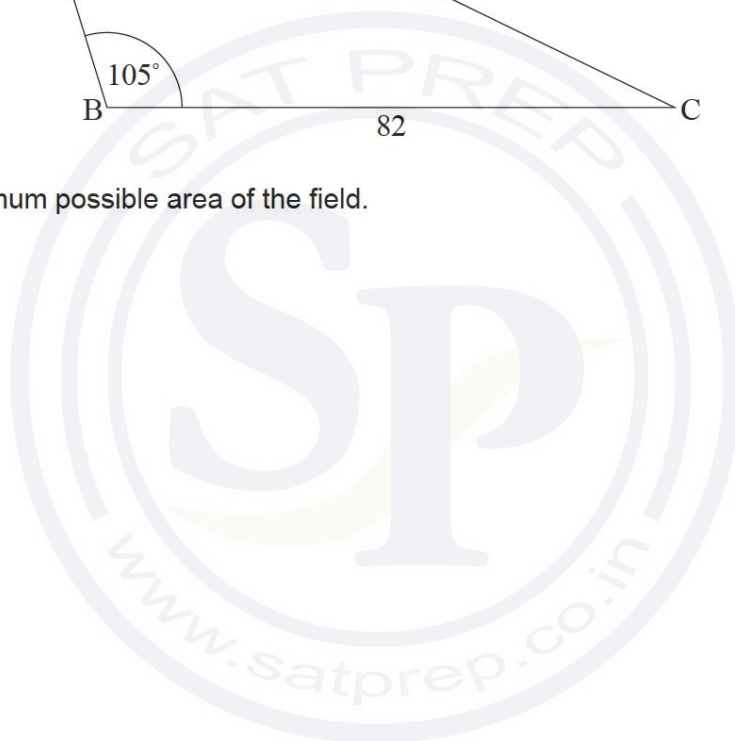
[Maximum mark: 5]

A triangular field ABC is such that  $AB = 56\text{m}$  and  $BC = 82\text{m}$ , each measured correct to the nearest metre, and the angle at B is equal to  $105^\circ$ , measured correct to the nearest  $5^\circ$ .

diagram not to scale



Calculate the maximum possible area of the field.



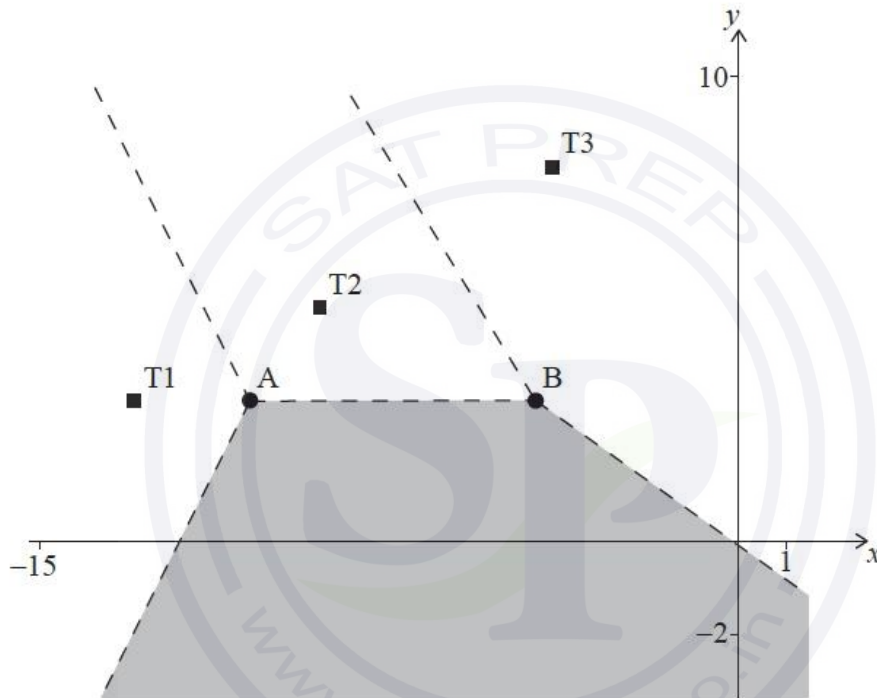
### Question 8

[Maximum mark: 6]

The Voronoi diagram below shows three identical cellular phone towers, T1, T2 and T3. A fourth cellular phone tower, T4 is located in the shaded region. The dashed lines in the diagram below represent the edges in the Voronoi diagram.

