Subject – Math AI(Standard Level)
Topic - Geometry and Trigonometry
Year - May 2021 – Nov 2022
Paper -2
Answers

Question 1

(a) (i) 0.909 (0.909181…) ***A2***

(ii) (very) strong and positive ***A1A1***

Note: Award ***A1*** for (very) strong ***A1*** for positive.

[4 marks]

(b) $y = 1.14x + 0.578$ ($y = 1.14033\ldots x + 0.578183\ldots$) ***A1A1***

[2 marks]

(c) (i) $1.14 \times 10 + 0.578$ ***M1***

12.0 (11.9814…) ***A1***

(ii) no the estimate is not reliable ***A1***

outside the known data range ***R1***

OR

a score greater than 10 is not possible ***R1***

(d)

Competitors	A	B	C	D	E	F	G	H
Stan's rank	7	8	6	4	2	4	1	4
Minsun's rank	7	8	6	4.5	3	2	1	4.5

A1A1

[2 marks]

(e) (i) 0.933 (0.932673…) ***A2***

(ii) Stan and Minsun strongly agree on the ranking of competitors. ***A1A1***

[4 marks]

(f) decreasing the score to 9.1, does not change the rank of competitor G ***A1***

[1 mark]

Total [17 marks]

Question 2

(a) $4\times\frac{360^\circ}{12}$ **OR** $4\times 30^\circ$ ***(M1)***

120° ***A1***

[2 marks]

(b) substitution in cosine rule ***(M1)***

$AB^2 = 10^2+6^2-2\times 10\times 6\times\cos(120^\circ)$ ***(A1)***

$AB = 14$ cm ***A1***

Note: Follow through marks in part (b) are contingent on working seen.

[3 marks]

(c) $\theta = 13\times 6$ ***(M1)***

$= 78^\circ$ ***A1***

[2 marks]

(d) substitution into the formula for arc length ***(M1)***

$l=\frac{78}{360}\times 2\times\pi\times 10$ **OR** $l=\frac{13\pi}{30}\times 10$

$=13.6$ cm $\left(13.6135\ldots,\ 4.33\pi,\ \frac{13\pi}{3}\right)$ ***A1***

[2 marks]

(e) substitution into the area of a sector ***(M1)***

$A=\frac{78}{360}\times\pi\times 10^2$ **OR** $l=\frac{1}{2}\times\frac{13\pi}{30}\times 10^2$

$=68.1\text{ cm}^2$ $\left(68.0678\ldots,\ 21.7\pi,\ \frac{65\pi}{3}\right)$ ***A1***

[2 marks]

(f) 23 ***A1***

[1 mark]

(g) correct substitution *(M1)*

$h = 10\cos(160^\circ) + 13$

$= 3.60 \text{ cm} \ (3.60307...)$ *A1*

[2 marks]

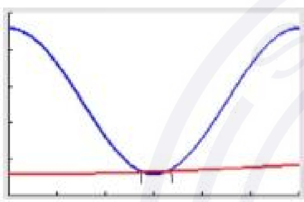
(h) 10 *A1*

[1 mark]

(i) **EITHER**

$10 \times \cos(\theta) + 13 = -10 \times \cos\left(\frac{\theta}{12}\right) + 13$ *(M1)*

OR



(M1)

Note: Award ***M1*** for equating the functions. Accept a sketch of $h(\theta)$ and $g(\theta)$ with point(s) of intersection marked.	

THEN

$k = 196^\circ \ (196.363...)$ *A1*

Note: The answer 166.153… is incorrect but the correct method is implicit. Award ***(M1)A0***.	

[2 marks]

Total [17 marks]

Question 3

(a) use of cosine rule *(M1)*

$\hat{ACB} = \cos^{-1}\left(\frac{1005^2 + 1225^2 - 650^2}{2 \times 1005 \times 1225}\right)$ *(A1)*

$= 32.0^\circ \ (31.9980...)$ *A1*

[3 marks]

(b) use of sine rule *(M1)*

$\frac{DE}{\sin 31.9980...^\circ} = \frac{210}{\sin 100^\circ}$ *(A1)*

$(DE =)\ 113 \text{ m} \ (112.9937...)$ *A1*

[3 marks]

