

Subject - Math AI(Standard Level)
Topic - Statistics and Probability
Year - May 2021 - Nov 2022
Paper -1
Answers

Question 1

(a) $\frac{6}{15} \left(0.4, \frac{2}{5} \right)$

A1

[1 mark]

(b) $P(X = 8)$

(M1)

Note: Award **(M1)** for evidence of recognizing binomial probability.

eg, $P(X = 8), X \sim B \left(20, \frac{6}{15} \right)$.

0.180 (0.179705...)

A1

[2 marks]

(c) $P(\text{male}) = \frac{9}{15} (0.6)$

A1

$P(X \leq 9) = 0.128 (0.127521...)$

(M1)A1

Note: Award **(M1)** for evidence of correct approach eg, $P(X \leq 9)$.

[3 marks]

Total [6 marks]

Question 2

(a) $\frac{4}{18} \left(\frac{2}{9} \right)$

A1

[1 mark]

(b) $-3 \times \frac{1}{18} + (-1) \times \frac{4}{18} + 0 \times \frac{3}{18} + \dots + 5 \times \frac{7}{18}$

(M1)

Note: Award (M1) for their correct substitution into the formula for expected value.

$$= 1.83 \left(\frac{33}{18}, 1.83333\dots \right)$$

A1

[2 marks]

(c) $2 \times \frac{1}{18} \times \frac{3}{18}$

(M1)(M1)

Note: Award (M1) for $\frac{1}{18} \times \frac{3}{18}$, award (M1) for multiplying their product by 2.

$$= \frac{1}{54} \left(\frac{6}{324}, 0.0185185\dots, 1.85\% \right)$$

A1

[3 marks]

Total [6 marks]

Question 3

(a) (i) $\mu_1 - \mu_2 = 0$

A1

(ii) $\mu_1 - \mu_2 \neq 0$

A1

Note: Accept equivalent statements in words.

[2 marks]

(b) 0.296 (0.295739...)

A2

[2 marks]

(c) $0.296 > 0.1$

R1

fail to reject the null hypothesis, there is no difference between the mean height of male and female students

A1

Note: Award (R1) for a correct comparison of their p -value to the test level, award (A1) for the correct interpretation from that comparison. Do not award ROA1.

[2 marks]

Total [6 marks]

Question 4

- (a) number of salad meals per week is independent of a person's position in the university

A1

Note: Accept "not associated" instead of independent.

[1 mark]

- (b) 0.0201 (0.0201118...)

A2

[2 marks]

- (c) $0.0201 < 0.05$

R1

the null hypothesis is rejected

A1

[2 marks]

Total [5 marks]

Question 5

- (a) discrete

A1

[1 mark]

- (b) $\frac{24 + 60 + 3k + 40 + 15 + 6}{88 + k} = 2$

M1A1

Note: Award **M1** for substitution into the formula for the mean, award **A1** for a correct equation.

attempt to solve their equation

(M1)

$$k = 31$$

A1

[4 marks]

- (c) systematic

A1

[1 mark]

Total [6 marks]

Question 6

(a) 210g

A1
[1 mark]

(b) 240g

A1
[1 mark]

(c) 240 – 190
= 50g

(M1)
A1
[2 marks]

(d) 240 + 1.5 × (50)
= 315g

M1
A1
[2 marks]

Total [6 marks]

Question 7

(a) $\left(\frac{74+97+91+86+112}{5}\right) = 92$

A1
[1 mark]

(b) (i) 4

A1

(ii) $\chi^2_{\text{calc}} = 8.54$ (8.54347...) **OR** $p\text{-value} = 0.0736$ (0.0735802...)

A2

$8.54 < 9.49$ **OR** $0.0736 > 0.05$

R1

therefore there is insufficient evidence to reject H_0

A1

(i.e. the data satisfies the model)

Note: Do not award **R0A1**. Accept “accept” or “do not reject” in place of “insufficient evidence to reject”. Award the **R1** for comparing their p -value with 0.05 or their χ^2 value with 9.49 and then **FT** their final conclusion.

[5 marks]

Total [6 marks]

Question 8

- (a) (let μ_c = population mean for chinchilla rabbits, μ_s = population mean for sable rabbits)

$$H_0 : \mu_c = \mu_s$$

A1

$$H_1 : \mu_c > \mu_s$$

A1

[2 marks]

- (b) p -value = 0.0408 (0.0408065...)

