

Subject - Math AI(Standard Level)

Topic - Number and Algebra

Year - May 2021 - Nov 2022

Paper -1

Answers

Question 1

(a) $10\log_{10}(6.4 \times 10^{-3} \times 10^{12})$

(M1)

$= 98.1(\text{dB}) (98.06179\dots)$

A1

[2 marks]

(b) $112 = 10\log_{10}(S \times 10^{12})$

(M1)

$0.158(\text{W m}^{-2})(0.158489\dots(\text{W m}^{-2}))$

A1

[2 marks]

Total [4 marks]

Question 2

(a) $(d =) - 250$

A1

[1 mark]

(b) $(u_{16} =) 6800 + (16 - 1)(- 250)$

M1

$(\text{¥}) 3050$

A1

[2 marks]

(c) $(S_{16} =) \left(\frac{16}{2}\right)(2 \times 6800 + (16 - 1)(- 250)) \times 2$

M1M1

OR

$(S_{16} =) \left(\frac{16}{2}\right)(6800 + 3050) \times 2$

M1M1

$(\text{¥}) 158000 (157600)$

A1

[3 marks]

Total [6 marks]

Question 3

(a) **METHOD 1**

$$\begin{aligned}N &= 5 \\I\% &= 2.75 \\PV &= -1500 \\PMT &= 0 \\P/Y &= 1 \\C/Y &= 2\end{aligned}$$

OR

$$\begin{aligned}N &= 10 \\I\% &= 2.75 \\PV &= -1500 \\PMT &= 0 \\P/Y &= 2 \\C/Y &= 2\end{aligned}$$

(M1)(A1)

Note: Award **M1** for an attempt to use a financial app in their technology, **A1** for all entries correct.

METHOD 2

$$1500 \left(1 + \frac{2.75}{2 \times 100} \right)^{2 \times 5}$$

1719.49 euro

(M1)(A1)

A1

[3 marks]

(b) **METHOD 1**

$$\begin{aligned}N &= 5 \\PV &= \pm 1500 \\FV &= \mp 2250 \\PMT &= 0 \\P/Y &= 1 \\C/Y &= 4\end{aligned}$$

OR

$$\begin{aligned}N &= 20 \\PV &= \pm 1500 \\FV &= \mp 2250 \\PMT &= 0 \\P/Y &= 4 \\C/Y &= 4\end{aligned}$$

(M1)(A1)

Note: Award **M1** for an attempt to use a financial app in their technology, **A1** for all entries correct. *PV* and *FV* must have opposite signs.

METHOD 2

$$1500 \left(1 + \frac{r}{4 \times 100} \right)^{4 \times 5} = 2250 \quad \text{OR} \quad \left(1 + \frac{r}{4 \times 100} \right)^{4 \times 5} = 1.5$$

(M1)(A1)

Note: Award **M1** for substitution in compound interest formula, **A1** for correct substitution and for equating to 2250 (if using LHS equation) or to 1.5 (if using RHS equation).

$$r = 8.19 \text{ (8.19206...)}$$

A1

Note: Accept $r = 8.19\%$.

Accept a trial and error method which leads to $r = 8.19$.

[3 marks]

Total [6 marks]

Question 4

- (a) (i) attempt to find u_{20} using an arithmetic sequence **(M1)**
e.g. $u_1 = 500$ and $d = 100$ **OR** $u_{20} = 500 + 1900$ **OR** 500, 600, 700, ...

(Charlie ran) 2400 m **A1**
- (ii) ($r =$) 1.02 **(A1)**
attempt to find u_{20} using a geometric sequence **(M1)**
e.g. identifying $u_1 = 500$ and a value for r **OR** $500 \times r^{19}$ **OR** 500, 510, 520.2, ...

(Daniella ran) 728 m (728.405...) **A1**
[5 marks]
- (b) $500 \times 1.02^{n-1} > 500 + (n-1) \times 100$ **(M1)**

attempt to solve inequality **(M1)**
 $n > 184.215...$
 $n = 185$ **A1**
[3 marks]
Total [8 marks]

Question 5

- (a) $\pi \approx 3 + \frac{1}{6 + \frac{13}{16}}$

 $= 3.14678... \left(\frac{343}{109}, 3\frac{16}{109} \right)$ **(A1)**
 $= 3.1468$ **A1**
[2 marks]
- (b) $\left| \frac{3.1468 - \pi}{\pi} \right| \times 100$ **(M1)**

 $= 0.166(\%)$ (0.165754...) **A1**
[2 marks]
Total [4 marks]

Question 6

- (a) $N = 360$
 $I\% = 3.8$
 $PV = (\pm)170\,000$
 $FV = 0$
 $P/Y = 12$
 $C/Y = 12$

(M1)(A1)

Note: Award **(M1)** for an attempt to use a financial app in their technology with at least two entries seen, award **A1** for all entries correct. Accept a positive or negative value for PV .

