

Subject - Math AI(Standard Level)
Topic - Geometry and Trigonometry
Year - May 2021 - Nov 2024
Paper -1
Answers

Question 1

(a) (i) 0.909 (0.909181...) **A2**

(ii) (very) strong and positive **A1A1**

Note: Award **A1** for (very) strong **A1** for positive.

[4 marks]

(b) $y = 1.14x + 0.578$ ($y = 1.14033...x + 0.578183...$) **A1A1**

[2 marks]

(c) (i) $1.14 \times 10 + 0.578$ **M1**
 12.0 (11.9814...) **A1**

(ii) no the estimate is not reliable **A1**
 outside the known data range **R1**

OR

a score greater than 10 is not possible **R1**

(d)

Competitors	A	B	C	D	E	F	G	H
Stan's rank	7	8	6	4	2	4	1	4
Minsun's rank	7	8	6	4.5	3	2	1	4.5

A1A1

[2 marks]

(e) (i) 0.933 (0.932673...) **A2**

(ii) Stan and Minsun strongly agree on the ranking of competitors. **A1A1**

[4 marks]

(f) decreasing the score to 9.1, does not change the rank of competitor G **A1**

[1 mark]

Total [17 marks]

Question 2

(a) $4 \times \frac{360^\circ}{12}$ OR $4 \times 30^\circ$

(M1)

120°

A1

[2 marks]

(b) substitution in cosine rule

(M1)

$$AB^2 = 10^2 + 6^2 - 2 \times 10 \times 6 \times \cos(120^\circ)$$

(A1)

$$AB = 14 \text{ cm}$$

A1

Note: Follow through marks in part (b) are contingent on working seen.

[3 marks]

(c) $\theta = 13 \times 6$

(M1)

$= 78^\circ$

A1

[2 marks]

(d) substitution into the formula for arc length

(M1)

$$l = \frac{78}{360} \times 2 \times \pi \times 10 \quad \text{OR} \quad l = \frac{13\pi}{30} \times 10$$

$$= 13.6 \text{ cm} \left(13.6135\dots, 4.33\pi, \frac{13\pi}{3} \right)$$

A1

[2 marks]

(e) substitution into the area of a sector

(M1)

$$A = \frac{78}{360} \times \pi \times 10^2 \quad \text{OR} \quad l = \frac{1}{2} \times \frac{13\pi}{30} \times 10^2$$

$$= 68.1 \text{ cm}^2 \left(68.0678\dots, 21.7\pi, \frac{65\pi}{3} \right)$$

A1

[2 marks]

(f) 23

A1

[1 mark]

(g) correct substitution
 $h = 10 \cos(160^\circ) + 13$
 $= 3.60 \text{ cm } (3.60307\dots)$

(M1)

A1

[2 marks]

(h) 10

A1

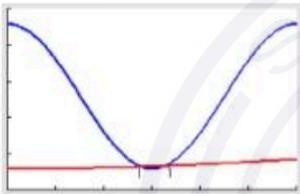
[1 mark]

(i) EITHER

$$10 \times \cos(\theta) + 13 = -10 \times \cos\left(\frac{\theta}{12}\right) + 13$$

(M1)

OR



(M1)

Note: Award **M1** for equating the functions. Accept a sketch of $h(\theta)$ and $g(\theta)$ with point(s) of intersection marked.

THEN

$$k = 196^\circ (196.363\dots)$$

A1

Note: The answer $166.153\dots$ is incorrect but the correct method is implicit. Award **(M1)A0**.

[2 marks]

Total [17 marks]

Question 3

(a) use of cosine rule

$$\hat{A}CB = \cos^{-1}\left(\frac{1005^2 + 1225^2 - 650^2}{2 \times 1005 \times 1225}\right)$$

$$= 32.0^\circ \text{ (31.9980...)}$$

(M1)

(A1)

A1

[3 marks]

(b) use of sine rule

$$\frac{DE}{\sin 31.9980\dots^\circ} = \frac{210}{\sin 100^\circ}$$

$$(DE =) 113 \text{ m (112.9937...)}$$

(M1)

(A1)

A1

[3 marks]

(c) **METHOD 1**

$$180^\circ - (100^\circ + \text{their part (a)})$$

$$= 48.0019\dots^\circ \text{ OR } 0.837791\dots$$

substituted area of triangle formula

$$\frac{1}{2} \times 112.9937\dots \times 210 \times \sin 48.002^\circ$$

$$8820 \text{ m}^2 \text{ (8817.18...)}$$

(M1)

(A1)

(M1)

(A1)

A1

METHOD 2

$$\frac{CE}{\sin(180 - 100 - \text{their part (a)})} = \frac{210}{\sin 100}$$

$$(CE =) 158.472\dots$$

substituted area of triangle formula

(M1)

(A1)

(M1)

EITHER

$$\frac{1}{2} \times 112.993\dots \times 158.472\dots \times \sin 100$$

(A1)

OR

$$\frac{1}{2} \times 210\dots \times 158.472\dots \times \sin(\text{their part (a)})$$

(A1)

THEN

$$8820 \text{ m}^2 \text{ (8817.18...)}$$

A1

METHOD 3

$$CE^2 = 210^2 + 112.993\dots^2 - (2 \times 210 \times 112.993\dots \times \cos(180 - 100 - \text{their part } (a))) \quad (M1)$$

$$(CE \Rightarrow) 158.472\dots \quad (A1)$$

substituted area of triangle formula (M1)

$$\frac{1}{2} \times 112.993\dots \times 158.472\dots \times \sin 100 \quad (A1)$$

$$8820 \text{ m}^2 \text{ (8817.18\dots)} \quad A1$$

[5 marks]

(d) $1005 - 210$ **OR** 795 (A1)

equating answer to part (c) to area of a triangle formula (M1)

$$8817.18\dots = \frac{1}{2} \times DF \times (1005 - 210) \times \sin 48.002\dots \quad (A1)$$

$$(DF \Rightarrow) 29.8 \text{ m (29.8473\dots)} \quad A1$$

[4 marks]

Total [15 marks]



Question 5

(a) $\tan(\theta) = \frac{6}{10}$ **(M1)**

$(\theta =) 31.0^\circ (30.9637\dots^\circ)$ **OR** $0.540 (0.540419\dots)$ **A1**

[2 marks]

(b) (i) (CV =) $40 \tan(\theta)$ **OR** (CV =) 4×6 **(M1)**

Note: Award **(M1)** for an attempt at trigonometry or similar triangles (e.g. ratios).