A2

- (c) $0.0408 < 0.05$.

R1

(there is sufficient evidence to) reject (or not accept) H_0

A1

(there is sufficient evidence to suggest that chinchilla rabbits are heavier than sable rabbits)

[2 marks]

Total [6 marks]

Question 9

- (a) (i) 2

A1

- (ii) 6

A1

- (iii) 8

A1

[3 marks]

- (b) **EITHER**

Each of these percentages represent approximately 25% of the employees.

R1

OR

The diagram is not explicit enough to show what is happening at the quartiles regarding 6 and 11 / we do not have the data points

R1

OR

Discrete data not clear how to interpret "fewer".

R1

THEN

Hence, Paul is not correct (**OR** no such inference can be made).

A1

Note: Do not award **R0A1**.

[2 marks]

Total [5 marks]

Question 10

i. (a)

t	1	2	3	4	5	6
$P(T=t)$	$\frac{1}{36}$ (0.027777...)	$\frac{3}{36}$ (0.083333...)	$\frac{5}{36}$ (0.138888...)	$\frac{7}{36}$ (0.194444...)	$\frac{9}{36}$ (0.25)	$\frac{11}{36}$ (0.305555...)

A2

Note: Award **A1** if three to five probabilities are correct.

[2 marks]

(b) (i) $\frac{32}{36} \left(\frac{8}{9}, 0.888888..., 88.9\% \right)$

(A1)

(ii) use of conditional probability
e.g. denominator of 32 **OR** denominator of 0.888888..., etc.

(M1)

$\frac{11}{32} (0.34375, 34.4\%)$

A1

[3 marks]

(c) $\frac{1 \times 1 + 3 \times 2 + 5 \times 3 + \dots + 11 \times 6}{36}$

(M1)

$= \frac{161}{36} (4\frac{17}{36}, 4.47, 4.47222...)$

A1

[2 marks]

Total [7 marks]

Question 11

(a) **EITHER**

H_0 : The population mean weight of eggs from (her/the) black geese is equal to/the same as the population mean weight of eggs from (her/the) white geese.

OR

H_0 : The population mean weight of eggs from (her/the) black geese is not less than the population mean weight of eggs from (her/the) white geese.

A1

[1 mark]

(b) $p\text{-value} = 0.177 (0.176953...)$

A2

[2 marks]

(c) $0.177 > 0.1$

R1

(insufficient evidence to reject H_0)

Arriane's claim is not supported by the evidence

A1

[2 marks]

Total [5 marks]

Question 12

(a) 14

A1
[1 mark]

(b) $\frac{14+15+\dots}{10}$
= 13.1

(M1)
A1
[2 marks]

(c) 2.21 (2.21133...)

A1
[1 mark]

Total [4 marks]

Question 13

(a) $a = 0.42$

A1
[1 mark]

(b) $(P(B' \cap F) =) b \times 0.68$

A1
[1 mark]

(c) (i) $0.32 \times 0.58 + 0.68b = 0.41$

(M1)

Note: Award (M1) for setting up equation for fair-haired or equivalent.

$b = 0.33$

A1

(ii) $c = 0.67$

A1
[3 marks]
Total: [5 marks]

Question 14

- (a) 75 A1
[1 mark]
- (b) recognition that all entries add up to 120 (M1)
 $a = 120 - 6 - 13 - 26 - b$ OR $a = 75 - b$ A1
[2 marks]
- (c) (i) $\frac{6 \times 1 + 13 \times 2 + 26 \times 3 + (75 - b) \times 4 + b \times 5}{120} = 3.65$ (M1)(A1)

Note: Award (M1) for attempt to substitute into mean formula, LHS expression is sufficient for the M mark. Award (A1) for correct substitutions in one variable OR in two variables, followed by evidence of solving simultaneously with $a + b = 75$.

- (b =) 28 A1
- (ii) 120 – their part (c)(i) seen (e.g. 92 indicated on graph) (M1)
84 A1
[5 marks]
- Total: [8 marks]**

Question 15

- (a) $r = 0.933$ (0.933419...) A2
[2 marks]
- (b) strong A1
- Note:** Answer may include “positive”, however this is not necessary for the mark. [1 mark]
- (c) $t = 0.228x + 24.3$ ($t = 0.227703...x + 24.3153...$) A1
- Note:** Condone y in place of t . Answer must be an equation. [1 mark]
- (d) ($t =$) $0.227703... \times 57 + 24.3153...$ (M1)

Note: Award (M1) for correct substitution into their regression line.