$(PMT =) 792.13 \text{ AUD}$

A1

Note: Accept an answer of -792.13 . Do not award final **A1** if answer is not given correct to 2 dp

[3 marks]

- (b) (i) $N = 120$
 $I\% = 3.8$
 $PV = (\pm)170\,000$
 $PMT = (\mp)792.13$
 $P/Y = 12$
 $C/Y = 12$

(M1)(A1)

Note: Award **(M1)** for an attempt to use a financial app in their technology with at least two entries seen, award **A1** for all entries correct. PV and PMT must have opposite signs.

$(FV =) 133019.94 \text{ AUD}$

A1

Note: Do not award final **A1** if answer is not given correct to 2 dp, unless already penalized in part (a). Accept 133020.30 from use of exact value for PMT .

- (ii) amount of money paid: $120 \times 792.13 (= 95055.60)$ **(M1)**
loan paid off: $170\,000 - 133019.94 (= 36980.06)$ **(M1)**
interest paid: $(95055.60 - 36980.06 =) 58075.54 \text{ AUD}$ **A1**

Note: Allow 58075.60 or 58075.90 from use of some exact values from parts (a) and (b)(i). If their answer to part (b)(i) is greater than 170 000 then award at most **(M1)(M1)(A0)** for follow through in part (b)(ii).

[6 marks]
Total: [9 marks]

Question 7

- (a) attempt at using trapezoidal rule formula

(M1)

$$\frac{1}{2} \left(\frac{2-0}{5} \right) (30 + 50 + 2(50 + 60 + 40 + 20))$$

(A1)

(total carbon =) 84 tonnes

A1

[3 marks]

(b) $\left| \frac{84-72}{72} \right| \times 100\%$

(M1)

Note: Award (M1) for correct substitution of final answer in part (a) into percentage error formula.

$$= 16.7\% \text{ (16.6666...%)}$$

A1

[2 marks]

Total: [5 marks]

Question 8

- (a) use of geometric sequence with $r = 0.85$

M1

EITHER

$$(0.85)^6(1.8) \text{ OR } 0.678869... \text{ OR } (0.85)^5(1.53)$$
$$= 0.68 \text{ m}$$
$$= 68 \text{ cm}$$

A1

AG

OR

$$(0.85)^6(1.80) \text{ OR } (0.85)^5(1.53)$$
$$= 68 \text{ cm}$$

A1

AG

[2 marks]

- (b) **EITHER**

$$(0.85)^n(1.8) > 0.1 \text{ OR } (0.85)^{n-1}(1.53) > 0.1$$

(M1)

17

A1

OR

$$(0.85)^{17}(1.8) = 0.114 \text{ m and } (0.85)^{18}(1.8) = 0.0966 \text{ m}$$

17

(M1)

A1

OR

$$\text{solving } (0.85)^n(1.8) = 0.1 \text{ to find } n = 17.8$$

(M1)

17

A1

[2 marks]

(c) **EITHER**

distance (in one direction) travelled between first and fourth bounce

$$= \frac{(1.8 \times 0.85)(1 - 0.85^3)}{1 - 0.85} (= 3.935925) \quad (\text{A1})$$

recognizing distances are travelled twice except first distance

$$1.8 + 2(3.935925) \quad (\text{M1})$$

$$= 9.67 \text{ m (9.67185... m)} \quad \text{A1}$$

OR

distance (in one direction) travelled between drop and fourth bounce

$$= \frac{(1.8)(1 - 0.85^4)}{1 - 0.85} (= 5.735925) \quad (\text{A1})$$

recognizing distances are travelled twice except first distance

$$2(5.735925) - 1.8 \quad (\text{M1})$$

$$= 9.67 \text{ m (9.67185... m)} \quad \text{A1}$$

[3 marks]

[Total 7 marks]

Question 9

(a) $\log_{10} 100 = a - 3$

$$a = 5$$

(M1)

A1

[2 marks]

(b) **EITHER**

$$N = 10^{5-M}$$

$$= \frac{10^5}{10^M} \left(= \frac{100000}{10^M} \right) \quad (\text{M1})$$

OR

$$100 = \frac{b}{10^3} \quad (\text{M1})$$

THEN

$$b = 100000 (= 10^5) \quad \text{A1}$$

[2 marks]

(c) $0.001 < N < 100000$ ($10^{-3} < N < 10^5$)

A1A1

[2 marks]

(d) $N = \frac{10^5}{10^{7.2}}$ ($= 0.0063095...$)

(M1)

$$\text{length of time} = \frac{1}{0.0063095...} = 10^{2.2}$$

$$= 158 \text{ years}$$

A1

[2 marks]

[Total 8 marks]

Question 10

- (a) $x + y + z = 600$ **A1**
 $15x + 10y + 12z = 7816$ **A1**
 $x = 2y$ **A1**

Note: Condone other labelling if clear, e.g. a (adult), c (child) and s (student).
Accept equivalent, distinct equations e.g. $2y + y + z = 600$.