(CV =) 24 m **A1**

(ii) $(V =) \frac{1}{3} 80^2 \times 24 - \frac{1}{3} 60^2 \times 18$ **M1A1A1**

Note: Award **M1** for finding the difference between the volumes of two pyramids, **A1** for each correct volume expression. The final **A1** is contingent on correct working leading to the given answer.
If the correct final answer is not seen, award at most **M1A1A0**. Award **M0A0A0** for any height derived from $V = 29600$, including 18.875 or 13.875.

$(V =) 29600 \text{ m}^3$ **AG**

[5 marks]

(c) **METHOD 1**
 $\left(\frac{29600}{80} =\right) 370$ (days) **A1**

$(370 > 366)$ Joshua is correct **A1**

Note: Award **A0A0** for unsupported answer of "Joshua is correct". Accept $1.01\dots > 1$ for the first **A1** mark.

METHOD 2

$80 \times 366 = 29280 \text{ m}^3$ **OR** $80 \times 365 = 29200 \text{ m}^3$ **A1**

$(29280 < 29600)$ Joshua is correct **A1**

Note: The second **A1** can be awarded for an answer consistent with their result.

[2 marks]

(d) height of trapezium is $\sqrt{10^2 + 6^2}$ (=11.6619...) (M1)

area of trapezium is $\frac{80+60}{2} \times \sqrt{10^2 + 6^2}$ (= 816.333...) (M1)(A1)

(SA \Rightarrow) $4 \times \left(\frac{80+60}{2} \times \sqrt{10^2 + 6^2} \right) + 60^2$ (M1)

Note: Award **M1** for adding 4 times their (MNOP) trapezium area to the area of the (60 \times 60) base.

(SA \Rightarrow) 6870 m² (6865.33 m²)

A1

Note: No marks are awarded if the correct shape is not identified.

[5 marks]
Total: [14 marks]



Question 5

- (a) (i) maximum $h = 130$ metres
 (ii) minimum $h = 50$ metres

A1
 A1
 [2 marks]

- (b) (i) $(60 \div 12 =)$ 5 seconds
 (ii) $360 \div 5$

A1
 (M1)

Note: Award (M1) for 360 divided by their time for one revolution.

$= 72^\circ$

A1
 [3 marks]

- (c) (i) (amplitude =) 40

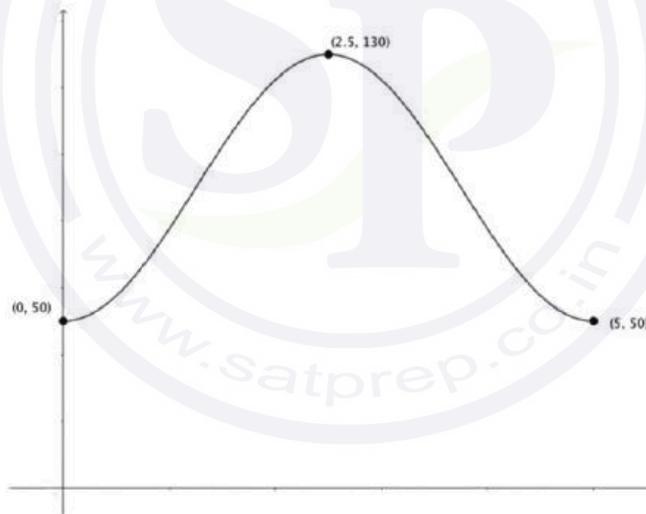
A1

- (ii) (period = $\frac{360}{72} =$) 5

A1

[2 marks]

- (d)



- Maximum point labelled with correct coordinates.
 At least one minimum point labelled. Coordinates seen for any minimum points must be correct.
 Correct shape with an attempt at symmetry and "concave up" evident as it approaches the minimum points. Graph must be drawn in the given domain.

A1
 A1
 A1

[3 marks]

(e) (i) $h = 90 - 40 \cos(144^\circ)$ (M1)
 $(h =) 122 \text{ m } (122.3606\dots)$ A1

(ii) evidence of $h = 100$ on graph OR $100 = 90 - 40 \cos(72t)$ (M1)
 t coordinates 3.55 (3.54892...) OR 1.45 (1.45107...) or equivalent (A1)

Note: Award A1 for either t -coordinate seen.

$= 2.10 \text{ seconds } (2.09784\dots)$ A1
 [5 marks]

(f) (i) $\frac{5 - 2.09784\dots}{5}$ (M1)
 $\frac{(2.902153\dots)}{5}$ (M1)
 $0.580 (0.580430\dots)$ A1

(ii) **METHOD 1**
 changing the frequency/dilation of the graph will not change the proportion of time that point C is visible. A1
 $0.580 (0.580430\dots)$ A1

METHOD 2
 correct calculation of relevant found values
 $\frac{(2.902153\dots)}{5/2}$ A1
 $0.580 (0.580430\dots)$ A1

Note: Award A0A1 for an unsupported correct probability.

[5 marks]
 Total: [20 marks]

Question 6

(a) (i) $\left(\frac{1}{2} \hat{A}OB = \right) \arccos\left(\frac{4}{4.5}\right) = 27.266\dots$ (M1)(A1)

$\hat{A}OB = 54.532\dots \approx 54.5^\circ$ (0.951764... \approx 0.952 radians) A1

Note: Other methods may be seen; award (M1)(A1) for use of a correct trigonometric method to find an appropriate angle and then A1 for the correct answer.

(ii) finding area of triangle
EITHER

area of triangle = $\frac{1}{2} \times 4.5^2 \times \sin(54.532\dots)$ (M1)

Note: Award M1 for correct substitution into formula.

= 8.24621... \approx 8.25 m² (A1)

OR

AB = $2 \times \sqrt{4.5^2 - 4^2} = 4.1231\dots$

area triangle = $\frac{4.1231\dots \times 4}{2}$ (M1)

= 8.24621... \approx 8.25 m² (A1)

finding area of sector

EITHER

area of sector = $\frac{54.532\dots}{360} \times \pi \times 4.5^2$ (M1)

= 9.63661... \approx 9.64 m² (A1)

OR

area of sector = $\frac{1}{2} \times 0.9517641\dots \times 4.5^2$ (M1)

= 9.63661... \approx 9.64 m² (A1)

THEN

area of segment = 9.63661... - 8.24621...