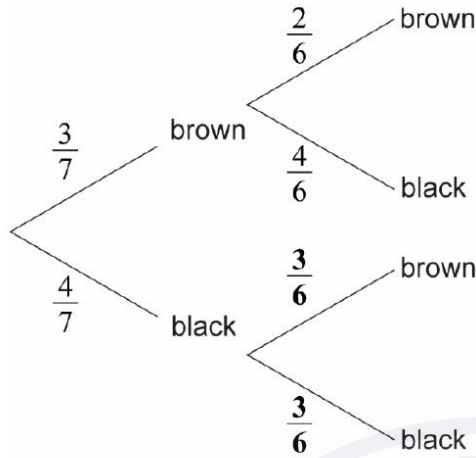
- ($t =$) 37.3 minutes (37.2944) A1

Note: Accept 37.1 and 37.4 from use of 2sf and/or 3sf values.

[2 marks]
Total: [6 marks]

Question 16

(a)



A1

[1 mark]

(b) multiplying along branches and then adding outcomes

(M1)

$$\frac{3}{7} \times \frac{2}{6} + \frac{4}{7} \times \frac{3}{6}$$

$$= \frac{18}{42} \left(= \frac{3}{7} \approx 0.429 \text{ (42.9\%)} \right)$$

A1

[2 marks]

(c) use of conditional probability formula

M1

$$\frac{\left(\frac{3}{7} \times \frac{2}{6} \right)}{\left(\frac{3}{7} \right)}$$

$$= \frac{6}{18} \left(= \frac{1}{3} \right) \left(\frac{252}{756}, 0.333, 33.3\% \right)$$

A1

A1

[3 marks]

[Total 6 marks]

Question 17

(a) 50%

A1

[1 mark]

(b) 0.0478 (0.0477903..., 4.78%)

A2

[2 marks]

(c) $P(X < k) = 0.98$ **OR** $P(X > k) = 0.02$

(M1)

506 g (506.161...)

A2

[3 marks]

[Total 6 marks]

Question 18

(a) H_0 : The die is fair **OR** $P(\text{any number}) = \frac{1}{6}$ **OR** probabilities are equal

H_1 : The die is not fair **OR** $P(\text{any number}) \neq \frac{1}{6}$ **OR** probabilities are not equal **A1**

[1 mark]

(b) 5

A1

[1 mark]

(c) 10

A1

[1 mark]

(d) (p -value \Rightarrow) 0.287 (0.28724163....)

A2

[2 marks]

(e) $0.287 > 0.05$

R1

EITHER

Insufficient evidence to reject the null hypothesis

A1

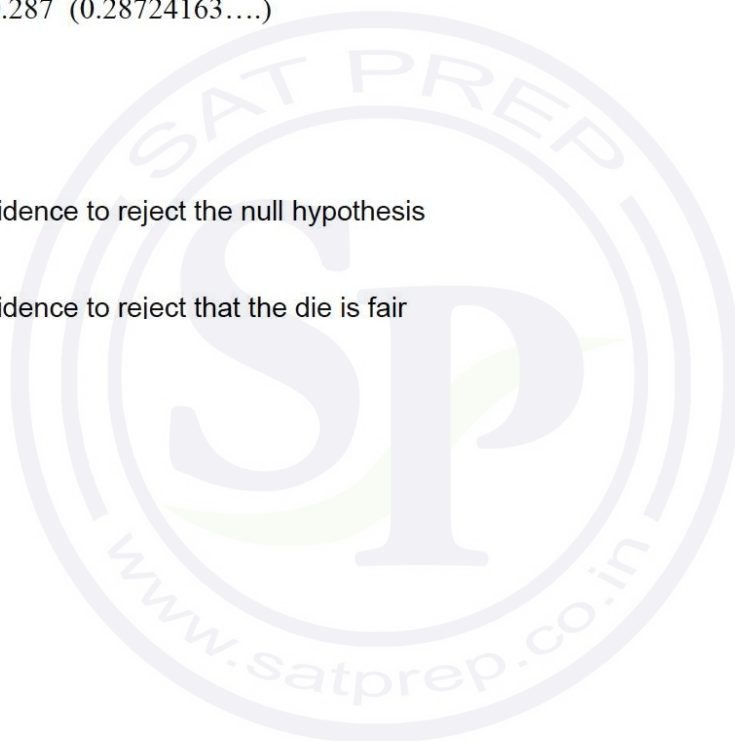
OR

Insufficient evidence to reject that the die is fair

A1

[2 marks]