Horizontal scale: 1 unit represents 1 km.

Vertical scale: 1 unit represents 1 km.



Tim stands inside the shaded region.

- (a) Explain why Tim will receive the strongest signal from tower T4. [1]

Tower T2 has coordinates  $(-9, 5)$  and the edge connecting vertices A and B has equation  $y = 3$ .

- (b) Write down the coordinates of tower T4. [2]

Tower T1 has coordinates  $(-13, 3)$ .

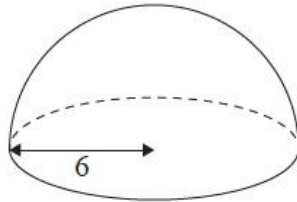
- (c) Find the gradient of the edge of the Voronoi diagram between towers T1 and T2. [3]



### Question 9

[Maximum mark: 6]

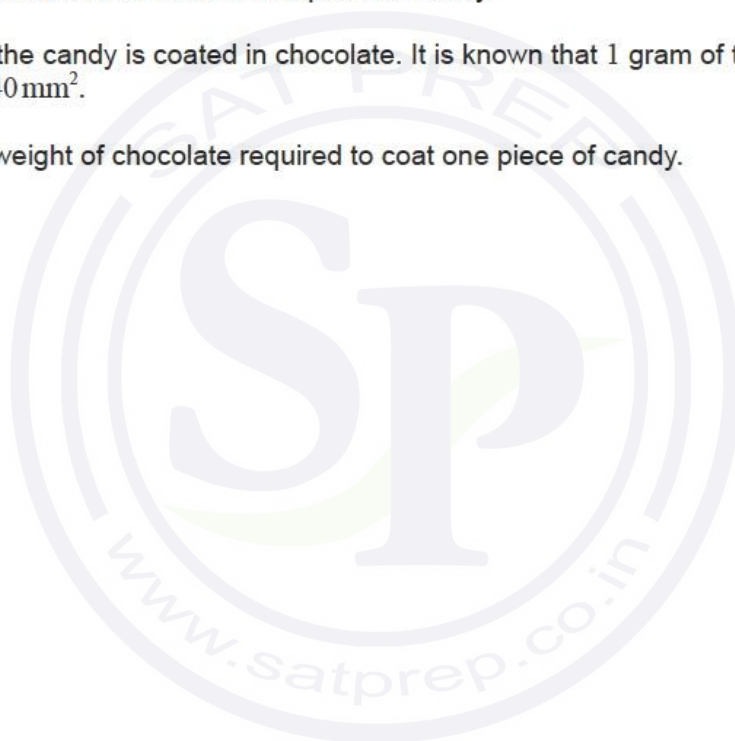
A piece of candy is made in the shape of a solid hemisphere. The radius of the hemisphere is 6 mm.



- (a) Calculate the **total** surface area of one piece of candy. [4]

The total surface of the candy is coated in chocolate. It is known that 1 gram of the chocolate covers an area of  $240 \text{ mm}^2$ .

