

Subject - Math AI(Standard Level)
Topic - Function
Year - May 2021 - Nov 2024
Paper -1
Answers

Question 1

(a) (i) $p(10)^2 + q(10) = 60$

M1

$10p + q = 6$ ($100p + 10q = 60$)

A1

(ii) $p = 1, q = -4$

A1A1

Note: If p and q are both incorrect then award **M1A0** for an attempt to solve simultaneous equations.

(b) $(2, -4)$

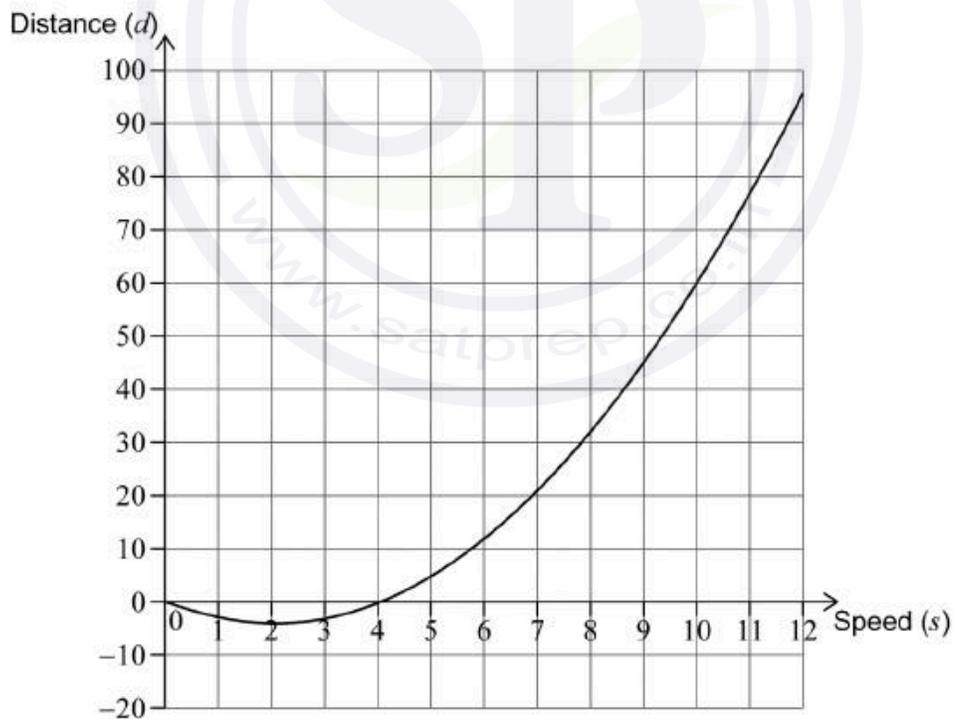
[4 marks]

A1A1

Note: Award **A1** for each correct coordinate.
Award **A0A1** if parentheses are missing.

[2 marks]

(c)



A3

Note: Award **A1** for smooth quadratic curve on labelled axes and within correct window. Award **A1** for the curve passing through (0, 0) and (10, 60). Award **A1** for the curve passing through their vertex. Follow through from part (b).

[3 marks]

(d) the graph indicates there are negative stopping distances (for low speeds) **R1**

Note: Award **R1** for identifying that a feature of their graph results in negative stopping distances (vertex, range of stopping distances...).

[1 mark]

(e) $0.95 \times 20^2 - 3.92 \times 20$
 $= 302(\text{m}) (301.6\dots)$

(M1)

A1

[2 marks]

(f) $\left| \frac{301.6 - 320}{320} \right| \times 100$
 $= 5.75(\%)$

M1

A1

[2 marks]

(g) $330 = 1.6 \times s + 0.95 \times s^2 - 3.92 \times s$

M1A1

Note: Award **M1** for an attempt to find an expression including stopping distance (model B) and reaction distance, equated to 330. Award **A1** for a completely correct equation.

$19.9(\text{ms}^{-1}) (19.8988\dots)$

A1

[3 marks]

Total [17 marks]

Question 2

(a) $(a =) 0$

A1

[1 mark]

(b) $(1.39, 5.24)$ $((1.38672\dots, 5.24025\dots))$

A1A1

Note: Award **A1** for each correct coordinate; do not accept 2 sf values. Award at most **A0A1** if parentheses are missing. Accept $x = 1.39$, $y = 5.24$.

[2 marks]

(c) any value greater than 5.24025...

A1

Note: Accept a specific value **OR** a description of the correct interval for all values.