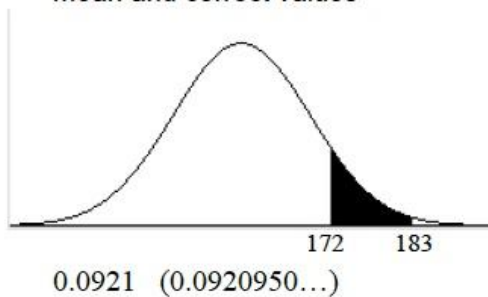
[Total 7 marks]



Question 19

- (a) sketch of normal curve with shaded region to the right of the mean and correct values

(M1)



A1
[2 marks]

- (b) **EITHER**
($P(x < 172)$)

0.906200...

(A1)

(0.906200... - 0.68)

0.226200...

(A1)

OR

($P(163 < x < 172)$)

0.406200...

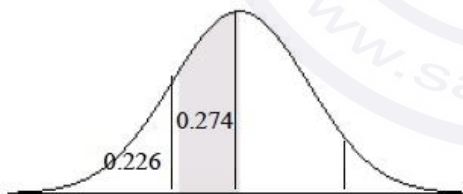
(A1)

0.5 - (0.68 - 0.406200...) **OR** 0.5 + (0.68 - 0.406200...)

0.226200... **OR** 0.773799...

(A1)

OR



(A1)(A1)

THEN

($k =$) 158 g (157.867...g)

A1
[3 marks]
Total [5 marks]

Question 20

- (a) Accept any one of the following (or equivalent):
one minimum and one maximum point
three x -intercepts or three roots (or zeroes)
one point of inflexion

R1

Note: Do not accept "S shape" as a justification.

[1 mark]

(b) (i) $(d =) -5$

A1

(ii) $8 = a + b + c$

$$4 = 8a + 4b + 2c$$

$$0 = 27a + 9b + 3c$$

A2

Note: Award A2 if all three equations are correct.
Award A1 if at least one is correct. Award A1 for three correct equations that include the letter "d".

(iii) $a = 2, b = -12, c = 18$

A1

[4 marks]

- (c) equating found expression to zero

$$0 = 2t^3 - 12t^2 + 18t - 5$$

$$t = 0.358216\dots, 1.83174\dots, 3.81003\dots$$

(M1)

(A1)

(so total time in debt is $3.81003\dots - 1.83174\dots + 0.358216 \approx$)

2.34 (2.33650...) years

A1

[3 marks]

Total [8 marks]

Question 21

(a) $(H_1:) \mu_1 - \mu_2 \neq 0$ ($\mu_1 \neq \mu_2$)

A1

Note: Accept an equivalent statement in words, however reference to "population mean" must be explicit for A1 to be awarded.

[1 mark]

(b) 0.0778 (0.0778465...)

A2

Note: Award A1 for an answer of 0.0815486... from not using a pooled estimate of the variance.

[2 marks]

(c) (i) $0.0778 < 0.1$
reject the null hypothesis

R1

A1

Note: Do not award R0A1.

(ii) there is (significant evidence of) a difference between the (population) mean reaction times

A1

Note: Their conclusion in (c)(ii) must match their conclusion in (c)(i) to earn A1. Award A0 if their conclusion refers to mean reaction times in the sample.

[3 marks]

Total [6 marks]

Question 22

(a) $(88 - 62) \times 1.5$ OR 26×1.5 seen anywhere OR 39 seen anywhere

(M1)

$62 - 39$

23

A1

$25 > 23$

so is not an outlier

R1

AG

[3 marks]

(b) The median score for the evening class is higher than the median score for the morning class.