- (b) Calculate the weight of chocolate required to coat one piece of candy. [2]

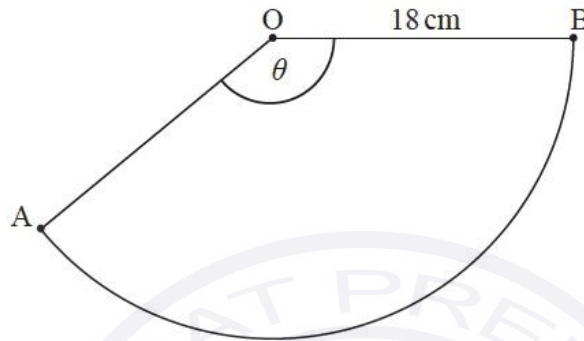


### Question 10

[Maximum mark: 5]

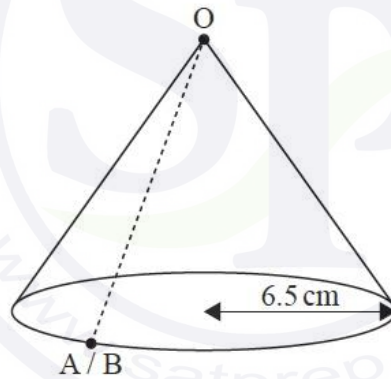
Joey is making a party hat in the form of a cone. The hat is made from a sector,  $AOB$ , of a circular piece of paper with a radius of 18 cm and  $\hat{AOB} = \theta$  as shown in the diagram.

diagram not to scale



To make the hat, sides  $[OA]$  and  $[OB]$  are joined together. The hat has a base radius of 6.5 cm.

diagram not to scale



- (a) (i) Write down the perimeter of the base of the hat in terms of  $\pi$ .
- (ii) Find the value of  $\theta$ . [3]
- (b) Find the surface area of the outside of the hat. [2]

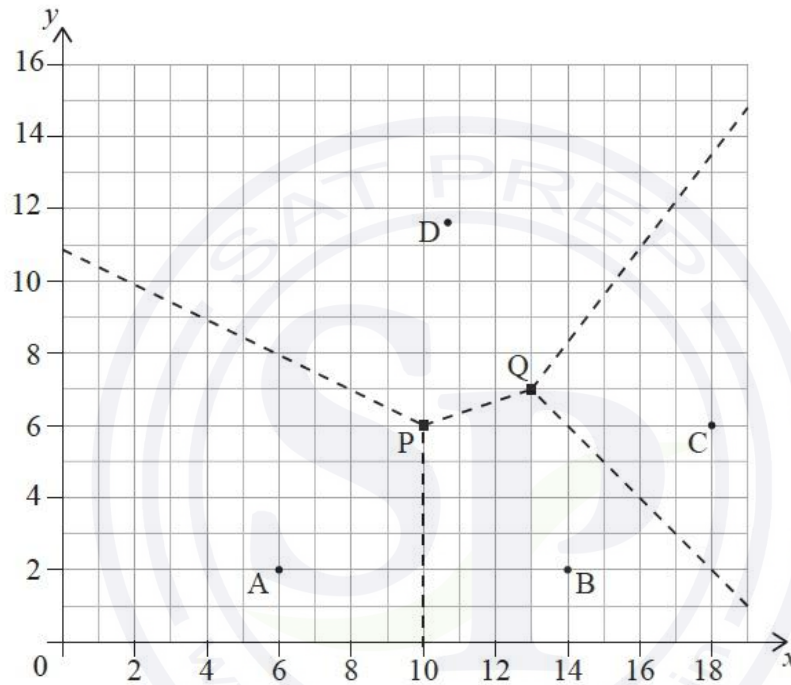
### Question 11

[Maximum mark: 6]

There are four stations used by the fire wardens in a national forest.

On the following Voronoi diagram, the coordinates of the stations are  $A(6, 2)$ ,  $B(14, 2)$ ,  $C(18, 6)$  and  $D(10.8, 11.6)$  where distances are measured in kilometres.

The dotted lines represent the boundaries of the regions patrolled by the fire warden at each station. The boundaries meet at  $P(10, 6)$  and  $Q(13, 7)$ .