[1 mark]

(d) **METHOD 1**

attempt to relate gradient of function to graph of f

R1

e.g. the function is increasing at $x = -4$

$m > 0$

A1

METHOD 2

attempt to find value of $f'(-4)$

R1

$\left(= \frac{25}{8} = 3.125 \right)$

$m > 0$

A1

METHOD 3

attempt to substitute $(-4, -12.75)$ into equation of tangent

$$-12.75 = m(-4) - \frac{1}{4} \text{ so } m = 3.125 \quad \text{R1}$$

$$m > 0 \quad \text{A1}$$

[2 marks]

(e) (i) attempt to substitute $(0, -5)$ or $(3, 4.5)$ into $g(x)$ **(M1)**

$$-5 = k \times p^0 - 9$$

$$(k =) 4 \quad \text{A1}$$

(ii) attempt to substitute $(3, 4.5)$ and their part (e)(i) into $g(x)$ **(M1)**

$$4.5 = 4 \times p^3 - 9$$

$$p^3 = 3.375$$

$$(p =) 1.5 \quad \text{A1}$$

[4 marks]

(f) $y = -9$

A2

[2 marks]

(g) $(x =) 4$

(M1)A1

[2 marks]

(h) **METHOD 1:**

$$f(-1) = 0 \Rightarrow h(-1) = g(-1) \quad (\text{This justification could be graphical.}) \quad \mathbf{R1}$$

yes **A1**

Note: Award **R1** for any reasoning based on the fact that -1 is the x -intercept of f .
Accept equivalent arguments in words. Do not award **R0A1**.

METHOD 2:

$$g(-1) = -\frac{19}{3} \quad (-6.33333\dots)$$

$$h(x) = 3x - 1 + 4x^{-2} + 4 \times 1.5^x - 9$$

$$h(-1) = 3(-1) - 1 + 4(-1)^{-2} + 4 \times 1.5^{-1} - 9$$

$$h(-1) = -\frac{19}{3} \quad (-6.33333\dots) \quad \mathbf{R1}$$

yes **A1**

Note: Award **R1** for showing that $g(-1) = -\frac{19}{3}$ and $h(-1) = -\frac{19}{3}$. Do not award

R0A1.

[2 marks]

[Total: 16 marks]

Question 3

- (a) (i) 1200 A1
 (ii) the initial population of the bacteria A1
[2 marks]
- (b) $1200 \times k^3 = 18750$ (A1)
 $(k =) 2.5$ A1
[2 marks]
- (c) $1200 \times 2.5^{1.5}$ (A1)
 4740 (4743.41...) A1

Note: Do not penalize if final answer is not given as an integer. Award **(A1)A0** for an answer of 3950 (3949.14...) from use of 1.3 in the exponent, but only if working is shown.

[2 marks]

- (d) equating $P(t)$ and $S(t)$ **OR** equating each function to a common variable (M1)
 $1200 \times 2.5^t = 5000 \times 1.65^t$; $1200 \times 2.5^t = x$ and $5000 \times 1.65^t = x$
 $t = 3.43$ (hours) (3.43456...) A1
[2 marks]

- (e) **METHOD 1**
 $5000 \times 1.65^t = 19000$ (M1)
 $(t =) 2.66586...$ **OR** $(t - 2 =) 0.66586...$ (seen) (A1)
 multiplying by 60 seen to convert to minutes (M1)
 $(m = 39.9521...)$
 $(m =) 40$ (minutes) **OR** 2 hours and 40 minutes A1

- METHOD 2**
 equating an expression for $S(t)$ to 19000 (M1)
 expressing t as $2 + \frac{m}{60}$ (A1)
 $5000 \times 1.65^{2 + \frac{m}{60}} = 19000$
 $2 + \frac{m}{60} = 2.66586...$ A1

- $(m =) 40$ (minutes) **OR** 2 hours and 40 minutes A1

Note: Award **(M1)(A1)(M1)A0** for an answer of 39.9521... or 39 with or without working.

[4 marks]

(f) **EITHER (find volume of all bacteria)**
multiplying total population of bacteria by the volume of bacterium **(M1)**
 $1 \times 10^{-18} \times (5000 \times 1.65^t)$

setting expression equal to 2.1×10^{-5} and attempt to solve (e.g. sketch) **(M1)**

OR (find total number of bacteria to fill container)
attempting to find the total number of bacteria to fill container **(M1)**

$$2.1 \times 10^{-5} = n \times 1 \times 10^{-18} \quad \text{OR} \quad \frac{2.1 \times 10^{-5}}{1 \times 10^{-18}} \quad \text{OR} \quad 2.1 \times 10^{13}$$

setting value equal to $S(t)$ and attempt to solve (e.g. sketch) **(M1)**

Note: If $\frac{2.1 \times 10^{-5}}{1 \times 10^{-18}}$ is seen but candidate has an incorrect total number of bacteria, the second **(M1)** can still be awarded for setting their incorrect value equal to $S(t)$ and attempting to solve.