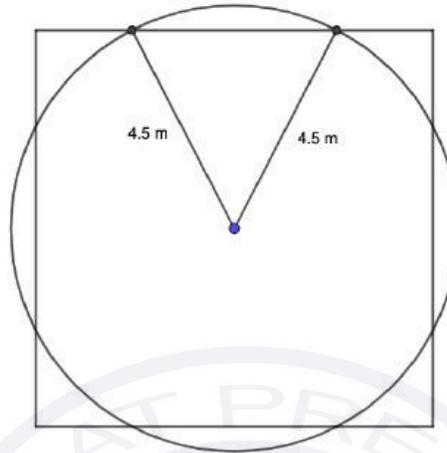
= 1.39 m² (1.39040...) A1

[8 marks]

(b) (i) $\pi \times 4.5^2$
 63.6 m^2 (63.6172...m²)

(M1)
 A1

(ii) **METHOD 1**



$4 \times 1.39040\dots$ (5.56160)
 subtraction of four segments from area of circle
 $= 58.1 \text{ m}^2$ (58.055...)

(A1)
 (M1)
 A1

METHOD 2

$4(0.5 \times 4.5^2 \times \sin 54.532\dots) + 4\left(\frac{35.4679}{360} \times \pi \times 4.5^2\right)$
 $= 32.9845\dots + 25.0707$
 $= 58.1 \text{ m}^2$ (58.055...)

(M1)
 (A1)
 A1

[5 marks]

(c) sketch of $\frac{dV}{dt}$ OR $\frac{dV}{dt} = 0.110363\dots$ OR attempt to find where $\frac{d^2V}{dt^2} = 0$
 $t = 1$ hour

(M1)
 A1

[2 marks]

[Total 15 marks]

Question 7

- (a) **EITHER**
annual cycle for daylight length **R1**
- OR**
there is a minimum length for daylight (cannot be negative) **R1**
- OR**
a quadratic could not have a maximum and a minimum or equivalent **R1**

Note: Do not accept "Paula's model is better".

[1 mark]

- (b) (i) 4 **A1**
- (ii) 12 **A1**
- (iii) $y = 12$ **A1A1**

Note: Award **A1** " $y = (\text{a constant})$ " and **A1** for that constant being 12.

[4 marks]

- (c) $f(t) = -4 \cos(30t) + 12$ **OR** $f(t) = -4 \cos(-30t) + 12$ **A1A1A1**

Note: Award **A1** for $b = 30$ (or $b = -30$), **A1** for $a = -4$, and **A1** for $d = 12$. Award at most **A1A1A0** if extra terms are seen or form is incorrect. Award at most **A1A1A0** if x is used instead of t .

[3 marks]

- (d) $10.5 = -4 \cos(30t) + 12$ **(M1)**

EITHER

$$t_1 = 2.26585\dots, t_2 = 9.73414\dots$$

(A1)(A1)

OR

$$t_1 = \frac{1}{30} \cos^{-1} \frac{3}{8}$$

(A1)

$$t_2 = 12 - t_1$$

(A1)

THEN

$$9.73414\dots - 2.26585\dots$$

$$7.47 \text{ (7.46828\dots) months (0.622356\dots years)}$$

A1

Note: Award **M1A1A1A0** for an unsupported answer of 7.46. If there is only one intersection point, award **M1A1A0A0**.

[4 marks]

Question 8

(a) (i) an attempt to find the amplitude

$$\frac{61.8}{2} \quad \text{OR} \quad \frac{64.5 - 2.7}{2}$$

$$(a =) 30.9 \text{ m}$$

(M1)

A1

Note: Accept an answer of $(a =) -30.9 \text{ m}$.

(ii) (period = $\frac{60}{1.5}$) = 40 (s)

$$((b =) \frac{360^\circ}{40})$$

$$(b =) 9$$

(A1)

A1

Note: Accept an answer of $(b =) -9$.

(iii) attempt to find d

$$(d =) 30.9 + 2.7 \quad \text{OR} \quad \frac{64.5 + 2.7}{2}$$

$$(d =) 33.6 \text{ m}$$

(M1)

A1

[6 marks]

(b) 12×1.5 OR $\frac{12 \times 60}{40}$

18 (revolutions per ride)

(M1)

A1

[2 marks]

(c) (i) $0 \leq t \leq 720$

(ii) $2.7 \leq h \leq 64.5$

A1

A1A1

Note: Award **A1** for correct endpoints of domain and **A1** for correct endpoints of range. Award **A1** for correct direction of both inequalities.

[3 marks]

(d) graph of $h(t)$ and $y = 16.7$ OR $h(t) = 16.7$

6.31596... and 33.6840...

27.4 (s) (27.3680...)

(M1)

(A1)

A1

[3 marks]

(e) (i) d A1

(ii) **EITHER**
 $d + 30.9 = 65.2$ (A1)

OR
 $65.2 - (61.8 + 2.7) = 0.7$ (A1)

OR
3.4 (new platform height) (A1)

THEN
 $(d =) 34.3 \text{ m}$ A1

[3 marks]
Total [17 marks]



Question 9

(a) $\left(\frac{2+6}{2}, \frac{2+0}{2}\right)$
(4, 1)

(M1)

A1

Note: Award A0 if parentheses are omitted in the final answer.

[2 marks]

(b) attempt to substitute values into gradient formula

(M1)

$$\left(\frac{0-2}{6-2}\right) = -\frac{1}{2}$$

(A1)

therefore the gradient of perpendicular bisector is 2

(M1)

so $y-1=2(x-4)$ ($y=2x-7$)

A1

[4 marks]

(c) identifying the correct equations to use:

(M1)

$y=2-x$ and $y=2x-7$

evidence of solving their correct equations or of finding intersection point graphically

(M1)

(3, -1)

A1

Note: Accept an answer expressed as " $x=3, y=-1$ ".

[3 marks]

(d) attempt to use distance formula

(M1)

$$YZ = \sqrt{(7-(-1))^2 + (7-3)^2}$$
$$= \sqrt{80} \quad (4\sqrt{5})$$

A1

[2 marks]

(e) **METHOD 1 (cosine rule)**

length of XZ is $\sqrt{80}$ ($4\sqrt{5}, 8.94427\dots$)

(A1)

Note: Accept 8.94 and 8.9.

attempt to substitute into cosine rule

(M1)

$$\cos \hat{X}YZ = \frac{80+32-80}{2 \times \sqrt{80} \sqrt{32}} \quad (= 0.316227\dots)$$

(A1)

Note: Award A1 for correct substitution of XZ, YZ, $\sqrt{32}$ values in the cos rule. Exact values do not need to be used in the substitution.

$(\hat{X}YZ =) 71.6^\circ$ ($71.5650\dots^\circ$)

A1

METHOD 2 (splitting isosceles triangle in half)

length of XZ is $\sqrt{80}$ ($4\sqrt{5}$, 8.94427...)

