

Subject – Math AI(Standard Level)
Topic - Function
Year - May 2021 – Nov 2024
Paper -1
Questions

Question 1

[Maximum mark: 17]

The braking distance of a vehicle is defined as the distance travelled from where the brakes are applied to the point where the vehicle comes to a complete stop.

The speed, $s \text{ m s}^{-1}$, and braking distance, $d \text{ m}$, of a truck were recorded. This information is summarized in the following table.

Speed, $s \text{ m s}^{-1}$	0	6	10
Braking distance, $d \text{ m}$	0	12	60

This information was used to create Model A, where d is a function of s , $s \geq 0$.

$$\text{Model A: } d(s) = ps^2 + qs, \text{ where } p, q \in \mathbb{Z}$$

At a speed of 6 m s^{-1} , Model A can be represented by the equation $6p + q = 2$.

- (a) (i) Write down a second equation to represent Model A, when the speed is 10 m s^{-1} .
(ii) Find the values of p and q [4]
- (b) Find the coordinates of the vertex of the graph of $y = d(s)$. [2]
- (c) Using the values in the table and your answer to part (b), sketch the graph of $y = d(s)$ for $0 \leq s \leq 10$ and $-10 \leq d \leq 60$, clearly showing the vertex. [3]
- (d) Hence, identify why Model A may not be appropriate at lower speeds. [1]

Additional data was used to create Model B, a revised model for the braking distance of a truck.

$$\text{Model B: } d(s) = 0.95s^2 - 3.92s$$

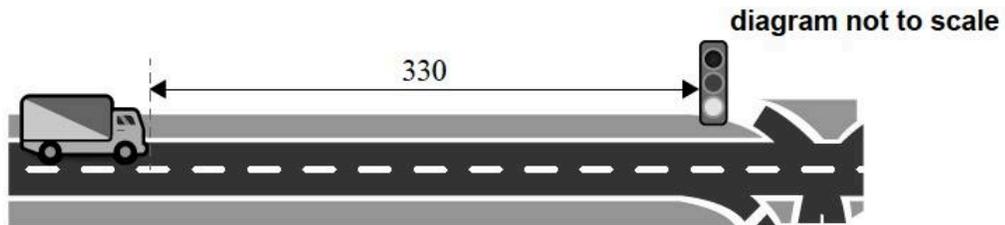
- (e) Use Model B to calculate an estimate for the braking distance at a speed of 20 m s^{-1} . [2]

The actual braking distance at 20 m s^{-1} is 320 m.

- (f) Calculate the percentage error in the estimate in part (e). [2]

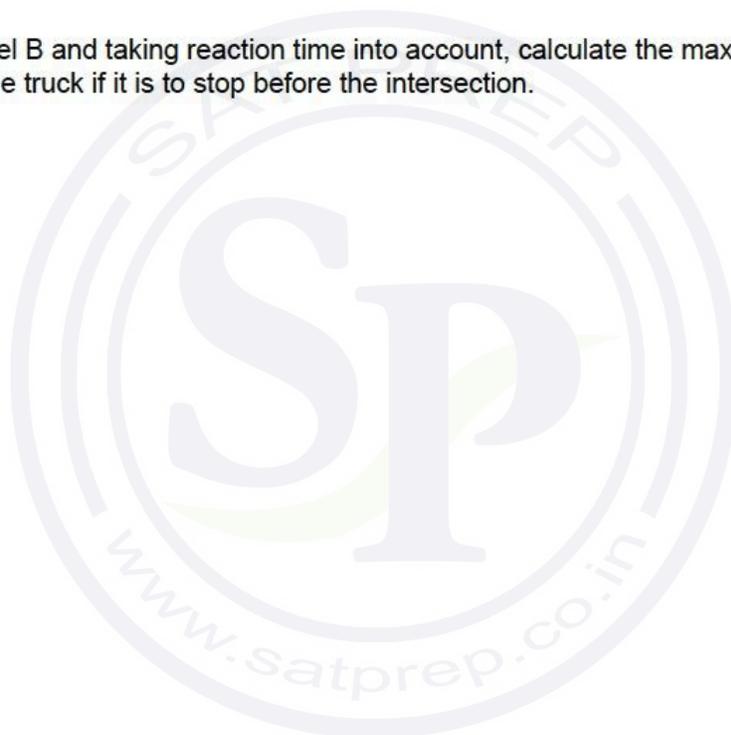
It is found that once a driver realizes the need to stop their vehicle, 1.6 seconds will elapse, on average, before the brakes are engaged. During this reaction time, the vehicle will continue to travel at its original speed.