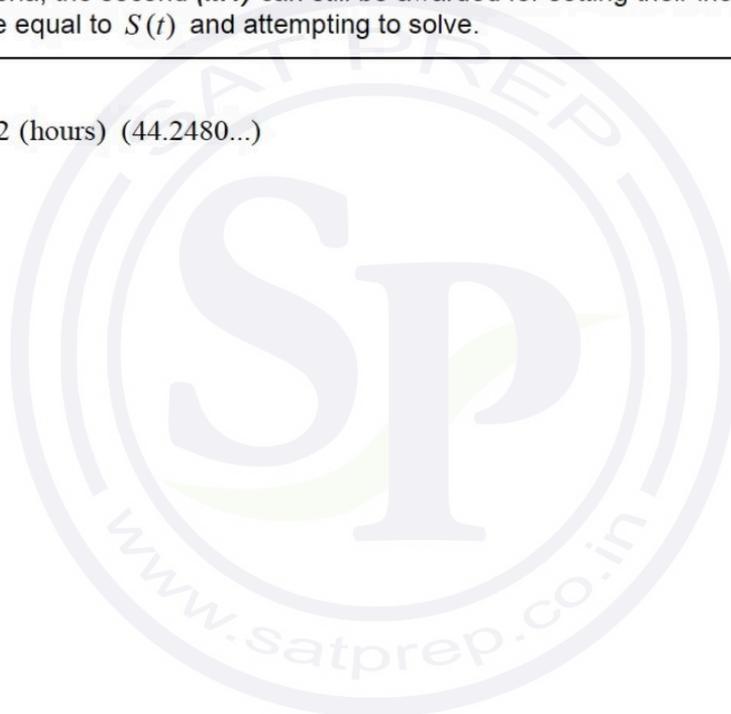
THEN

$$t = 44.2 \text{ (hours) (44.2480...)}$$

A1

[3 marks]

Total [15 marks]



Question 4

- (a) attempt to substitute 16 into the given formula **(M1)**

$$n = 20000 - 1000(16)$$

$$n = 4000$$

A1
[2 marks]

- (b) multiplying their answer to part (a) by 16
(average monthly income =) 16×4000
64000 (EUR)

(M1)
A1
[2 marks]

- (c) $R(x) = x(20000 - 1000x)$ **OR** $R(x) = 20000x - 1000x^2$

A1
[1 mark]

- (d) **EITHER**

attempt to find total costs (both fixed and variable) **AND** subtract from their 64000

(M1)
(A1)

$$64000 - (10000 + 10 \times 4000)$$

OR

attempt to find total profit from cases **AND** subtract fixed costs
 $(16 - 10) \times 4000 - 10000$

(M1)
(A1)

THEN

$$= 14000 \text{ (EUR)}$$

A1
[3 marks]

(e) **METHOD 1**

attempt to subtract total costs in terms of x from their $R(x)$

(M1)

$$(P(x) =)(20000x - 1000x^2) - (10000 + 10(20000 - 1000x))$$

correct intermediate step leading to given answer

A1

(e.g. correct expansion of $10(20000 - 1000x)$)

$$P(x) = -1000x^2 + 30000x - 210000$$

AG

Note: Do not award the **A1** mark if the **AG** line is not stated.

METHOD 2

attempt to express profit per case, and then subtract fixed monthly costs

(M1)