[3 marks]

- (b) $x = 308, y = 154, z = 138$ **A1A1**

Note: Award **A1** for all three correct values seen, **A1** for correctly labelled as x, y or z .
Accept answers written in words: e.g. 308 adult tickets.

[2 marks]

[Total 5 marks]

Question 11

- (a) **METHOD 1** – (with $FV = 4000$)

EITHER

$$N = 10$$

$$I = 1.5$$

$$FV = 4000$$

$$P/Y = 1$$

$$C/Y = 1$$

(A1)(M1)

Note: Award **A1** for $(3.5 - 2 \Rightarrow) 1.5$ seen and **M1** for all other entries correct.

OR

$$4000 = A(1 + 0.015)^{10}$$

(A1)(M1)

Note: Award **A1** for 1.5 or 0.015 seen, **M1** for attempt to substitute into compound interest formula **and** equating to 4000.

THEN

$$(PV \Rightarrow) \$3447$$

A1

METHOD 2 – (With FV including inflation)

calculate FV with inflation

$$4000 \times 1.02^{10} \\ (=4875.977\dots)$$

(A1)

EITHER

$$4000 \times 1.02^{10} = PV \times 1.035^{10}$$

(A1)

OR

$$N = 10$$

$$I = 3.5$$

$$FV = 4875.977\dots$$

$$P/Y = 1$$

$$C/Y = 1$$

(M1)

Note: Award **M1** for *their* FV and all other entries correct.

THEN

$$(PV =) \$3457$$

A1

Note: Award **A0** if not rounded to a whole number or a negative sign given.

METHOD 3 – (Using formula to calculate real rate of return)

(real rate of return =) 1.47058...(%)

(A1)

EITHER

$$4000 = PV \times 1.0147058\dots^{10}$$

(A1)

OR

$$N = 10$$

$$I = 1.47058\dots$$

$$FV = 4000$$

$$P/Y = 1$$

$$C/Y = 1$$

(M1)

Note: Award **M1** for all entries correct.

THEN

$$(PV =) \$3457$$

A1

[3 marks]

(b) **METHOD 1** – (Finding the future value of the investment using PV from part (a))

$$N = 10$$

$$I = 3.5$$

$$PV = 3446.66\dots(\text{from Method 1}) \text{ OR } 3456.67\dots(\text{from Methods 2, 3})$$

$$P/Y = 1$$

$$C/Y = 1$$

(M1)

Note: Award **M1** for interest rate 3.5 and answer to part (a) as PV.

$$(FV =) \$4861.87 \text{ OR } \$4875.97$$

(A1)

so payment required (from TVM) will be \$294 OR \$295

A1

Note: Award **A0** if a negative sign given, unless already penalized in part (a).

METHOD 2 – (Using FV)

$$N = 10$$

$$I = 3.5$$

$$PV = -1000$$

$$FV = 4875.977\dots$$

$$P/Y = 1$$

$$C/Y = 1$$

(A1)(M1)

Note: Award **A1** for $I=3.5$ and $FV = \pm 4875.977\dots$, **M1** for all other entries correct and opposite PV and FV signs.

$$(PMT =) \$295 \text{ (} 295.393\text{)}$$

A1

Note: Correct 3sf answer is 295, however accept an answer of 296 given that the context supports rounding up. Award **A0** if a negative sign given, unless already penalized in part (a).

[3 marks]
Total [6 marks]

Question 12

OR

referencing the graph

The graph of $y = -\log_{10}(x)$ shows that as the value of x increases, the value of y decreases.

M1

Since the C -value (x -value) of the unknown liquid is larger than that of the coffee, the pH level (y -value) is lower.

R1

The unknown liquid is more acidic (than coffee).

A1

- (a) (pH =) $-\log_{10}(1.3 \times 10^{-5})$
4.89 (4.88605...)

(M1)

A1

[2 marks]

- (b) **EITHER**
calculating pH
(pH =) $-\log_{10}(10 \times 1.3 \times 10^{-5})$
3.89 (3.88605...)