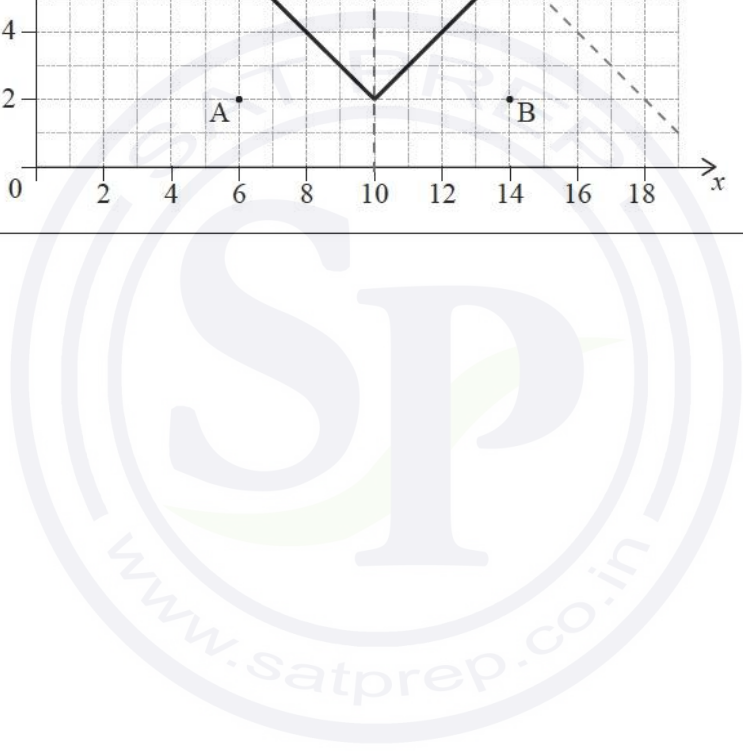
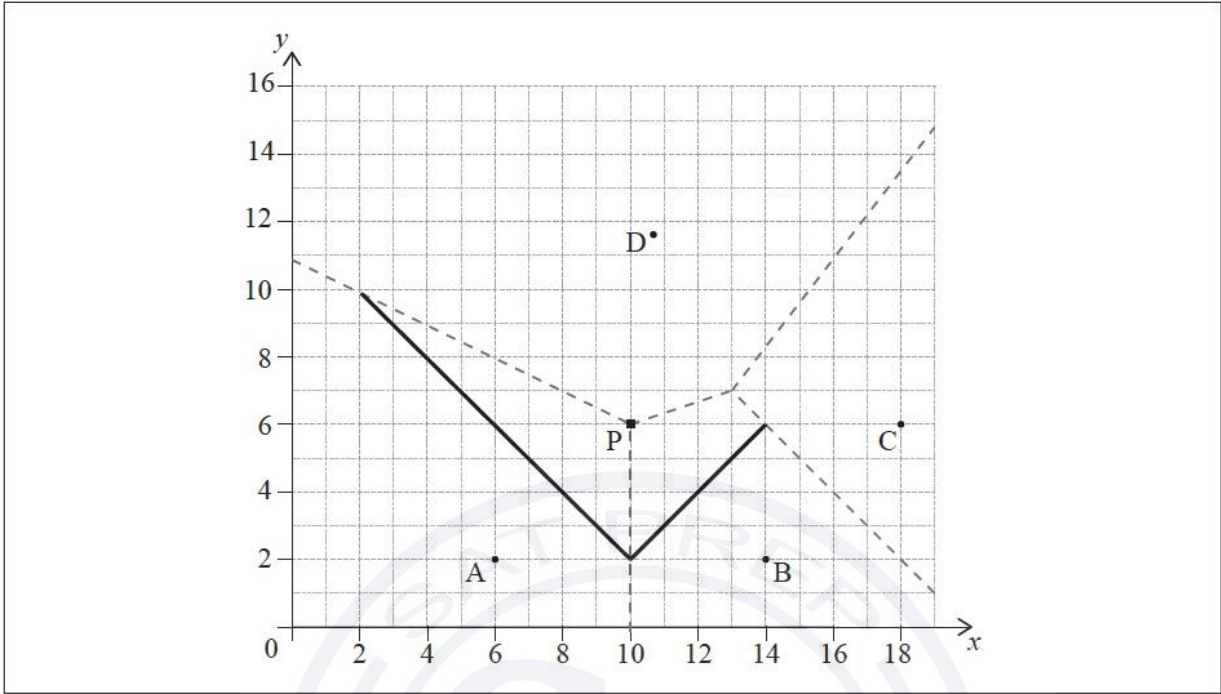


To reduce the areas of the regions that the fire wardens patrol, a new station is to be built within the quadrilateral  $ABCD$ . The new station will be located so that it is as far as possible from the nearest existing station.