(c) **METHOD 1**

$180^\circ - (100^\circ +$ their part $(a))$ *(M1)*

$= 48.0019...^\circ$ **OR** $0.837791...$ *(A1)*

substituted area of triangle formula *(M1)*

$\frac{1}{2} \times 112.9937... \times 210 \times \sin 48.002^\circ$ *(A1)*

$8820 \text{ m}^2 \ (8817.18...)$ *A1*

METHOD 2

$\frac{CE}{\sin(180 - 100 - \text{their part } (a))} = \frac{210}{\sin 100}$ *(M1)*

$(CE =)\ 158.472...$ *(A1)*

substituted area of triangle formula *(M1)*

EITHER

$\frac{1}{2} \times 112.993... \times 158.472... \times \sin 100$ *(A1)*

OR

$\frac{1}{2} \times 210... \times 158.472... \times \sin(\text{their part } (a))$ *(A1)*

THEN

$8820 \text{ m}^2 \ (8817.18...)$ *A1*

METHOD 3

$CE^2 = 210^2 + 112.993...^2 - (2 \times 210 \times 112.993... \times \cos(180 - 100 - \text{their part } (a)))$ ***(M1)***

$(CE =)\ 158.472\ldots$ ***(A1)***

substituted area of triangle formula ***(M1)***

$\frac{1}{2} \times 112.993... \times 158.472... \times \sin 100$ ***(A1)***

$8820\ \text{m}^2\quad (8817.18\ldots)$ ***A1***

[5 marks]

(d) $1005 - 210$ **OR** 795 ***(A1)***

equating answer to part (c) to area of a triangle formula ***(M1)***

$8817.18\ldots = \frac{1}{2} \times DF \times (1005 - 210) \times \sin 48.002...^{\circ}$ ***(A1)***

$(DF =)\ 29.8\ \text{m}\quad (29.8473...)$ ***A1***

[4 marks]

Total [15 marks]

Question 5

(a) $\tan(\theta) = \dfrac{6}{10}$ ***(M1)***

$(\theta =)\ 31.0^\circ\ (30.9637...^\circ)$ **OR** $0.540\ (0.540419\ldots)$ ***A1***

[2 marks]

(b) (i) $(\text{CV} =)\ 40\tan(\theta)$ **OR** $(\text{CV} =)\ 4 \times 6$ ***(M1)***

Note: Award ***(M1)*** for an attempt at trigonometry or similar triangles (e.g. ratios).

$(\text{CV} =)\ 24\ \text{m}$ ***A1***

(ii) $(V =)\ \dfrac{1}{3}80^2 \times 24 - \dfrac{1}{3}60^2 \times 18$ ***M1A1A1***

Note: Award ***M1*** for finding the difference between the volumes of two pyramids, ***A1*** for each correct volume expression. The final ***A1*** is contingent on correct working leading to the given answer. If the correct final answer is not seen, award at most ***M1A1A0***. Award ***M0A0A0*** for any height derived from $V = 29\,600$, including 18.875 or 13.875.

$(V =)\ 29\,600\ \text{m}^3$ ***AG***

[5 marks]

(c) **METHOD 1**

$\left(\dfrac{29\,600}{80} =\right)$ 370 (days) ***A1***

$(370 > 366)$ Joshua is correct ***A1***

Note: Award ***A0A0*** for unsupported answer of "Joshua is correct". Accept $1.01... > 1$ for the first ***A1*** mark.

METHOD 2

$80 \times 366 = 29280\ \text{m}^3$ **OR** $80 \times 365 = 29200\ \text{m}^3$ ***A1***

$(29280 < 29600)$ Joshua is correct ***A1***

Note: The second ***A1*** can be awarded for an answer consistent with their result.

[2 marks]

(d) height of trapezium is $\sqrt{10^2+6^2}$ $(=11.6619...)$ ***(M1)***

area of trapezium is $\frac{80+60}{2}\times\sqrt{10^2+6^2}$ $(=816.333...)$ ***(M1)(A1)***

$(SA=)\ 4\times\left(\frac{80+60}{2}\times\sqrt{10^2+6^2}\right)+60^2$ ***(M1)***

Note: Award ***M1*** for adding 4 times their (MNOP) trapezium area to the area of the (60×60) base.

$(SA=)\ 6870\ \text{m}^2\ \ (6865.33\ \text{m}^2)$ ***A1***

Note: No marks are awarded if the correct shape is not identified.

[5 marks]
Total: [14 marks]

Question 5

(a) (i) maximum $h = 130$ metres **A1**

(ii) minimum $h = 50$ metres **A1**

[2 marks]

(b) (i) $(60 \div 12 =)$ 5 seconds **A1**

(ii) $360 \div 5$ ***(M1)***

Note: Award ***(M1)*** for 360 divided by their time for one revolution.