(M1)

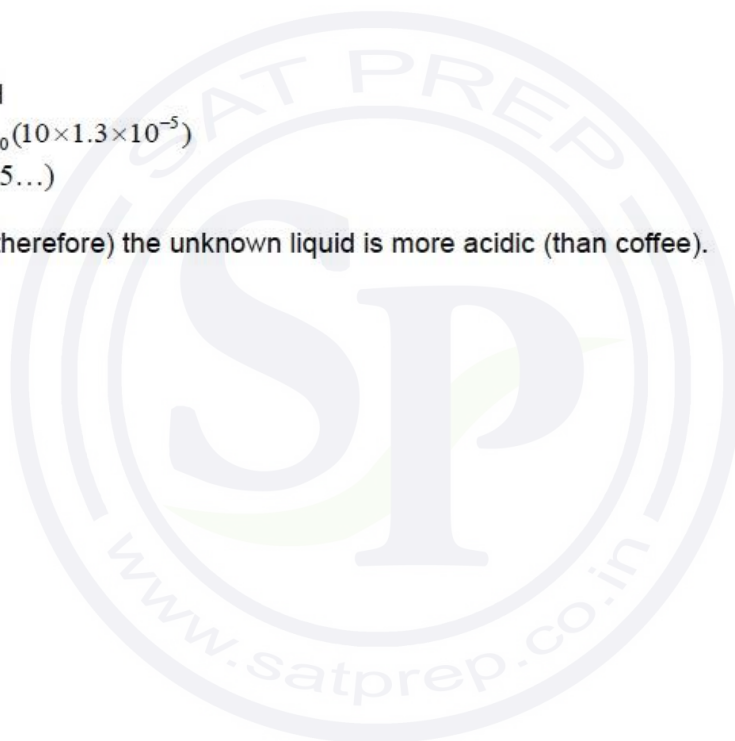
A1

(3.89 < 4.89, therefore) the unknown liquid is more acidic (than coffee).

A1

[3 marks]

Total [5 marks]



Question 13

- (a) $4.5 = 2(r)^{3-1}$ **M1**
 $r = \pm 1.5$, **R1**
(Some x -values are negative or direction from house changes each day)
 $r = -1.5$ **AG**

Note: Award **MOR0AG** for a verification approach $4.5 = 2(-1.5)^{3-1}$.

[2 marks]

- (b) $2(-1.5)^{6-1}$ **(M1)**
EITHER
 $(-15.2, 0)$ $(-15.1875\dots, 0)$ **A1**
OR
 $x = -15.2$ km **A1**
OR
15.2 km west (of the origin) **A1**

Note: Award **(M1)A0** for an answer of “-15.2 (km)” without indicating that it is the x -value.

[2 marks]

- (c) choosing $r = 1.5$ **(A1)**
 $\frac{2((1.5)^7 - 1)}{1.5 - 1}$ **(M1)**

Note: Award **M1** for an attempt at a substituted GP formula with $n = 7$. Award **A0M1A0** for substitution of $r = -1.5$, with $n = 7$ (this can be implied from a final answer of 14.4687...).

- 64.3... km (64.3437...) **A1**

[3 marks]
Total [7 marks]

Question 14

(a) $m = 1 - 2.5 \log_{10}(0.0525)$
 $= 4.20$ (4.19960...)

(M1)

A1

[2 marks]

(b) attempt to solve $7 = 1 - 2.5 \log_{10}(b)$

(M1)

Note: Accept a sketch from their GDC as an attempt to solve $7 = 1 - 2.5 \log_{10}(b)$.

$b = 0.00398$ (0.00398107...)

A1

[2 marks]

(c) $\frac{0.0525}{0.00398107}$
 $= 13.2$ (13.1874...)

(M1)

A1

[2 marks]

Total [6 marks]

Question 15

(a) $I\% = 7.5$
 $PV = \mp 800$
 $PMT = \mp 500$
 $FV = \pm 10000$
 $P/Y = 12$
 $C/Y = 12$

(M1)(A1)

17.3070...
($k =$) 18

(A1)

A1

[4 marks]

(b) $10389 - (18 \times 500 + 800)$ OR $10389 - (9800)$

(A1)(M1)

589 EUR

A1

[3 marks]

Total [7 marks]

Question 16

- (a) use of the n^{th} term of an arithmetic sequence formula

$$u_{15} = 85 + (15 - 1) \times 30$$

$$505$$

(M1)

(A1)

A1

[3 marks]

- (b) use of the sum of n terms of an arithmetic sequence formula

$$S_{15} = \frac{15}{2}(85 + 505) \quad \text{OR} \quad \frac{15}{2}(2 \times 85 + (15 - 1) \times 30)$$

$$4430 \quad (4425)$$

(M1)

A1

[2 marks]

- (c) $\frac{4425}{15}$ OR $85 + (15 - 1) \times 30$
295

(M1)

A1

Note: Accept 295.333... from use of 3sf value from part (b).

[2 marks]

Total [7 marks]