A truck approaches an intersection with speed $s \text{ m s}^{-1}$. The driver notices the intersection's traffic lights are red and they must stop the vehicle within a distance of 330 m.



- (g) Using model B and taking reaction time into account, calculate the maximum possible speed of the truck if it is to stop before the intersection.

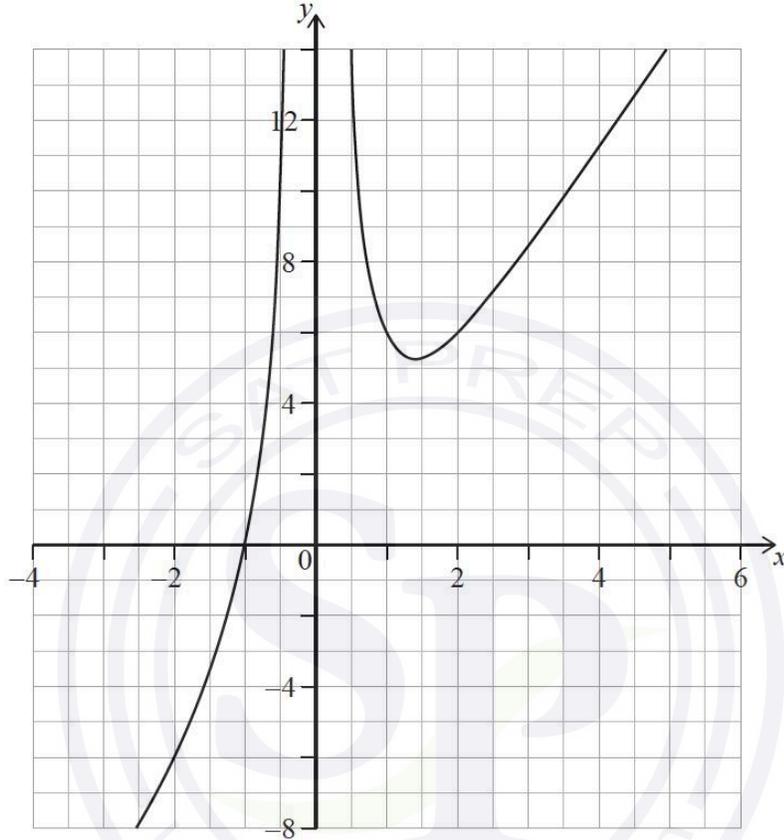
[3]



Question 2

[Maximum mark: 16]

Consider the function $f(x) = 3x - 1 + 4x^{-2}$. Part of the graph of $y = f(x)$ is shown below.



The function is defined for all values of x except for $x = a$.

(a) Write down the value of a . [1]

(b) Use your graphic display calculator to find the coordinates of the local minimum. [2]

The equation $f(x) = w$, where $w \in \mathbb{R}$, has three solutions.

(c) Identify one possible value for w . [1]

The line $y = mx - \frac{1}{4}$ is tangent to $f(x)$ when $x = -4$.

(d) Write down whether the value of m is positive or negative. Justify your answer. [2]

A second function is given by $g(x) = kp^x - 9$, where $p > 0$. The graph of $y = g(x)$ intersects the y -axis at point $A(0, -5)$ and passes through point $B(3, 4.5)$.

(e) Find the value of

(i) k ;

(ii) p .

[4]

(f) Write down the equation of the horizontal asymptote of $y = g(x)$.