(A1)

Note: Accept 8.94 and 8.9.

required angle is $\cos^{-1}\left(\frac{\sqrt{32}}{2\sqrt{80}}\right)$

(M1)(A1)

Note: Award **A1** for correct substitution of XZ (or YZ), $\frac{\sqrt{32}}{2}$ values in the cos rule.
Exact values do not need to be used in the substitution.

$(\hat{X}\hat{Y}\hat{Z} =) 71.6^\circ$ (71.5650°)

A1

Note: Last **A1** mark may be lost if prematurely rounded values of XZ, YZ and/or XY are used.

[4 marks]

(f) (area =) $\frac{1}{2}\sqrt{80}\sqrt{32}\sin 71.5650\dots$ **OR** (area =) $\frac{1}{2}\sqrt{32}\sqrt{72}$
= 24 km²

(M1)

A1

[2 marks]

(g) *Any sensible answer such as:*
There might be factors other than proximity which influence shopping choices.
A larger area does not necessarily result in an increase in population.
The supermarkets might be specialized / have a particular clientele who visit even if other shops are closer.
Transport links might not be represented by Euclidean distances.
etc.

R1

[1 mark]

Total [18 marks]

Question 10

- (a) (i) B A1
- (ii) F A1
[2 marks]
- (b) correct substitution into the midpoint formula (M1)

$$\frac{8+5}{2}$$

$$y = 6.5$$
 A1
- Note:** Answer must be an equation for the **A1** to be awarded.
- [2 marks]
- (c) midpoint = (5, 7) (A1)
- correct use of gradient formula (M1)

$$\frac{8-6}{7-3}$$
 gradient of BC = 0.5 (A1)
 negative reciprocal of gradient (M1)
 perpendicular gradient = -2

$$y - 7 = -2(x - 5) \text{ (or } y = -2x + 17)$$
 A1
- [5 marks]
- (d) (i) attempt to find the intersection of two perpendicular bisectors (BC & CD) (M1)
- Note:** This may be seen graphically or algebraically.
- $$6.5 - 7 = -2(x - 5) \text{ OR } 6.5 = -2x + 17$$
- Note:** Accept equivalent methods using the perpendicular bisector of BD, $y - 5.5 = 4(x - 5)$ OR $y = 4x - 14.5$
- $$x = 5.25, y = 6.5 \text{ OR } (5.25, 6.5)$$
- A1
- Note:** The x -coordinate must be exact or expressed to at least 3 sf.
- (ii) their correct substitution into distance formula (M1)

$$\sqrt{(5.25 - 7)^2 + (6.5 - 5)^2}$$

$$= 2.30 \text{ km } \left(2.30488\dots, \frac{\sqrt{85}}{4} \right)$$
 A1
- [4 marks]
Total [13 marks]

Question 11

(a) (i) 19° A1

(ii) $\frac{BD}{\sin 120^\circ} = \frac{40}{\sin 19^\circ}$ (M1)(A1)

Note: Award **M1** for substituted sine rule for BCD, **A1** for their correct substitution.

(BD =) 106 m (106.401...) A1

[4 marks]

(b) **METHOD 1 (cosine rule)**

$\cos \text{BAD} = \frac{85^2 + 85^2 - 106.401\dots^2}{2 \times 85 \times 85}$ (M1)(A1)

Note: Award **M1** for substituted cosine rule, **A1** for their correct substitution.

77.495 A1

Note: Accept an answer of 77.149 from use of 3 sf answer from part (a). The final answer must be correct to five significant figures.

METHOD 2 (right angled trig/isosceles triangles)

$\sin\left(\frac{\text{BAD}}{2}\right) = \frac{53.2008\dots}{85}$ (A1)(M1)

Note: Award **A1** for 53.2008... seen. Award **M1** for correctly substituted trig ratio. Follow through from part (a).

77.495... A1

Note: Use of 3 sf answer from part (a), results in 77.149.

[3 marks]

(c) **EITHER**

(Area =) $\frac{1}{2} \times 85 \times 85 \times \sin(77^\circ)$ (M1)(A1)

Note: Award **M1** for substituted area formula, **A1** for correct substitution.
Award at most **(M1)(A1)A0** if an angle other than 77° is used.

OR

(Area =) $\frac{1}{2} \times (2 \times 85 \times \sin(38.5^\circ)) \times (85 \times \cos(38.5^\circ))$ (M1)(A1)

Note: Award **M1** for substituted area formula $A = \frac{1}{2}bh$, **A1** for correct substitution.

3520 m² (3519.91...) A1

[3 marks]

(d) 85 m

A1

[1 mark]

(e) $85 + 85 + \frac{77}{360} \times 2\pi \times 85$

(M1)(M1)

Note: Award **M1** for correctly substituted into $\frac{\theta}{360} \times 2\pi \times r$, **M1** for addition of AB and AD.

284 m (284.231...)

A1

[3 marks]

(f) $\frac{77}{360} \times \pi \times (85)^2 - 3519.91...$

(M1)(M1)

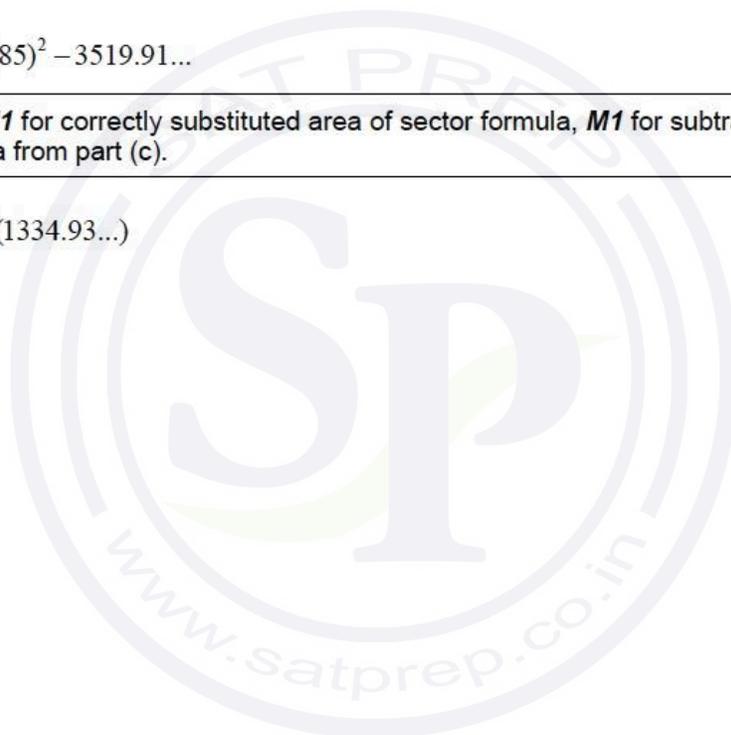
Note: Award **M1** for correctly substituted area of sector formula, **M1** for subtraction of their area from part (c).