- (a) Show that the new station should be built at  $P$ . [3]

The Voronoi diagram is to be updated to include the region around the new station at  $P$ . The edges defined by the perpendicular bisectors of  $[AP]$  and  $[BP]$  have been added to the following diagram.

- (b) (i) Write down the equation of the perpendicular bisector of  $[PC]$ .  
 (ii) Hence draw the missing boundaries of the region around  $P$  on the following diagram. [3]



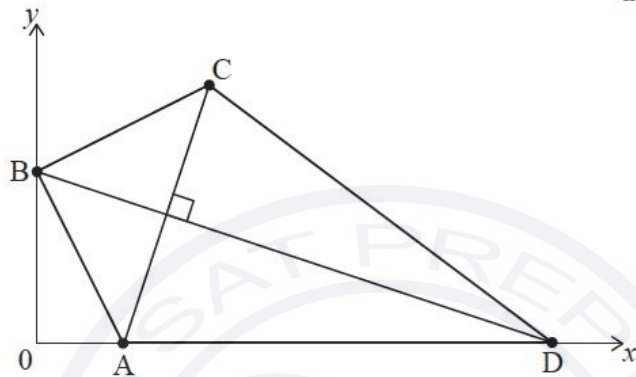
### Question 12

[Maximum mark: 6]

Dilara is designing a kite ABCD on a set of coordinate axes in which one unit represents 10 cm.

The coordinates of A, B and C are (2, 0), (0, 4) and (4, 6) respectively. Point D lies on the  $x$ -axis. [AC] is perpendicular to [BD]. This information is shown in the following diagram.

diagram not to scale



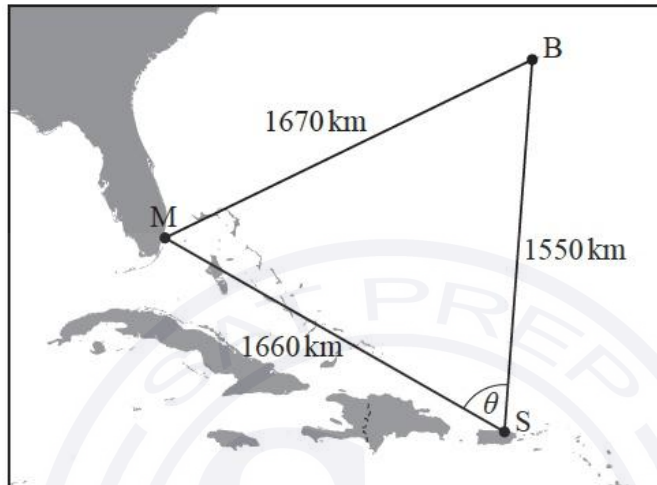
- (a) Find the gradient of the line through A and C. [2]
- (b) Write down the gradient of the line through B and D. [1]
- (c) Find the equation of the line through B and D. Give your answer in the form  $ax + by + d = 0$ , where  $a$ ,  $b$  and  $d$  are integers. [2]
- (d) Write down the  $x$ -coordinate of point D. [1]

### Question 13

[Maximum mark: 5]

The Bermuda Triangle is a region of the Atlantic Ocean with Miami (M), Bermuda (B), and San Juan (S) as vertices, as shown on the diagram.

diagram not to scale



The distances between M, B and S are given in the following table, correct to three significant figures.