A1

THEN

but the scores are more spread out in the evening class than in the morning class

A1

OR

the scores are more inconsistent in the evening class

A1

OR

the lowest scores are in the evening class

A1

OR

the interquartile range is lower in the morning class

A1

[2 marks]

Total [5 marks]

Question 23

(a) $(E(X) =) 10 \times 0.8$
8 (people)

(M1)
A1
[2 marks]

(b) recognition of binomial probability
0.0881 (0.0880803...)

(M1)
A1
[2 marks]

(c) 0.8 and 6 seen OR 0.2 and 3 seen
attempt to use binomial probability
0.121 (0.120873...)

(A1)
(M1)
A1
[3 marks]
Total [7 marks]

Question 24

(a) $\left(\frac{17+25}{130} =\right) \frac{42}{130} \left(\frac{21}{65}, 0.323076\dots\right)$

A1
[1 mark]

(b) $\left(\frac{17}{17+25} =\right) \frac{17}{42} (0.404761\dots)$

A1A1

Note: Award **A1** for correct numerator and **A1** for correct denominator.
Award **A1A0** for working of $\frac{17}{130}$ if followed by an
their answer to (a)
incorrect answer.

[2 marks]

(c) $\frac{41}{130} \times \frac{40}{129}$

A1M1

Note: Award **A1** for two correct fractions seen, **M1** for multiplying their fractions.

$$= \frac{1640}{16770} \approx 0.0978 \left(0.0977936\dots, \frac{164}{1677}\right)$$

A1
[3 marks]
Total [6 marks]

Question 25

(a) $0.5 \times 0.1 + 0.4 \times 0.4 + 0.1 \times 0.5$

(M1)(M1)(M1)

0.26

A1

[4 marks]

(b) $0 = -8 \times 0.5 + 4 \times 0.4 + 0.1k$

(M1)(M1)

(k =) 24 (points)

A1

[3 marks]

Total [7 marks]

Question 26

(a) $P(T < 55)$

(M1)

0.0912 (0.0912112...)

A1

Note: Award **M1** for a correct calculator notation such as normal cdf(0, 55, 59, 3) or normal cdf(-1⁹⁹, 55, 59, 3).

[2 marks]

(b) correct use of expected value
 $8.6 = 20 \times p$ OR ($p =$) 0.43 seen

(M1)

EITHER

correct probability statement

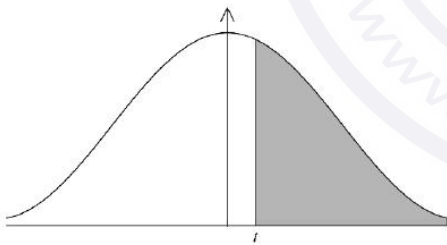
$P(T > t) = 0.43$ OR $P(T < t) = 0.57$

(M1)

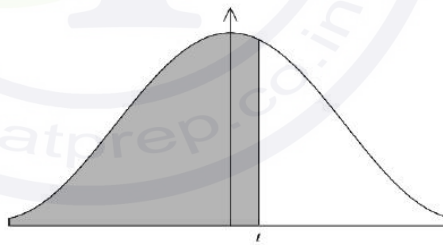
OR

t indicated on sketch to communicate correct area

(M1)



OR



THEN

($t =$) 59.5 (seconds) (59.5291...)

A1

[3 marks]

Total [5 marks]

Question 27

(a) $(H_1 :) \mu_1 \neq \mu_2$

A1

Note: Accept an equivalent statement in words referring to μ_1 and μ_2 as defined in the question.

[1 mark]

(b) 0.97652 (0.976516...)

A2

[2 marks]

(c) $0.97652 > 0.05$ ($0.977 > 0.05$)

R1

Annabelle's conclusion is correct.

A1

Note: Do not award **ROA1**. Answer must reference Annabelle's conclusion; do not accept an answer, without context, of "fail to reject H_0 " for the **A1** mark.

[2 marks]

Total [5 marks]

Question 28

(a) The favourite breakfast/berry (of adults) is independent of (their) income (level). **A1**

[1 mark]

(b) $\chi^2 = 2.27$ (2.26821...)

A2

[2 marks]

(c) **EITHER**
 $2.27 < 7.78$ **OR** $2.27 < \text{critical value}$
OR
 $0.687 > 0.1$ (using p -value)

R1

THEN

(Do not reject H_0)

Insufficient evidence (at the 10% significance level) that the favourite berry depends on income level.

A1

[2 marks]

Total [5 marks]