$= 72°$ **A1**

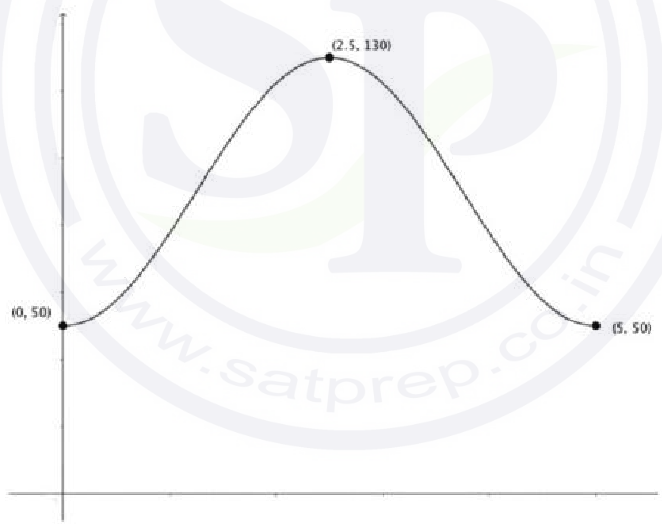
[3 marks]

(c) (i) (amplitude $=$) 40 **A1**

(ii) (period $= \dfrac{360}{72} =$) 5 **A1**

[2 marks]

(d)



Maximum point labelled with correct coordinates. **A1**

At least one minimum point labelled. Coordinates seen for any minimum points must be correct. **A1**

Correct shape with an attempt at symmetry and "concave up" evident as it approaches the minimum points. Graph must be drawn in the given domain. **A1**

[3 marks]

(e) (i) $h = 90 - 40\cos(144^{\circ})$ ***(M1)***

$(h =)\ 122\,\text{m}\ \ (122.3606.....)$ ***A1***

(ii) evidence of $h = 100$ on graph **OR** $100 = 90 - 40\cos(72t)$ ***(M1)***

t coordinates $3.55\ (3.54892...)$ **OR** $1.45\ (1.45107...)$ or equivalent ***(A1)***

Note: Award ***A1*** for either t-coordinate seen.

= 2.10 seconds $(2.09784\ldots)$ ***A1***

[5 marks]

(f) (i) $5 - 2.09784\ldots$ ***(M1)***

$\frac{(2.902153...)}{5}$ ***(M1)***

$0.580\ \ (0.580430...)$ ***A1***

(ii) **METHOD 1**

changing the frequency/dilation of the graph will not change the proportion of time that point C is visible. ***A1***

$0.580\ \ (0.580430...)$ ***A1***

METHOD 2

correct calculation of relevant found values

$\frac{(2.902153...)/2}{5/2}$ ***A1***

$0.580\ \ (0.580430...)$ ***A1***

Note: Award ***A0A1*** for an unsupported correct probability.

[5 marks]

Total: [20 marks]

Question 6

(a) (i) $\left(\frac{1}{2}\hat{AOB} =\right) \arccos\left(\frac{4}{4.5}\right) = 27.266...$ ***(M1)(A1)***

$\hat{AOB} = 54.532... \approx 54.5°$ $(0.951764... \approx 0.952$ radians$)$ ***A1***

Note: Other methods may be seen; award ***(M1)(A1)*** for use of a correct trigonometric method to find an appropriate angle and then ***A1*** for the correct answer.

(ii) finding area of triangle

EITHER

area of triangle = $\frac{1}{2} \times 4.5^2 \times \sin(54.532...)$ ***(M1)***

Note: Award ***M1*** for correct substitution into formula.

