Subject – Math AI(Standard Level)
Topic - Function
Year - May 2021 – Nov 2022
Paper -2
Questions

Question 1

[Maximum mark: 17]

The braking distance of a vehicle is defined as the distance travelled from where the brakes are applied to the point where the vehicle comes to a complete stop.

The speed, $s\,\text{m s}^{-1}$, and braking distance, $d\,\text{m}$, of a truck were recorded. This information is summarized in the following table.

Speed, $s\,\text{m s}^{-1}$	0	6	10
Braking distance, $d\,\text{m}$	0	12	60

This information was used to create Model A, where d is a function of s, $s \geq 0$.

Model A: $d(s) = ps^2 + qs$, where $p, q \in \mathbb{Z}$

At a speed of $6\,\text{m s}^{-1}$, Model A can be represented by the equation $6p + q = 2$.

(a) (i) Write down a second equation to represent Model A, when the speed is $10\,\text{m s}^{-1}$.

(ii) Find the values of p and q [4]

(b) Find the coordinates of the vertex of the graph of $y = d(s)$. [2]

(c) Using the values in the table and your answer to part (b), sketch the graph of $y = d(s)$ for $0 \leq s \leq 10$ and $-10 \leq d \leq 60$, clearly showing the vertex. [3]

(d) Hence, identify why Model A may not be appropriate at lower speeds. [1]

Additional data was used to create Model B, **a revised model** for the braking distance of a truck.

Model B: $d(s) = 0.95s^2 - 3.92s$

(e) Use Model B to calculate an estimate for the braking distance at a speed of $20\,\text{m s}^{-1}$. [2]

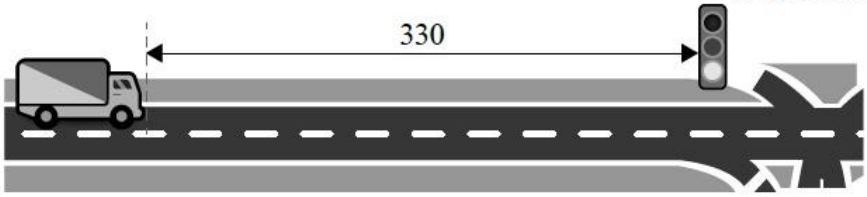
The actual braking distance at $20\,\text{m s}^{-1}$ is $320\,\text{m}$.

(f) Calculate the percentage error in the estimate in part (e). [2]

It is found that once a driver realizes the need to stop their vehicle, 1.6 seconds will elapse, on average, before the brakes are engaged. During this reaction time, the vehicle will continue to travel at its original speed.

A truck approaches an intersection with speed $s\,\mathrm{m\,s^{-1}}$. The driver notices the intersection's traffic lights are red and they must stop the vehicle within a distance of $330\,\mathrm{m}$.

diagram not to scale



(g) Using model B and taking reaction time into account, calculate the maximum possible speed of the truck if it is to stop before the intersection. [3]