Distance between Miami and Bermuda	1670 km
Distance between Bermuda and San Juan	1550 km
Distance between San Juan and Miami	1660 km

- (a) Calculate the value of  $\theta$ , the measure of angle  $M\hat{S}B$ . [3]
- (b) Find the area of the Bermuda Triangle. [2]

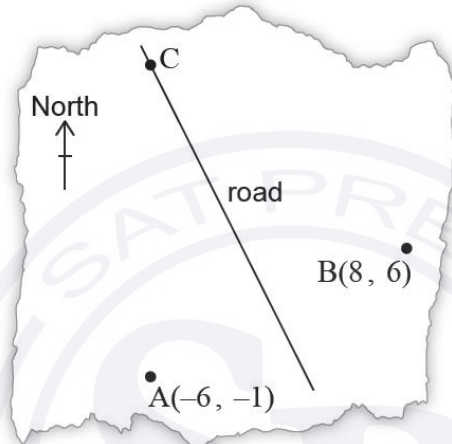
### Question 14

[Maximum mark: 7]

Three towns, A, B and C are represented as coordinates on a map, where the  $x$  and  $y$  axes represent the distances east and north of an origin, respectively, measured in kilometres.

Town A is located at  $(-6, -1)$  and town B is located at  $(8, 6)$ . A road runs along the perpendicular bisector of  $[AB]$ . This information is shown in the following diagram.

**diagram not to scale**



- (a) Find the equation of the line that the road follows. [5]

Town C is due north of town A and the road passes through town C.

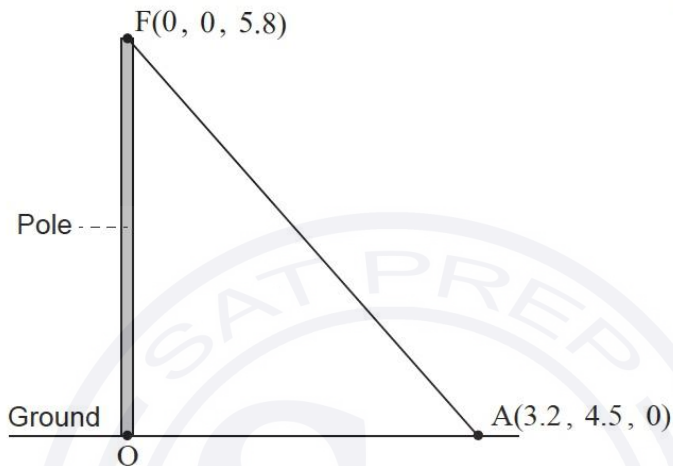
- (b) Find the  $y$ -coordinate of town C. [2]

### Question 15

[Maximum mark: 4]

A vertical pole stands on horizontal ground. The bottom of the pole is taken as the origin,  $O$ , of a coordinate system in which the top,  $F$ , of the pole has coordinates  $(0, 0, 5.8)$ . All units are in metres.

diagram not to scale



The pole is held in place by ropes attached at  $F$ .

One of the ropes is attached to the ground at a point  $A$  with coordinates  $(3.2, 4.5, 0)$ . The rope forms a straight line from  $A$  to  $F$ .

- (a) Find the length of the rope connecting  $A$  to  $F$ . [2]
- (b) Find  $\hat{FAO}$ , the angle the rope makes with the ground. [2]

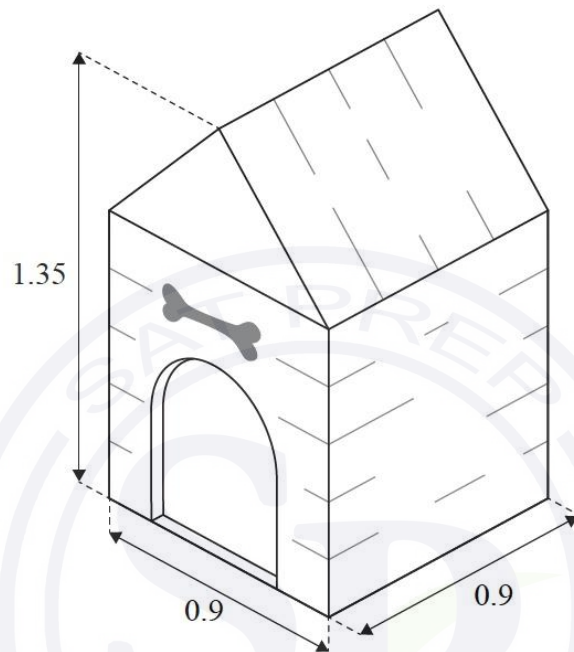


### Question 16

[Maximum mark: 5]