$= 8.24621... \approx 8.25 \text{ m}^2$ ***(A1)***

OR

$AB = 2 \times \sqrt{4.5^2 - 4^2} = 4.1231...$

area triangle $= \frac{4.1231... \times 4}{2}$ ***(M1)***

$= 8.24621... \approx 8.25 \text{ m}^2$ ***(A1)***

finding area of sector

EITHER

area of sector = $\frac{54.532...}{360} \times \pi \times 4.5^2$ ***(M1)***

$= 9.63661... \approx 9.64 \text{ m}^2$ ***(A1)***

OR

area of sector = $\frac{1}{2} \times 0.9517641.. \times 4.5^2$ ***(M1)***

$= 9.63661... \approx 9.64 \text{ m}^2$ ***(A1)***

THEN

area of segment = $9.63661... - 8.24621...$

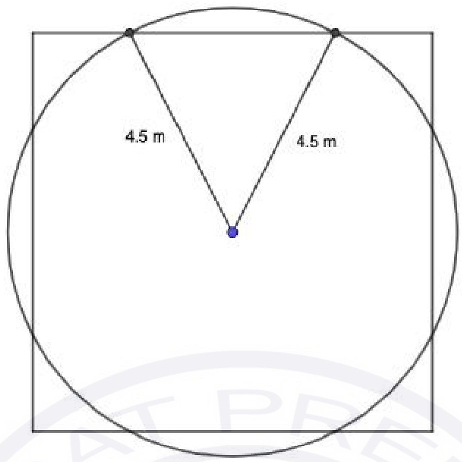
$= 1.39 \text{ m}^2$ $(1.39040...)$ ***A1***

[8 marks]

(b) (i) $\pi \times 4.5^2$ **(M1)**

$63.6\ \text{m}^2\ \ (63.6172\ldots\text{m}^2)$ **A1**

(ii) **METHOD 1**



$4 \times 1.39040...\ \ (5.56160)$ **(A1)**

subtraction of four segments from area of circle **(M1)**

$= 58.1\ \text{m}^2\ \ (58.055...)$ **A1**

METHOD 2

$4(0.5 \times 4.5^2 \times \sin 54.532\ldots) + 4\left(\frac{35.4679}{360} \times \pi \times 4.5^2\right)$ **(M1)**

$= \ 32.9845\ldots + 25.0707$ **(A1)**

$= 58.1\ \text{m}^2\ \ (58.055\ldots)$ **A1**

[5 marks]

(c) sketch of $\frac{\mathrm{d}V}{\mathrm{d}t}$ **OR** $\frac{\mathrm{d}V}{\mathrm{d}t} = 0.110363...$ **OR** attempt to find where $\frac{\mathrm{d}^2V}{\mathrm{d}t^2} = 0$ **(M1)**

$t = 1$ hour **A1**

[2 marks]

[Total 15 marks]

Question 7

(a) **EITHER**
annual cycle for daylight length ***R1***

OR
there is a minimum length for daylight (cannot be negative) ***R1***

OR
a quadratic could not have a maximum and a minimum or equivalent ***R1***

Note: Do not accept "Paula's model is better".

[1 mark]

(b) (i) 4 ***A1***

(ii) 12 ***A1***

(iii) $y = 12$ ***A1A1***

Note: Award ***A1*** "$y =$ (a constant)" and ***A1*** for that constant being 12.

[4 marks]

(c) $f(t) = -4\cos(30t) + 12$ **OR** $f(t) = -4\cos(-30t) + 12$ ***A1A1A1***

Note: Award ***A1*** for $b = 30$ (or $b = -30$), ***A1*** for $a = -4$, and ***A1*** for $d = 12$. Award at most ***A1A1A0*** if extra terms are seen or form is incorrect. Award at most ***A1A1A0*** if x is used instead of t.

[3 marks]

(d) $10.5 = -4\cos(30t) + 12$ ***(M1)***

EITHER
$t_1 = 2.26585\ldots,\ t_2 = 9.73414\ldots$ ***(A1)(A1)***

OR
$t_1 = \frac{1}{30}\cos^{-1}\frac{3}{8}$ ***(A1)***
$t_2 = 12 - t_1$ ***(A1)***

THEN
$9.73414\ldots - 2.26585\ldots$
7.47 (7.46828…) months (0.622356…years) ***A1***

Note: Award ***M1A1A1A0*** for an unsupported answer of 7.46. If there is only one intersection point, award ***M1A1A0A0***.

[4 marks]

Question 8

(a) (i) an attempt to find the amplitude **(M1)**

$\frac{61.8}{2}$ **OR** $\frac{64.5-2.7}{2}$

$(a=)\ 30.9\text{ m}$ **A1**

Note: Accept an answer of $(a=)\ -30.9\text{ m}$.

(ii) (period $=\frac{60}{1.5}=$) $40\ (\text{s})$ **(A1)**

$((b=)\ \frac{360^{\circ}}{40})$

$(b=)\ 9$ **A1**

Note: Accept an answer of $(b=)\ -9$.