1330 m² (1334.93...)

A1

[3 marks]

Total [17 marks]



Question 12

(a) $\frac{18-4}{2}$ (M1)

(a) = 7 A1

[2 marks]

(b) $\frac{18+4}{2}$ OR $18-7$ OR $4+7$ (M1)

(d) = 11 A1

[2 marks]

(c) (time between high and low tide is) 6h15m OR 375 minutes (A1)

multiplying by 2 (M1)

750 minutes A1

[3 marks]

(d) EITHER

$\frac{360^\circ}{b} = 750$ (A1)

OR

$7\cos(b \times 375) + 11 = 4$ (A1)

THEN

(b =) 0.48 A1

Note: Award **A1A0** for an answer of $\frac{2\pi}{750}$ ($= \frac{\pi}{375} = 0.00837758\dots$)

[2 marks]

(e) equating their cos function to 6 or graphing their cos function and 6

(M1)

$7\cos(0.48t) + 11 = 6$

$\Rightarrow t = 282.468\dots$ (minutes) (A1)

$= 4.70780\dots$ (hr) OR 4hr 42 mins (4hr 42.4681... mins) (A1)

so the time is 10:42 A1

[4 marks]

- (f) next solution is $t = 467.531\dots$ (A1)
 $467.531\dots - 282.468\dots$
 185 (mins) (185.063...) A1

Note: Accept an (unsupported) answer of 186 (from correct 3 sf values for t)

[2 marks]

[Total: 15 marks]

Question 13

- (a) attempt to use sine rule (M1)

$$\frac{\sin \hat{A}BO}{25.9} = \frac{\sin 10^\circ}{6.36}$$
 (A1)
 45.0° (45.0036...°) A1

Note: Accept an answer of 45° for full marks.

[3 marks]

- (b) ($\hat{O}AB =$) $124.996\dots^\circ$ (A1)
 attempt to use area of triangle formula (M1)

$$\frac{1}{2} \times 25.9 \times 6.36 \times \sin(124.996\dots^\circ)$$
 (A1)
 67.5 m^2 (67.4700... m^2) A1

Note: Units are required. The final **A1** is only awarded if the correct units are seen in their answer; hence award **(A1)(M1)(A1)A0** for an unsupported answer of 67.5. Accept $67.4670\dots\text{m}^2$ from use of 3 sf values.
 Full follow through marks can be awarded for this part even if their $\hat{O}AB$ is not obtuse, provided that all working is shown.

[4 marks]

- (c) attempt to use cosine rule (M1)
 $(BK =) \sqrt{12^2 + 6.36^2 - 2 \times 12 \times 6.36 \times \cos 45^\circ}$ (A1)
 8.75 (m) (8.74738...(m)) A1

Note: Award **(M1)(A1)(A0)** for radian answer of 10.2 (m) (10.2109...(m)) with or without working shown.

[3 marks]

(d) **METHOD 1**

attempt to use sine rule with measurements from triangle OKX

(M1)

$$\frac{OX}{\sin 51.1^\circ} = \frac{22.2}{\sin 53.8^\circ}$$

(A1)

(OX =) 21.4 (m) (21.4099...)(m)

A1

(21.4 (m) < 22.2 (m))

Odette is closer to the football / Khemil is further from the football

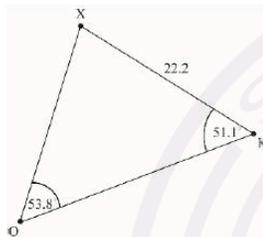
A1

Note: For the final **A1** to be given, 21.4 (21.4099...) must be seen. Follow through within question part for final **A1** for a consistent comparison with their OX.

METHOD 2

sketch of triangle OXK with vertices, angles and lengths

(A1)



51.1° is smallest angle in triangle OXK
opposite side (OX) is smallest length
therefore Odette is closest

R1

R1

A1

[4 marks]

(e) attempt to use length of arc formula

(M1)

$$\frac{135}{360} \times 2\pi \times 12$$

(A1)

28.3(m) (9π, 28.2743...) (m)

A1

[3 marks]

Total [17 marks]

Question 14

- (a) 45 (m) A1
[1 mark]
- (b) (i) recognition of need to use Pythagoras theorem (M1)
 $BE^2 = 32^2 + 45^2$
 (BE =) 55.2 (55.2177..., $\sqrt{3049}$) (m) A1
- (ii) correct use of trig ratio for \hat{BEM} (M1)
 (\hat{BEM} =) $\tan^{-1}\left(\frac{45}{32}\right)$ or equivalent
 (\hat{BEM} =) 54.6 (54.5829...) A1
[4 marks]
- (c) attempt to use arc length formula (M1)
 (arc length =) $\frac{2 \times 54.5829...}{360} \times 2\pi(55.2177...)$ (A1)
 (arc length =) 105 (105.206...) (m) A1
[3 marks]
- (d) 59.2177... (seen anywhere) (A1)
 use of area of sector formula (M1)
 recognition of subtracting areas of two sectors (M1)
 (area =) $\frac{109.165...}{360} \times \pi((59.2177...)^2 - (55.2177...)^2)$
 (area =) 436 (m²) (436.068...) A1
[4 marks]
- (e) multiplying their area from part (d) by 0.15 or 15 (M1)
 0.15 (m) seen **OR** 4360688 (cm²) seen (A1)
 436.068... \times 0.15 **OR** 4360688 \times 15
 65.4 (65.4103...) m³ **OR** 65400000 (65410332) cm³ A1
[3 marks]
[Total 15 marks]

Question 15

- (a) 25 (m) A1
[1 mark]
- (b) (i) recognition of need to use Pythagoras theorem (M1)
 $BF^2 = 20^2 + 25^2$
 (BF =) 32.0 (32.0156..., $\sqrt{1025}$, $5\sqrt{41}$) (m) A1
- (ii) correct use of trig ratio for $\hat{B}\hat{F}M$ (M1)
 ($\hat{B}\hat{F}M$ =) $\tan^{-1}\left(\frac{25}{20}\right)$ or equivalent
 ($\hat{B}\hat{F}M$ =) 51.3 (51.3401...) A1

Note: Accept an answer of 51.4 from use of 3sf answer to part (b)(i) and then either cosine rule or inverse sine.

[4 marks]

- (c) attempt to use arc length formula (M1)
 (arc length =) $\frac{2 \times 51.3401...}{360} \times 2\pi(32.0156...)$ (A1)
 (arc length =) 57.4 (57.3755...) (m) A1

Note: Accept 57.3 from use of 3 sf. values of their answers from parts (b)(i) and (b)(ii).

