Subject – Math AI(Standard Level)
Topic - Calculus
Year - May 2021 – Nov 2022
Paper -2
Questions

Question 1

(a) $2(8\times4+3\times4+3\times8)$ ***M1***

$=136\ (\text{cm}^2)$ ***A1***

[2 marks]

(b) $\sqrt{8^2+4^2+3^2}$ ***M1***

$(\text{AG} =)\ 9.43\ (\text{cm})\ (9.4339\ldots,\ \sqrt{89})$ ***A1***

[2 marks]

(c) $-2x+220=0$ ***M1***

$x=110$ ***A1***

$110\,000$ (boxes) ***A1***

[3 marks]

(d) $P(x)=\int -2x+220\,\mathrm{d}x$ ***M1***

Note: Award ***M1*** for evidence of integration.

$P(x)=-x^2+220x+c$ ***A1A1***

Note: Award ***A1*** for either $-x^2$ or $220x$ award ***A1*** for both correct terms and constant of integration.

$1700=-(20)^2+220(20)+c$ ***M1***

$c=-2300$

$P(x)=-x^2+220x-2300$ ***A1***

[5 marks]

(e) $-x^2+220x-2300=0$ ***M1***

$x=11.005$ ***A1***

$11\,006$ (boxes) ***A1***

[3 marks]

Total [15 marks]

Question 2

(a) (i) evidence of power rule (at least one correct term seen) ***(M1)***

$\frac{dy}{dx} = -0.3x^2 + 1.6x$ ***A1***

(ii) $-0.3x^2 + 1.6x = 0$ ***M1***

$x = 5.33 \left(5.33333..., \frac{16}{3}\right)$ ***A1***

$y = -0.1 \times 5.33333...^3 + 0.8 \times 5.33333...^2$ ***(M1)***

Note: Award ***M1*** for substituting their zero for $\frac{dy}{dx}$ $(5.333\ldots)$ into y.

7.59 m $(7.58519...)$ ***A1***

Note: Award ***M0A0M0A0*** for an unsupported 7.59. Award at most ***M0A0M1A0*** if only the last two lines in the solution are seen. Award at most ***M1A0M1A1*** if their $x = 5.33$ is not seen.

[6 marks]

(b) $A = \frac{1}{2} \times 2\left((2.4 + 0) + 2(6.4 + 7.2)\right)$ ***(A1)(M1)***

Note: Award ***A1*** for $h = 2$ seen. Award ***M1*** for correct substitution into the trapezoidal rule (the zero can be omitted in working).

$= 29.6 \text{ m}^2$ ***A1***

[3 marks]

(c) (i) $A = \int_2^8 -0.1x^3 + 0.8x^2 \, dx$ **OR** $A = \int_2^8 y \, dx$ ***A1A1***

Note: Award ***A1*** for a correct integral, ***A1*** for correct limits in the correct location. Award at most ***A0A1*** if dx is omitted.

(ii) $A = 32.4 \text{ m}^2$ ***A2***

Note: As per the marking instructions, ***FT*** from their integral in part (c)(i). Award at most ***A1FTA0*** if their area is >48, this is outside the constraints of the question (a 6x8 rectangle).

[4 marks]

Total [13 marks]

Question 3

(a) evidence of splitting diagram into equilateral triangles **_M1_**

area $=6\left(\frac{1}{2}x^2\sin 60^\circ\right)$ **_A1_**

$=\frac{3\sqrt{3}x^2}{2}$ **_AG_**

Note: The ***AG*** line must be seen for the final ***A1*** to be awarded.

[2 marks]

(b) total surface area of prism $1200=2\left(3x^2\frac{\sqrt{3}}{2}\right)+6xh$ **_M1A1_**

Note: Award ***M1*** for expressing total surface areas as a sum of areas of rectangles and hexagons, and ***A1*** for a correctly substituted formula, equated to 1200.

$h=\frac{400-\sqrt{3}x^2}{2x}$ **_A1_**

volume of prism $=\frac{3\sqrt{3}}{2}x^2\times h$ **_(M1)_**

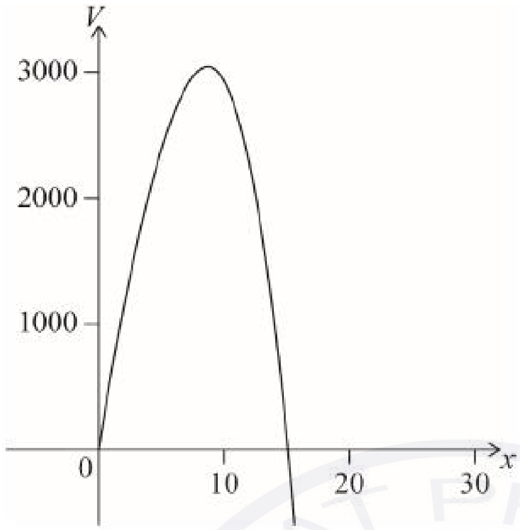
$=\frac{3\sqrt{3}}{2}x^2\left(\frac{400-\sqrt{3}x^2}{2x}\right)$ **_A1_**

$=300\sqrt{3}x-\frac{9}{4}x^3$ **_AG_**

Note: The ***AG*** line must be seen for the final ***A1*** to be awarded.