$$(P(x) =)(x - 10)(20000 - 1000x) - 10000$$

correct expansion leading to the given answer

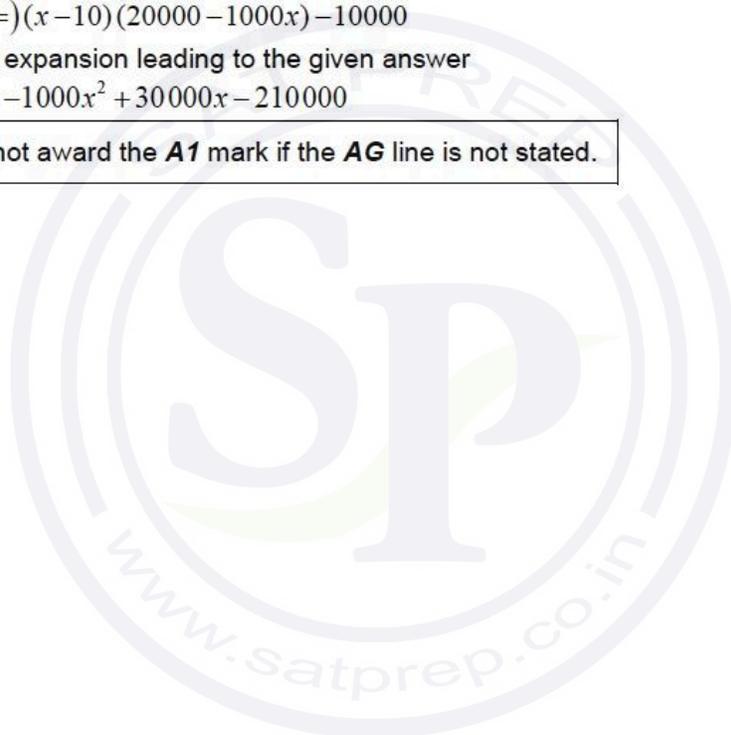
A1

$$P(x) = -1000x^2 + 30000x - 210000$$

AG

Note: Do not award the **A1** mark if the **AG** line is not stated.

[2 marks]



(f) (i) $(P'(x) =) -2000x + 30000$ **A1A1**

Note: Award at most **A1A0** if additional terms are seen.

(ii) **METHOD 1**

$$P'(x) = 0 \text{ OR sketch of } P(x) \text{ OR use of } x = -\frac{b}{2a}$$

$x = 15$ is the maximum, not 16

hence salesman's price is not the optimum

A1R1
AG

Note: Award **A1** for $x = 15$, and **R1** for comparing it to 16 **OR** making a statement that is some version of the **AG** line. It is possible to award **A1R0**.

METHOD 2

$$P'(16) = -2000 \neq 0$$

hence salesman's price is not the optimum

A1R1
AG

Note: Award **A1** for finding an appropriate value, and **R1** for comparing it to zero. It is possible to award **A1R0**. To award the **R1** a statement that is some version of the **AG** line must also be given.

METHOD 3

finding $P(x)$ for any value from $14 < x < 16$

comparing this value to their part (d)

hence salesman's price is not the optimum

A1
R1
AG

Note: It is possible to award **A1R0**. To award the **R1** a statement that is some version of the **AG** line must also be given.

[4 marks]

(g) substituting the expression for n into cost function, $C(n)$. **(M1)**

$$(\text{cost} =) 10 - 0.0001(20000 - 1000x) = 8 + 0.1x$$

substituting $C(x)$ into the total cost expression and subtracting for $R(x)$ **(M1)**

$$(\text{New } P(x) =) (20000x - 1000x^2) - (10000 + (8 + 0.1x)(20000 - 1000x)) \quad \mathbf{A1}$$

$$((\text{New } P(x) =) -900x^2 + 26000x - 170000)$$

[3 marks]

(h) 14.4 (EUR) $(14.4444\dots, \frac{130}{9})$ **A2**

[2 marks]

[Total 19 marks]

Question 5

- (a) attempt to substitute 16 into the given formula (M1)
- $$n = 20000 - 1000(16)$$
- $$n = 4000$$
- A1
[2 marks]
- (b) multiplying their answer to part (a) by 16 (M1)
(average monthly income =) 16×4000
64000 (EUR)
- A1
[2 marks]
- (c) $R(x) = x(20000 - 1000x)$ OR $R(x) = 20000x - 1000x^2$ A1
[1 mark]
- (d) **EITHER**
attempt to find total costs (both fixed and variable) AND subtract from their 64000 (M1)
 $64000 - (10000 + 10 \times 4000)$ (A1)
- OR**
attempt to find total profit from mugs AND subtract fixed costs (M1)
 $(16 - 10) \times 4000 - 10000$ (A1)
- THEN**
= 14000 (EUR) A1
[3 marks]
- (e) **METHOD 1**
attempt to subtract total costs in terms of x from their $R(x)$ (M1)
 $(P(x) =)(20000x - 1000x^2) - (10000 + 10(20000 - 1000x))$
- correct intermediate step leading to given answer A1
(e.g. correct expansion of $10(20000 - 1000x)$)
- $$P(x) = -1000x^2 + 30000x - 210000$$
- AG
- Note:** Do not award the A1 mark if the AG line is not stated.
- METHOD 2**
attempt to express profit per mug, and then subtract fixed monthly costs (M1)
 $(P(x) =)(x - 10)(20000 - 1000x) - 10000$
- correct expansion leading to the given answer A1
 $P(x) = -1000x^2 + 30000x - 210000$ AG
- Note:** Do not award the A1 mark if the AG line is not stated.
- [2 marks]

(f) (i) $(P'(x) =) -2000x + 30000$ **A1A1**

Note: Award at most **A1A0** if additional terms are seen.