[3 marks]

- (d) 34.0156... (seen anywhere) (A1)
 use of area of sector formula (M1)
 recognition of subtracting areas of two sectors (M1)
 (area =) $\frac{102.680...}{360} \times \pi((34.0156...)^2 - (32.0156...)^2)$
 (area =) 118 (m²) (118.335...) A1

[4 marks]

- (e) multiplying their area from part (d) by 0.12 or 12 (M1)
 0.12 (m) seen **OR** 1183350 (cm²) seen (A1)
 118.335... \times 0.12 **OR** 1183350 \times 12
 14.2 (14.2002...) m³ **OR** 14200000 (14200236) cm³ A1

[3 marks]
 [Total 15 marks]

Question 16

- (a) (i) correct approach to find missing length (A1)
 $\sqrt{4^2 - 1^2} (= \sqrt{15})$
 attempt to find cross-section (M1)
 e.g. use of area of trapezoid formula or rectangle+triangle or rectangle – triangle
 use of volume of prism formula (M1)
 (their cross-section multiplied by 3)
 $3 \left[\frac{1}{2} (10+11) (\sqrt{4^2 - 1^2}) \right]$
 $= 122 (\text{m}^3) (121.998\dots)$ A1

- (ii) correct approach to find missing height (A1)
 $\sqrt{4^2 - 3.2^2} (= 2.4)$
 attempt to find volume (M1)
 (multiplication by 3.2 and 3 seen)
 $3 \left[\frac{1}{2} (10+10 + \sqrt{4^2 - 3.2^2}) (3.2) \right]$
 $= 108 (\text{m}^3) (107.52\dots)$ A1

- (iii) correct approach to find missing lengths (A1)
 $\sin(60^\circ)$ and $\cos(60^\circ)$ OR $\sin(60^\circ)$ and Pythagoras etc seen in work
 $3 \left[\frac{1}{2} (10+10 + 4 \cos(60^\circ)) 4 \sin(60^\circ) \right]$
 $= 114 (\text{m}^3) (114.315\dots)$ A1

[9 marks]

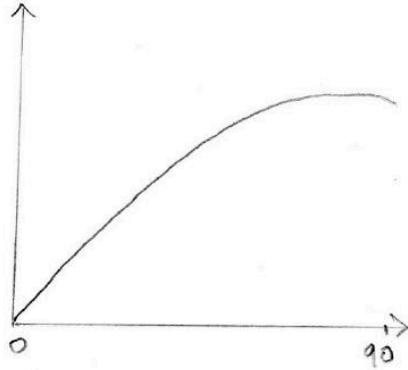
- (b) $V = 3 \left[\frac{1}{2} (10+10 + 4 \cos(\theta)) 4 \sin(\theta) \right]$ A1
 all correct intermediate working leading to given answer A1
 e.g. $V = 6 \sin(\theta)(20 + 4 \cos(\theta))$
 $V = 24 \sin(\theta)(5 + \cos(\theta))$ AG

Note: The AG line must be seen for the final A1 to be awarded.

[2 marks]

- (c) accept any reasoning along the lines: “skip would have zero volume” or
 “if the angle is zero, then the contents would fall out” R1
 [1 mark]

(d) (i)



A1A1

Note: Award **A1** for the correct shape and **A1** for the graph on the correct, labelled, domain. Condone omission of θ / V labels (or x/y).

(ii) $\theta = 79.3^\circ$ (79.2723...°) (1.38 (1.38356...)), $V_{\max} = 122$ (122.292...)

A1A1

Note: Award **A0A1** if values are reversed and **A0A0** for a coordinate pair.

[4 marks]

[Total: 16 marks]



Question 17

(a) $BC = 20$ (m)

A1

[1 mark]

(b) use of Pythagoras

(M1)

$$AB = \sqrt{12^2 + 4^2}$$

$$= 12.6 \text{ (m) } (12.6491\dots, \sqrt{160})$$

A1

[2 marks]

(c) **METHOD 1 – finding angle ABC**

correct use of a trig ratio to find $\hat{A}BC$ (or finding the bearing of B from A)

(A1)

$$\text{e.g. } \tan(\hat{A}BC) = \frac{12}{4}, \cos \hat{A}BC = \frac{20^2 + 12.649^2 - 20^2}{2 \times 20 \times 12.649}, \cos \hat{A}BC = \frac{6.3245}{20}$$

$$\hat{A}BC = 71.6 \text{ (71.5650\dots)}$$

(A1)

Note: Angle $\hat{A}BC$ can be 71.5 or 72.2 depending on their working out. Bearings should be given in degrees.

$$180 + 71.5650\dots = 252^\circ \text{ (251.565\dots)}$$

A1

Note: The final **A1** can be awarded for 180 plus their 71.6. If radians used, award **A1A1** for 1.24904... or 4.39063... seen, and then **A0** for the radian answer.

METHOD 2 – finding angle that AB makes with the horizontal (angle H)

correct use of a trig ratio to find H , the angle AB makes with horizontal **(A1)**

$$\text{e.g. } \tan \hat{H} = \frac{4}{12}, \cos \hat{H} = \frac{12^2 + 12.649^2 - 4^2}{2 \times 12 \times 12.649}$$

$$\hat{H} = 18.4 \text{ (18.4349\dots)}$$

(A1)

Note: Accept 18.5 (18.5078...) from use of 3sf answer from part (b). Bearings should be given in degrees.

$$270 - 18.4348\dots = 252^\circ \text{ (251.565\dots)}$$

A1

Note: The final **A1** can be awarded for 270 minus their 18.4. If radians used, award **A1A1** for 0.321750... or 4.39063... seen, and then **A0** for the radian answer.

[3 marks]

(d) (i) $-\frac{4}{3} \left(-\frac{16}{12} \right)$ **A1**

(ii) (6, 8) **A1A1**

Note: Award **A1A0** if parentheses are missing.

(iii) gradient of (their) perp line = $\frac{3}{4}$ **(M1)**

equation of perpendicular bisector of AC **(A1)**

e.g. $(y-8) = \frac{3}{4}(x-6)$ **OR** $y = \frac{3}{4}x + 3.5$

EITHER

equation of perpendicular bisector of BC is $y = 10$ **(A1)**

OR

equation of perpendicular bisector of AB is $y = -3x + 36$ **(A1)**

Note: The **A1** is for either equation of perpendicular bisector of BC or AB.

point of intersection $\left(8\frac{2}{3}, 10 \right)$ **OR** $(8.67, 10)$ $[(8.666\dots, 10)]$ **(M1)A1**

Note: Award **M1** for an attempt to equate their perpendicular bisectors
Award the final **A1** for the correct coordinate pair – parentheses omitted or not.