The front view of a doghouse is made up of a square with an isosceles triangle on top. The doghouse is 1.35 m high and 0.9 m wide, and sits on a square base.

diagram not to scale



The top of the rectangular surfaces of the roof of the doghouse are to be painted.

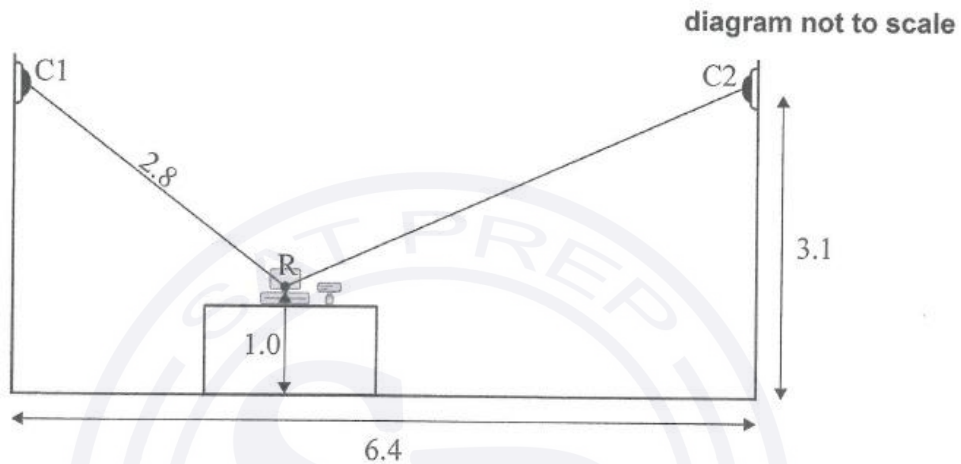
Find the area to be painted.

### Question 17

[Maximum mark: 8]

The owner of a convenience store installs two security cameras, represented by points C1 and C2. Both cameras point towards the centre of the store's cash register, represented by the point R.

The following diagram shows this information on a cross-section of the store.



The cameras are positioned at a height of 3.1 m, and the horizontal distance between the cameras is 6.4 m. The cash register is sitting on a counter so that its centre, R, is 1.0 m above the floor.

The distance from Camera 1 to the centre of the cash register is 2.8 m.

- (a) Determine the angle of depression from Camera 1 to the centre of the cash register. Give your answer in degrees. [2]
- (b) Calculate the distance from Camera 2 to the centre of the cash register. [4]
- (c) Without further calculation, determine which camera has the largest angle of depression to the centre of the cash register. Justify your response. [2]