(iii) attempt to find d **(M1)**

$(d=)\ 30.9+2.7$ **OR** $\frac{64.5+2.7}{2}$

$(d=)\ 33.6\text{ m}$ **A1**

[6 marks]

(b) 12×1.5 **OR** $\frac{12\times60}{40}$ **(M1)**

18 (revolutions per ride) **A1**

[2 marks]

(c) (i) $0\le t\le720$ **A1**

(ii) $2.7\le h\le64.5$ **A1A1**

Note: Award ***A1*** for correct endpoints of domain and ***A1*** for correct endpoints of range. Award ***A1*** for correct direction of both inequalities.

[3 marks]

(d) graph of $h(t)$ and $y=16.7$ **OR** $h(t)=16.7$ **(M1)**

$6.31596\ldots$ and $33.6840\ldots$ **(A1)**

$27.4\ (\text{s})\ \ (27.3680\ldots)$ **A1**

[3 marks]

(e) (i) d ***A1***

(ii) **EITHER**

$d + 30.9 = 65.2$ ***(A1)***

OR

$65.2 - (61.8 + 2.7) = 0.7$ ***(A1)***

OR

3.4 (new platform height) ***(A1)***

THEN

$(d =)\ 34.3\ \text{m}$ ***A1***

[3 marks]

Total [17 marks]

Question 9

(a) $\left(\frac{2+6}{2}, \frac{2+0}{2}\right)$ ***(M1)***

$(4, 1)$ ***A1***

Note: Award ***A0*** if parentheses are omitted in the final answer.

[2 marks]

(b) attempt to substitute values into gradient formula ***(M1)***

$\left(\frac{0-2}{6-2}=\right)-\frac{1}{2}$ ***(A1)***

therefore the gradient of perpendicular bisector is 2 ***(M1)***

so $y-1=2(x-4)$ $\quad (y=2x-7)$ ***A1***

[4 marks]

(c) identifying the correct equations to use: ***(M1)***

$y=2-x$ and $y=2x-7$

evidence of solving their correct equations or of finding intersection point graphically ***(M1)***

$(3, -1)$ ***A1***

Note: Accept an answer expressed as "$x=3,\ y=-1$".

[3 marks]

(d) attempt to use distance formula ***(M1)***

$\text{YZ}=\sqrt{(7-(-1))^2+(7-3)^2}$

$=\sqrt{80}\ \left(4\sqrt{5}\right)$ ***A1***

[2 marks]

(e) **METHOD 1 (cosine rule)**

length of XZ is $\sqrt{80}\ \left(4\sqrt{5}, 8.94427\ldots\right)$ ***(A1)***

Note: Accept 8.94 and 8.9.

attempt to substitute into cosine rule ***(M1)***

$\cos \text{X}\hat{\text{Y}}\text{Z}=\frac{80+32-80}{2\times\sqrt{80}\sqrt{32}}$ $\quad (=0.316227\ldots)$ ***(A1)***

Note: Award ***A1*** for correct substitution of XZ, YZ, $\sqrt{32}$ values in the cos rule. Exact values do not need to be used in the substitution.

$\left(\text{X}\hat{\text{Y}}\text{Z}=\right)\ 71.6°$ $\quad (71.5650\ldots°)$ ***A1***

METHOD 2 (splitting isosceles triangle in half)

length of XZ is $\sqrt{80}$ $\left(4\sqrt{5}, 8.94427...\right)$ ***(A1)***

Note: Accept 8.94 and 8.9.

required angle is $\cos^{-1}\left(\frac{\sqrt{32}}{2\sqrt{80}}\right)$ ***(M1)(A1)***

Note: Award ***A1*** for correct substitution of XZ (or YZ), $\frac{\sqrt{32}}{2}$ values in the cos rule. Exact values do not need to be used in the substitution.

$\left(X\hat{Y}Z =\right)$ $71.6°$ $(71.5650°)$ ***A1***

Note: Last ***A1*** mark may be lost if prematurely rounded values of XZ, YZ and/or XY are used.

[4 marks]

(f) (area =) $\frac{1}{2}\sqrt{80}\sqrt{32}\sin 71.5650\ldots$ **OR** (area =) $\frac{1}{2}\sqrt{32}\sqrt{72}$ ***(M1)***

$= 24 \text{ km}^2$ ***A1***

[2 marks]

(g) *Any sensible answer such as:*
There might be factors other than proximity which influence shopping choices.
A larger area does not necessarily result in an increase in population.
The supermarkets might be specialized / have a particular clientele who visit even if other shops are closer.
Transport links might not be represented by Euclidean distances.
etc. ***R1***

[1 mark]

Total [18 marks]

Question 10

(a) (i) B *A1*

(ii) F *A1*

[2 marks]

(b) correct substitution into the midpoint formula *(M1)*

$\frac{8+5}{2}$

$y = 6.5$ *A1*

Note: Answer must be an equation for the ***A1*** to be awarded.