[2]

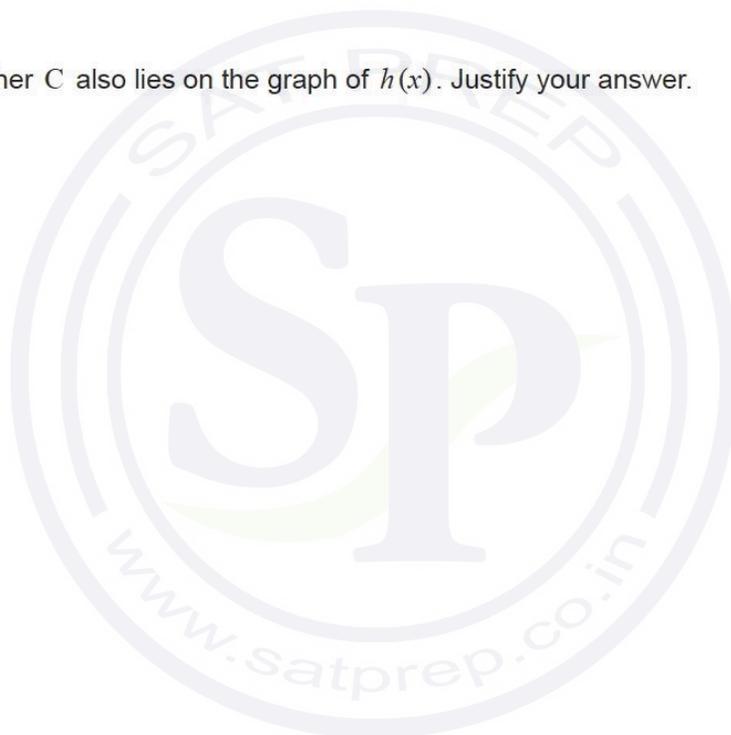
(g) Find the solution of $f(x) = g(x)$ when $x > 0$.

[2]

Consider a third function, h , where $h(x) = f(x) + g(x)$. The point $C(-1, q)$ lies on the graph of $g(x)$.

(h) State whether C also lies on the graph of $h(x)$. Justify your answer.

[2]



Question 3

[Maximum mark: 15]

A scientist is conducting an experiment on the growth of a certain species of bacteria.

The population of the bacteria, P , can be modelled by the function

$$P(t) = 1200 \times k^t, \quad t \geq 0,$$

where t is the number of hours since the experiment began, and k is a positive constant.

- (a) (i) Write down the value of $P(0)$.
- (ii) Interpret what this value means in this context. [2]

3 hours after the experiment began, the population of the bacteria is 18 750.

- (b) Find the value of k . [2]
- (c) Find the population of the bacteria 1 hour and 30 minutes after the experiment began. [2]

The scientist conducts a second experiment with a different species of bacteria. The population of this bacteria, S , can be modelled by the function

$$S(t) = 5000 \times 1.65^t, \quad t \geq 0,$$

where t is the number of hours since both experiments began.

- (d) Find the value of t when the two populations of bacteria are equal. [2]

It takes 2 hours and m minutes for the number of bacteria in the second experiment to reach 19 000.

- (e) Find the value of m , giving your answer as an integer value. [4]

The bacteria in the second experiment are growing inside a container. The scientist models the volume of each bacterium in the second experiment to be $1 \times 10^{-18} \text{ m}^3$, and the available volume inside the container is $2.1 \times 10^{-5} \text{ m}^3$.

- (f) Determine how long it would take for the bacteria to fill the container. [3]

Question 4

[Maximum mark: 19]

Mosaic Industries is an international company that sells cell phone cases. Their market research predicts that the average number of cases they will sell each month is modelled by the equation

$$n = 20\,000 - 1000x$$

where x represents the selling price, in euro (EUR), of each case.

A salesperson suggests that Mosaic Industries should sell the cases for 16EUR each.

- (a) Find the average number of cases that this model predicts Mosaic Industries will sell at this price. [2]
- (b) Calculate Mosaic Industries' average monthly income, before any expenses, at this selling price. [2]
- (c) Hence, write down the function $R(x)$ that can be used to predict Mosaic Industries' average monthly income, before expenses, at any selling price, x . [1]

Mosaic Industries has 10000EUR of fixed monthly operational costs. Additionally, Mosaic Industries must pay their phone case supplier 10EUR for each case.