[8 marks]
[Total: 14 marks]

Question 18

(a) attempt to use Pythagoras' theorem

(M1)

$$\sqrt{3.4^2 - 2^2}$$

$$= 2.75 \text{ (2.74954...)} \text{ (m)}$$

A1
[2 marks]

(b) (i) **METHOD 1** (Use of $\frac{1}{2} \times a \times b \times \sin(\theta)$)

$$60^\circ$$

(A1)

attempt to find area of one triangle using $\frac{1}{2} \times a \times b \times \sin(\theta)$

(M1)

$$\frac{1}{2} \times 2 \times 2 \times \sin(60^\circ)$$

$$\left(6 \times \frac{1}{2} \times 2 \times 2 \times \sin(60^\circ) \right) = 10.4 \text{ (10.3923..., } 6\sqrt{3}) \text{ (m}^2\text{)}$$

A1

Note: Award **AOM0A0** for $\frac{1}{2} \times 2 \times 2$ or equivalent.

METHOD 2 (Use of altitude)

(altitude is) $\sqrt{3}$

(A1)

attempt to find the area of one triangle using $\frac{1}{2} \times b \times h$ with their altitude.

(M1)

$$\frac{1}{2} \times 2 \times \sqrt{3}$$

$$\left(6 \times \frac{1}{2} \times 2 \times \sqrt{3} \right) = 10.4 \text{ (10.3923..., } 6\sqrt{3}) \text{ (m}^2\text{)}$$

A1

Note: Award **AOM0A0** for $\frac{1}{2} \times 2 \times 2$ or equivalent.

METHOD 3 (Finding the area of a trapezoid)

(altitude of one trapezoid is) $\sqrt{3}$

(A1)

attempt to find area of one trapezoid using $\frac{1}{2} \times (a + b) h$

(M1)

$$\frac{1}{2} \times (2 + 4) \sqrt{3} \quad (3\sqrt{3})$$

$$\left(2 \times \frac{1}{2} \times (2 + 4) \sqrt{3} \right) = 10.4 \text{ (10.3923..., } 6\sqrt{3}) \text{ (m}^2\text{)}$$

A1

(ii) $\frac{1}{3} \times 10.3923... \times 2.74954...$ (A1)
 $= 9.52 \text{ m}^3$ (9.52470...) (A1)

Note: Units must be seen.

[5 marks]

(c) $\cos(\hat{M}\hat{A}\hat{T}) = \frac{2}{3.4}$ or correct equivalent (A1)
 $(\hat{M}\hat{A}\hat{T} =) 54.0^\circ$ (53.9681...) (A1)

[2 marks]

(d) Angle $Y\hat{A}X = 180 - 53.9681... = 126.031...^\circ$ (A1)
 Angle $Y\hat{X}A = 180 - 35 - 126.031... = 18.9681...^\circ$ (A1)

Note: These angles may be seen in the sine rule.

Attempt to substitute into sine rule (M1)

$$\frac{AY}{\sin(18.9681...)} = \frac{2.6}{\sin(126.031)}$$

$AY = 1.05$ (1.04503...) (m)

(A1)
 [4 marks]

(e) attempt to substitute into cosine rule to form a quadratic for YZ (M1)
 $0.9^2 = YZ^2 + 1.04503...^2 - 2 \times 1.04503... \times YZ \times \cos(35)$ (A1)
 $YZ = 0.185$ (0.184692...) (m), 1.53 (1.52739...) (m)

(A1A1)
 [4 marks]

[Total: 17 marks]

Question 19

(a) (i) $\frac{45-8}{2}$ **A1**
 18.5 (m) **AG**

Note: Use of 18.5 in the calculation is reverse engineering, and falls short of the demand of the command term "Show that". In such cases, award **A0**.

(ii) $(2\pi \times 18.5 =) 116$ (116.2389..., 37π) (m) **A1**
[2 marks]

(b) recognition the distance is the arc length **(M1)**

$\frac{10^\circ}{360^\circ} \times 2\pi \times 18.5$ **OR** $\frac{116.2389\dots}{36}$ **(A1)**

$= 3.23 \left(3.228859\dots, \frac{37\pi}{36} \right)$ (m) **A1**
[3 marks]

(c) (i) $\left(b = \frac{360}{15} = \right) 24$ **A1**

(ii) $\frac{\text{max} + \text{min}}{2}$ **(A1)**

$(d =) \frac{45+8}{2}$ **OR** $45 - 18.5$ **OR** $8 + 18.5$ **A1**
 $= 26.5$

(iii) $h(t) = 18.5 \sin(24t) + 26.5$ **A1A1**

Note: Award **A1** for 18.5 seen as parameter a , **A1** for a completely correct equation (including LHS).

[5 marks]

(d) attempt to equate 33 with $h(t)$ **OR** sketch graph of curve and line **(M1)**

$h(t) (= 18.5 \sin(24t) + 26.5) = 33$

$t = 0.857$ (0.857083...) and $t = 6.64$ (6.64291...) (minutes) **A1A1**
[3 marks]

(e) 4 (revolutions per hour) seen **OR** 32 (revolutions per day) seen **(A1)**

$(4 \times 8 \times 3000 =) 96000$ **(A1)**

9.6×10^4 **A1**
[3 marks]

[Total 16 marks]

Question 20

(a) (2, 2)

A1

Note: Award **A0** if parentheses are omitted.

[1 mark]

(b) (i) attempt to use midpoint formula (at least one correct)

(M1)

$$\left(\frac{9+10}{2}, \frac{0+6}{2} \right)$$

(9.5, 3)

A1

(ii) $\left(m = \frac{6-0}{10-9} = 6 \right)$

(A1)

finding negative reciprocal of their gradient

(M1)

$$m_{\perp} = -\frac{1}{6}$$

attempt to substitute their midpoint and their gradient into equation of straight line (M1)

eg. $y - 3 = -\frac{1}{6} \left(x - \frac{19}{2} \right)$ OR $3 = -\frac{1}{6}(9.5) + c$

$$y = -\frac{1}{6}x + \frac{55}{12} \quad \text{OR} \quad y = -0.167x + 4.58 \quad (y = -0.166666\dots x + 4.58333\dots)$$

A1

Note: Substituting $m = 6$ (no negative reciprocal) and their point into the equation of straight line would receive at most **A1M0M1A0**.