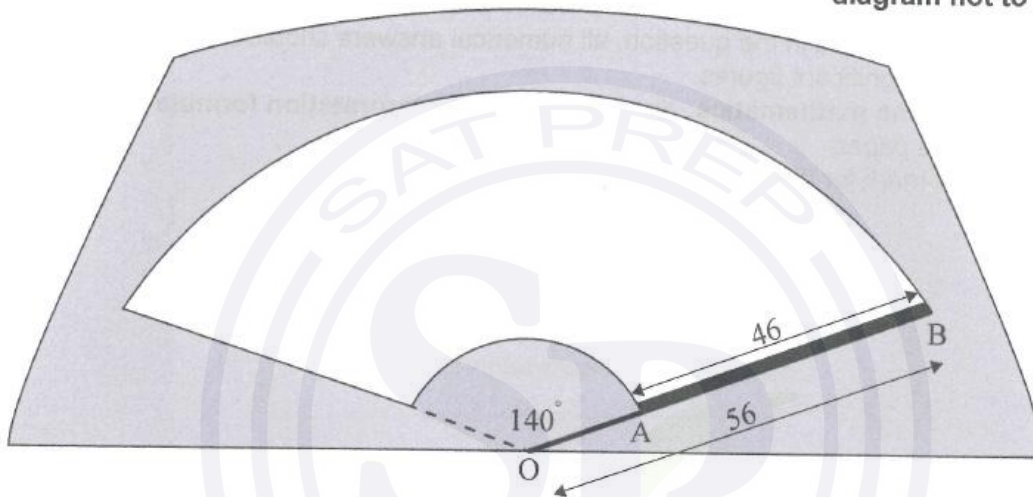
### Question 18

[Maximum mark: 5]

The straight metal arm of a windscreen wiper on a car rotates in a circular motion from a pivot point,  $O$ , through an angle of  $140^\circ$ . The windscreen is cleared by a rubber blade of length 46 cm that is attached to the metal arm between points  $A$  and  $B$ . The total length of the metal arm,  $OB$ , is 56 cm.

The part of the windscreen cleared by the rubber blade is shown unshaded in the following diagram.

diagram not to scale

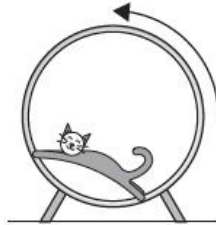


- (a) Calculate the length of the arc made by  $B$ , the end of the rubber blade. [2]
- (b) Determine the area of the windscreen that is cleared by the rubber blade. [3]

### Question 19

[Maximum mark: 6]

A cat runs inside a circular exercise wheel, making the wheel spin at a constant rate in an anticlockwise direction. The height,  $h$  cm, of a fixed point, P, on the wheel can be modelled by  $h(t) = a \sin(bt) + c$  where  $t$  is the time in seconds and  $a, b, c \in \mathbb{R}^+$ .



When  $t = 0$ , point P is at a height of 78 cm.

(a) Write down the value of  $c$ . [1]

When  $t = 4$ , point P first reaches its maximum height of 143 cm.

(b) Find the value of

(i)  $a$ .

(ii)  $b$ . [3]

(c) Write down the minimum height of point P. [1]

Later, the cat is tired, and it takes twice as long for point P to complete one revolution at a new constant rate.

(d) Write down the new value of  $b$ . [1]

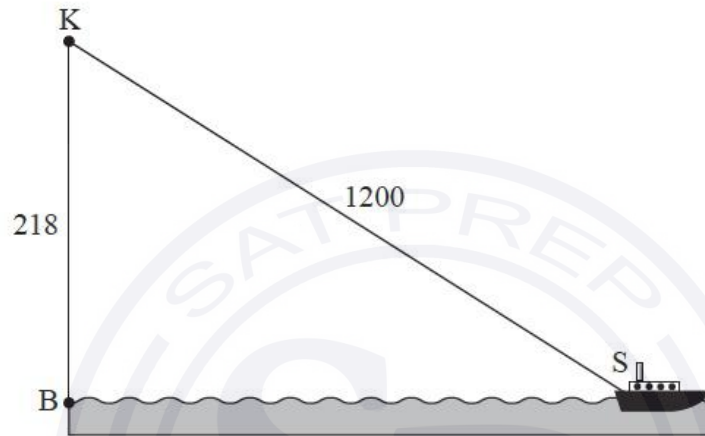
### Question 20

[Maximum mark: 6]

Kacheena stands at point K, the top of a 218 m vertical cliff. The base of the cliff is located at point B. A ship is located at point S, 1200 m from Kacheena.

This information is shown in the following diagram.

diagram not to scale



- (a) Find the angle of elevation from the ship to Kacheena. [2]
- (b) Find the horizontal distance from the base of the cliff to the ship. [2]
- (c) Write down your answer to part (b) in the form  $a \times 10^k$  where  $1 \leq a < 10$  and  $k \in \mathbb{Z}$ . [2]