[5 marks]

(c)



A1A1

> **Note:** Award ***A1*** for correct shape, ***A1*** for roots in correct place with some indication of scale (indicated by a labelled point).

[2 marks]

(d) $\frac{\mathrm{d}V}{\mathrm{d}x} = 300\sqrt{3} - \frac{27}{4}x^2$ **A1A1**

> **Note:** Award ***A1*** for a correct term.

[2 marks]

(e) from the graph of V or $\frac{\mathrm{d}V}{\mathrm{d}x}$ **OR** solving $\frac{\mathrm{d}V}{\mathrm{d}x} = 0$ ***(M1)***

$x = 8.77$ $(8.77382...)$ ***A1***

[2 marks]

(f) from the graph of V **OR** substituting their value for x into V *(M1)*

$V_{\max} = 3040\text{ cm}^3$ (3039.34...) ***A1***

[2 marks]

(g) **EITHER**

wasted space / spheres do not pack densely (tesselate) ***A1***

OR

the model uses exterior values / assumes infinite thinness of materials and hence the modelled volume is not the true volume ***A1***

[1 mark]

Total [16 marks]

Question 4

(a) (i) $f'(x)=\frac{-2x}{50}+2\ \left(=\frac{-x}{25}+2,\ -0.04x+2\right)$ ***A1A1***

Note: Award ***A1*** for each correct term. Award at most ***A0A1*** if extra terms are seen.

(ii) $0=\frac{-x}{25}+2$ **OR** sketch of $f'(x)$ with x-intercept indicated ***M1***

$x=50$ ***A1***

$y=80$ ***A1***

$(50,\ 80)$

Note: Award ***M0A0A1*** for the coordinate $(50,\ 80)$ seen either with no working or found from a graph of $f(x)$.

[5 marks]

(b) (i) $\int_0^{70}\frac{-x^2}{50}+2x+30\ \mathrm{d}x$ ***A1A1***

Note: Award ***A1*** for a correct integral, ***A1*** for correct limits in the correct location. Award at most ***A0A1*** if $\mathrm{d}x$ is omitted.

(ii) (Area =) $4710\ \mathrm{m}^2\ \left(4713.33\ldots,\ \frac{14140}{3}\right)$ ***A2***

[4 marks]

(c) (i) $\frac{11.4}{4713.33\ldots}\times 100\%$ **OR** $\left|\frac{4701.93\ldots-4713.33\ldots}{4713.33\ldots}\right|\times 100\%$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution into the percentage error formula.

$0.242\%\ \ (0.241867\ldots\%)$ ***A1***

Note: Percentage sign is required. Accept $0.242038\ldots\%$ if 4710 is used.

(ii) **EITHER**

reduce the width of the intervals (trapezoids) ***A1***

OR

increase the number of intervals (trapezoids) ***A1***

Note: Accept equivalent statements. Award ***A0*** for the ambiguous answer "increase the intervals".

[3 marks]

(d) (i) width of the square is $70-x$ **OR** the length of the square is $\frac{-x^2}{50}+2x+30$

(M1)

Note: Award ***(M1)*** for $70-x$ seen anywhere. Accept $\frac{-x^2}{50}+2x+30$ but only if this expression is explicitly identified as a dimension of the square.

in term of x, equating the length to the width ED ***(M1)***

$\frac{-x^2}{50}+2x+30=70-x$

$(x=14.7920\ldots$ or $135.21)$

$(x=)\ 14.8\ \text{m}\ (14.7920\ldots)$ ***A1***

Note: Award ***M0M0A0*** for an unsupported answer of 15. Award at most ***M1M0A0*** for an approach which leads to $A'(x)=0$. This will lead to a square base which extends beyond the east boundary of the property. Similar for any solution where F is not on the northern boundary, or GH is not on the east boundary.

(ii) **EITHER**

$(70-14.7920\ldots)^2$ ***(M1)***

OR

$(55.2079\ldots)^2$ ***(M1)***

OR

$\left(\frac{-(14.7920\ldots)^2}{50}+2(14.7920\ldots)+30\right)^2$ ***(M1)***

THEN

$(\text{Area}=)\ 3050\ \text{m}^2\ (3047.92\ldots)$ ***A1***

Note: Follow through from part (d)(i), provided x is between 0 and 70. Award at most ***M1A0*** if their answer is outside the range of their $[0, 4713.33\ldots]$ from part (b).

[5 marks]
Total [17 marks]