[2 marks]

(c) midpoint $= (5, 7)$ *(A1)*

correct use of gradient formula *(M1)*

$\frac{8-6}{7-3}$

gradient of $\text{BC} = 0.5$ *(A1)*

negative reciprocal of gradient *(M1)*

perpendicular gradient $= -2$

$y - 7 = -2(x - 5)$ (or $y = -2x + 17$) *A1*

[5 marks]

(d) (i) attempt to find the intersection of two perpendicular bisectors (BC & CD) *(M1)*

Note: This may be seen graphically or algebraically.

$6.5 - 7 = -2(x - 5)$ **OR** $6.5 = -2x + 17$

Note: Accept equivalent methods using the perpendicular bisector of BD, $y - 5.5 = 4(x - 5)$ **OR** $y = 4x - 14.5$

$x = 5.25, y = 6.5$ **OR** $(5.25, 6.5)$ *A1*

Note: The x-coordinate must be exact or expressed to at least 3 sf.

(ii) their correct substitution into distance formula *(M1)*

$\sqrt{(5.25-7)^2 + (6.5-5)^2}$

$= 2.30 \text{ km} \left(2.30488\ldots, \frac{\sqrt{85}}{4}\right)$ *A1*

[4 marks]

Total [13 marks]

Question 11

(a) (i) 19° **A1**

(ii) $\dfrac{\text{BD}}{\sin 120^\circ} = \dfrac{40}{\sin 19^\circ}$ ***(M1)(A1)***

Note: Award ***M1*** for substituted sine rule for BCD, ***A1*** for their correct substitution.

$(\text{BD} =)\ 106\text{ m}\ \ (106.401...)$ ***A1***

[4 marks]

(b) **METHOD 1 (cosine rule)**

$\cos \text{BAD} = \dfrac{85^2 + 85^2 - 106.401...^2}{2 \times 85 \times 85}$ ***(M1)(A1)***

Note: Award ***M1*** for substituted cosine rule, ***A1*** for their correct substitution.

77.495 ***A1***

Note: Accept an answer of 77.149 from use of 3 sf answer from part (a). The final answer must be correct to five significant figures.

METHOD 2 (right angled trig/isosceles triangles)

$\sin\left(\dfrac{\text{BAD}}{2}\right) = \dfrac{53.2008...}{85}$ ***(A1)(M1)***

Note: Award ***A1*** for $53.2008...$ seen. Award ***M1*** for correctly substituted trig ratio. Follow through from part (a).

$77.495...$ ***A1***

Note: Use of 3 sf answer from part (a), results in 77.149.

[3 marks]

(c) **EITHER**

$(\text{Area} =)\ \dfrac{1}{2} \times 85 \times 85 \times \sin(77^\circ)$ ***(M1)(A1)***

Note: Award ***M1*** for substituted area formula, ***A1*** for correct substitution. Award at most ***(M1)(A1)A0*** if an angle other than 77° is used.

OR

$(\text{Area} =)\ \dfrac{1}{2} \times \left(2 \times 85 \times \sin(38.5^\circ)\right) \times \left(85 \times \cos(38.5^\circ)\right)$ ***(M1)(A1)***

Note: Award ***M1*** for substituted area formula $A = \dfrac{1}{2}bh$, ***A1*** for correct substitution.

$3520\text{ m}^2\ (3519.91...)$ ***A1***

[3 marks]

(d) 85 m **A1**

[1 mark]

(e) $85+85+\frac{77}{360}\times 2\pi\times 85$ ***(M1)(M1)***

Note: Award ***M1*** for correctly substituted into $\frac{\theta}{360}\times 2\pi\times r$, ***M1*** for addition of AB and AD.

284 m (284.231...) **A1**

[3 marks]

(f) $\frac{77}{360}\times\pi\times(85)^2-3519.91...$ ***(M1)(M1)***

Note: Award ***M1*** for correctly substituted area of sector formula, ***M1*** for subtraction of their area from part (c).

1330 m^2 (1334.93...) **A1**

[3 marks]
Total [17 marks]