[6 marks]

Question 21

(a) attempt to use Pythagoras' theorem

(M1)

$$\sqrt{4.2^2 - 3^2}$$

$$= 2.94 \text{ (2.93938...)} \text{ (m)}$$

A1
[2 marks]

(b) (i) **METHOD 1** (Use of $\frac{1}{2} \times a \times b \times \sin(\theta)$)

$$60^\circ$$

(A1)

attempt to find area of one triangle using $\frac{1}{2} \times a \times b \times \sin(\theta)$

(M1)

$$\frac{1}{2} \times 3 \times 3 \times \sin(60^\circ)$$

$$\left(6 \times \frac{1}{2} \times 3 \times 3 \times \sin(60^\circ) \right) = 23.4 \text{ (23.3826..., } \frac{27\sqrt{3}}{2} \text{)} \text{ (m}^2\text{)}$$

A1

Note: Award **AOMOA0** for $\frac{1}{2} \times 3 \times 3$ or equivalent.

METHOD 2 (Use of altitude)

$$\text{(altitude is) } \frac{3}{2}\sqrt{3} \text{ (2.59807...)}$$

(A1)

attempt to find area of one triangle using $\frac{1}{2} \times b \times h$ with their altitude

(M1)

$$\frac{1}{2} \times 3 \times \frac{3}{2}\sqrt{3}$$

$$\left(6 \times \frac{1}{2} \times 3 \times \frac{3}{2}\sqrt{3} \right) = 23.4 \text{ (23.3826..., } \frac{27\sqrt{3}}{2} \text{)} \text{ (m}^2\text{)}$$

A1

Note: Award **AOMOA0** for $\frac{1}{2} \times 2 \times 2$ or equivalent.

METHOD 3 (Finding the area of a trapezoid)

$$\text{(altitude of one trapezoid is) } \frac{3}{2}\sqrt{3} \text{ (2.59807...)}$$

(A1)

attempt to find area of one trapezoid using $\frac{1}{2} \times (a+b)h$

(M1)

$$\frac{1}{2}(3+6)\left(\frac{3}{2}\sqrt{3}\right) \text{ OR } \frac{27}{4}\sqrt{3}$$

$$\left(2 \times \frac{1}{2}(3+6)\left(\frac{3}{2}\sqrt{3}\right) \right) = 23.4 \text{ (23.3826..., } \frac{27\sqrt{3}}{2} \text{)} \text{ (m}^2\text{)}$$

A1

$$(ii) \quad \frac{1}{3} \times 23.3826... \times 2.93938... \quad (A1)$$

$$= 22.9 \text{ m}^3 \text{ (22.9102...)} \quad A1$$

Note: Units must be seen.

[5 marks]

$$(c) \quad \cos(\hat{OAT}) = \frac{3}{4.2} \text{ or correct equivalent} \quad (A1)$$

$$(\hat{OAT} =) 44.4^\circ \text{ (44.4153..., 0.775193... radians)} \quad A1$$

[2 marks]

$$(d) \quad \text{Angle QAP} = 180 - 44.4... = 135.584...^\circ \quad (A1)$$

$$\text{Angle QPA} = 180 - 30 - 135.584... = 14.4153...^\circ \quad (A1)$$

Note: These angles may be seen in the sine rule.

Attempt to substitute into sine rule

(M1)

$$\frac{AQ}{\sin(14.4153...)} = \frac{3.5}{\sin(135.584...)}$$

$$AQ = 1.25 \text{ (1.24500...)} \text{ (m)}$$

A1

Note: Accept 1.24, from use of a 3sf value for \hat{QAP} and \hat{QPA} .

[4 marks]

$$(e) \quad \text{attempt to substitute into cosine rule to form a quadratic for QR} \quad (M1)$$

$$0.8^2 = QR^2 + 1.24500...^2 - 2 \times 1.24500... \times QR \times \cos(30) \quad (A1)$$

$$QR = 0.576 \text{ (0.575717...)} \text{ (m)}, 1.58 \text{ (1.58068...)} \text{ (m)} \quad A1A1$$

[4 marks]

[Total: 17 marks]

Question 22

(a) (i) $\frac{45-8}{2}$ A1
 18.5 (m) AG

Note: Use of 18.5 in the calculation is reverse engineering, and falls short of the demand of the command term "Show that". In such cases, award **A0**.

(ii) $(2\pi \times 18.5) = 116$ (116.2389..., 37π) (m). A1
[2 marks]

(b) recognition the distance is the arc length (M1)
 $\frac{10^\circ}{360^\circ} \times 2\pi \times 18.5$ OR $\frac{116.2389\dots}{36}$ (A1)
 $= 3.23 \left(3.228859\dots, \frac{37\pi}{36} \right)$ (m) A1
[3 marks]

(c) (i) $\left(b = \frac{360}{15} = \right) 24$ A1

(ii) $\frac{\max + \min}{2}$ (A1)
 $(d =) \frac{45+8}{2}$ OR $45 - 18.5$ OR $8 + 18.5$
 $= 26.5$ A1

(iii) $h(t) = 18.5 \sin(24t) + 26.5$ A1A1

Note: Award **A1** for 18.5 seen as parameter a , **A1** for a completely correct equation (including LHS).

[5 marks]

(d) attempt to equate 33 with $h(t)$ OR sketch graph of curve and line (M1)
 $h(t) (= 18.5 \sin(24t) + 26.5) = 33$
 $t = 0.857$ (0.857083...) and $t = 6.64$ (6.64291...) (minutes) A1A1
[3 marks]

(e) 4 (revolutions per hour) seen OR 32 (revolutions per day) seen (A1)
 $(4 \times 8 \times 3000 =) 96000$ (A1)
 9.6×10^4 A1
[3 marks]
[Total 16 marks]

Question 23

(a) (2, 6)

A1

Note: Award **A0** if parentheses are omitted.

[1 mark]

(b) (i) attempt to use midpoint formula (at least one correct)

(M1)

$$\left(\frac{11+12}{2}, \frac{0+6}{2} \right)$$

(11.5, 3)

A1

(ii) $\left(m = \frac{6-0}{12-11} = 6 \right)$

(A1)

finding negative reciprocal of their gradient

(M1)

$$m_{\perp} = -\frac{1}{6}$$

attempt to substitute their midpoint and their gradient into equation of straight line (M1)

eg. $y - 3 = -\frac{1}{6} \left(x - \frac{23}{2} \right)$ OR $3 = -\frac{1}{6}(11.5) + c$

$$y = -0.167x + 4.92 \left(y = -0.166666\dots x + 4.91666\dots, y = -\frac{1}{6}x + \frac{59}{12} \right)$$

A1

Note: Substituting $m = 6$ (no negative reciprocal) and their point into the equation of straight line would receive at most **A1M0M1A0**.

[6 marks]