- (d) Calculate Mosaic Industries' average monthly profit if they sell each case at a price of 16EUR. [3]
- (e) Show that the average monthly profit for any selling price, x , can be found using the function $P(x) = -1000x^2 + 30\,000x - 210\,000$. [2]
- (f) (i) Find $P'(x)$.
(ii) Show that the salesperson's selling price does not maximize their average monthly profit. [4]

Mosaic Industries negotiates a new deal with their phone case supplier. Under the new deal, the supplier agrees to discount the cost of each case based on the number of cases purchased by Mosaic Industries. The cost charged by the supplier for each case can be found using the function

$$C(n) = 10 - 0.0001n$$

where n represents the number of cases sold by Mosaic Industries.

- (g) Find the function that can be used to find Mosaic Industries' average monthly profit using the new deal from the supplier. [3]
- (h) Hence, find the selling price, per case, that Mosaic Industries should choose in order to maximize their average monthly profit under the new deal. [2]

Question 5

[Maximum mark: 19]

Howell Industries is an international company that sells travel coffee mugs. Their market research predicts that the average number of mugs they will sell each month is modelled by the equation

$$n = 20\,000 - 1000x$$

where x represents the selling price, in euro (EUR), of each mug.

A salesperson suggests that Howell Industries should sell the mugs for 16 EUR each.

- (a) Find the average number of mugs that this model predicts Howell Industries will sell at this price. [2]
- (b) Calculate Howell Industries' average monthly income, before any expenses, at this selling price. [2]
- (c) Hence, write down the function $R(x)$ that can be used to predict Howell Industries' average monthly income, before expenses, at any selling price, x . [1]

Howell Industries has 10 000 EUR of fixed monthly operational costs. Additionally, Howell Industries must pay their mug supplier 10 EUR for each coffee mug.

- (d) Calculate Howell Industries' average monthly profit if they sell each mug at a price of 16 EUR. [3]
- (e) Show that the average monthly profit for any selling price, x , can be found using the function $P(x) = -1000x^2 + 30\,000x - 210\,000$. [2]
- (f) (i) Find $P'(x)$.
(ii) Show that the salesperson's selling price does not maximize their average monthly profit. [4]

Howell Industries negotiates a new deal with their mug supplier. Under the new deal, the supplier agrees to discount the cost of each mug based on the number of mugs purchased by Howell Industries. The cost charged by the supplier for each mug can be found using the function

$$C(n) = 10 - 0.0001n$$

where n represents the number of mugs sold by Howell Industries.

- (g) Find the function that can be used to find Howell Industries' average monthly profit using the new deal from the supplier. [3]
- (h) Hence, find the selling price, per mug, that Howell Industries should choose in order to maximize their average monthly profit under the new deal. [2]

Question 6

[Maximum mark: 16]

In a given week, the number of students in a particular primary school that were absent due to headlice (H), influenza (I) and/or chickenpox (C) were recorded as follows.

The primary school has 500 students.

- 35 students had headlice only
- 20 students had influenza only
- 5 students had chickenpox only
- 4 students had headlice and influenza but not chickenpox
- 2 students had headlice and chickenpox but not influenza
- 3 students had influenza and chickenpox but not headlice
- 1 student had headlice, influenza and chickenpox

- (a) Draw a Venn diagram to represent this information. [3]
- (b) Calculate the number of students who did not have headlice or influenza or chickenpox. [2]

A student is chosen at random from all the students in the school.

- (c) Find the probability that this student has
- (i) headlice.
 - (ii) influenza given that the student has headlice. [4]

Diego is a teacher in the school. He believes that the number of students, n , who have had influenza during the first t days of the school year, can be modelled by the function

$$n(t) = 250 - 240(2)^{kt}, \quad k \in \mathbb{R}.$$

- (d) Use Diego's model to calculate the number of students who started the school year with influenza. [2]

It is known that 130 students have had influenza during the first 10 days of the school year.

- (e) Find the value of k . [2]
- (f) Using this model, calculate how many days it will take for 200 students to have had influenza since the start of the school year. [2]

By the last day of the school year, it is known that 300 students have had influenza.

- (g) Comment on the appropriateness of Diego's model. [1]