(ii) **METHOD 1**

$$P'(x) = 0 \text{ OR sketch of } P(x) \text{ OR use of } x = -\frac{b}{2a}$$

$x = 15$ is the maximum, not 16

hence salesman's price is not the optimum

A1R1
AG

Note: Award **A1** for $x = 15$, and **R1** for either comparing it to 16 **OR** making a statement that is some version of the **AG** line. It is possible to award **A1R0**.

METHOD 2

$$P'(16) = -2000 \neq 0$$

hence salesman's price is not the optimum

A1R1
AG

Note: Award **A1** for finding an appropriate value, and **R1** for comparing it to zero. It is possible to award **A1R0**. To award the **R1** a statement that is some version of the **AG** line must also be given.

METHOD 3

finding $P(x)$ for any value from $14 < x < 16$

comparing this value to their part (d)

hence salesman's price is not the optimum

A1
R1
AG

Note: It is possible to award **A1R0**. To award the **R1** a statement that is some version of the **AG** line must also be given.

[4 marks]

(g) substituting the expression for n into cost function, $C(n)$. **(M1)**

$$(\text{cost} =) 10 - 0.0001(20000 - 1000x) = 8 + 0.1x$$

substituting $C(x)$ into the total cost expression and subtracting for $R(x)$ **(M1)**

$$(\text{New } P(x) =) (20000x - 1000x^2) - (10000 + (8 + 0.1x)(20000 - 1000x)) \quad \mathbf{A1}$$

$$((\text{New } P(x) =) -900x^2 + 26000x - 170000)$$

[3 marks]

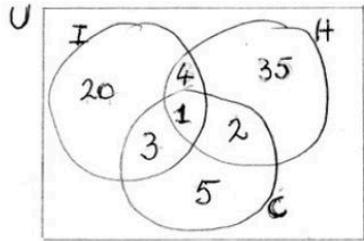
(h) 14.4 (EUR) $(14.4444\dots, \frac{130}{9})$ **A2**

[2 marks]

[Total 19 marks]

Question 6

(a)



A3

Note: Award **A1** for 1 in correct place, **A1** for 3, 2 and 4 correct, **A1** for 35, 20 and 5 correct. Award at most **A0A1A1** if the rectangle is omitted. Condone the omission of the 430, as explicitly asked for in part (b).

[3 marks]

(b) $35 + 4 + 20 + 3 + 1 + 2 + 5$
430

(M1)
A1

Note: The 430 may be seen in the Venn diagram.

[2 marks]

(c) (i) $\frac{42}{500}$ ($\frac{21}{250}$, 0.084, 8.4%)

A2

Note: Award **A1** for numerator, **A1** for denominator.

(ii) $\frac{5}{42}$ (0.119, 0.119047..., 11.9%)

A2

Note: **A1** for numerator, **A1** for denominator.

The first **A1** can be awarded for an attempt to use conditional probability with 0.084 on the denominator.

[4 marks]

(d) substituting $t = 0$ into given expression
10

(M1)
A1

[2 marks]

(e) $130 = 250 - 240(2)^{10k}$
 $k = -0.1$

(M1)
A1

[2 marks]

(f) $200 = 250 - 240(2)^{-0.1t}$
 $t = 22.6$ (22.6303..., 23) (days)

(M1)
A1

[2 marks]

- (g) **EITHER**
model does not predict n to go above 250 / reach 300 **A1**
- OR**
 $250 - 240 \times 2^{-0.1 \times 365} = 250$, so does not reach 300 **A1**
- OR**
there is no solution to $n(t) = 300$ **A1**
- OR**
correct sketch graph, with 250 and/or 300 labelled, and a supporting comment **A1**
- THEN**
hence Diego's model is not appropriate.

Note: Do not credit reasoning based on selecting arbitrary high values of t and finding the associated n value.

[1 mark]
[Total: 16 marks]

