Subject – Math AI(Standard Level)
Topic - Geometry and Trigonometry
Year - May 2021 – Nov 2022
Paper -2
Questions

Question 1

[Maximum mark: 17]

The Malvern Aquatic Center hosted a 3 metre spring board diving event. The judges, Stan and Minsun awarded 8 competitors a score out of 10. The raw data is collated in the following table.

Competitors	A	B	C	D	E	F	G	H
Stan's score (x)	4.1	3	4.3	6	7.1	6	7.5	6
Minsun's score (y)	4.7	4.6	4.8	7.2	7.8	9	9.5	7.2

(a) (i) Write down the value of the Pearson's product–moment correlation coefficient, r.

(ii) Using the value of r, interpret the relationship between Stan's score and Minsun's score. [4]

(b) Write down the equation of the regression line y on x. [2]

(c) (i) Use your regression equation from part (b) to estimate Minsun's score when Stan awards a perfect 10.

(ii) State whether this estimate is reliable. Justify your answer. [4]

The Commissioner for the event would like to find the Spearman's rank correlation coefficient.

(d) **Copy** and complete the information in the following table. [2]

Competitors	A	B	C	D	E	F	G	H
Stan's Rank		8					1	4
Minsun's Rank		8					1	4.5

(e) (i) Find the value of the Spearman's rank correlation coefficient, r_s.

(ii) Comment on the result obtained for r_s. [4]

The Commissioner believes Minsun's score for competitor G is too high and so decreases the score from 9.5 to 9.1.

(f) Explain why the value of the Spearman's rank correlation coefficient r_s does not change. [1]

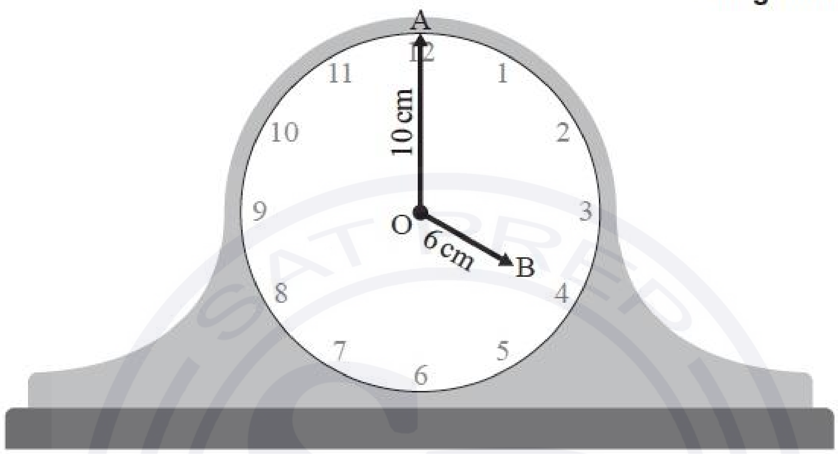
Question 2

[Maximum mark: 17]

The diagram below shows a circular clockface with centre O. The clock's minute hand has a length of $10\,\text{cm}$. The clock's hour hand has a length of $6\,\text{cm}$.

At 4:00 pm the endpoint of the minute hand is at point A and the endpoint of the hour hand is at point B.

diagram not to scale

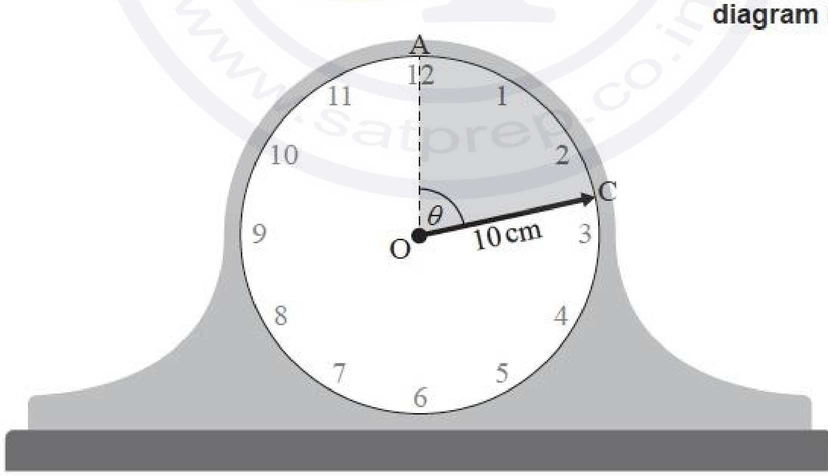


(a) Find the size of angle $\text{A}\hat{\text{O}}\text{B}$ in degrees. [2]

(b) Find the distance between points A and B. [3]

Between 4:00 pm and 4:13 pm, the endpoint of the **minute hand** rotates through an angle, θ, from point A to point C. This is illustrated in the diagram.

diagram not to scale



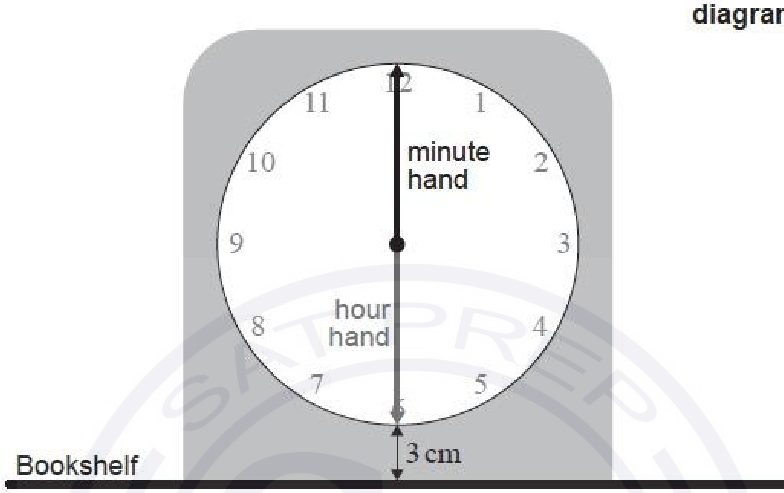
(c) Find the size of angle θ in degrees. [2]

(d) Calculate the length of arc AC. [2]

(e) Calculate the area of the shaded sector, AOC. [2]

A **second** clock is illustrated in the diagram below. The clock face has radius $10\,\mathrm{cm}$ with minute and hour hands both of length $10\,\mathrm{cm}$. The time shown is 6:00 am. The bottom of the clock face is located $3\,\mathrm{cm}$ above a horizontal bookshelf.

diagram not to scale



(f) Write down the height of the endpoint of the minute hand above the bookshelf at 6:00 am. [1]

The height, h centimetres, of the endpoint of the minute hand above the bookshelf is modelled by the function

$$h(\theta) = 10\cos\theta + 13\,,\ \theta \geq 0\,,$$

where θ is the angle rotated by the minute hand from 6:00 am.

(g) Find the value of h when $\theta = 160^\circ$. [2]

The height, g centimetres, of the endpoint of the **hour hand** above the bookshelf is modelled by the function

$$g(\theta) = -10\cos\left(\frac{\theta}{12}\right) + 13\,,\ \theta \geq 0\,,$$

where θ is the angle in degrees rotated by the minute hand from 6:00 am.

(h) Write down the amplitude of $g(\theta)$. [1]

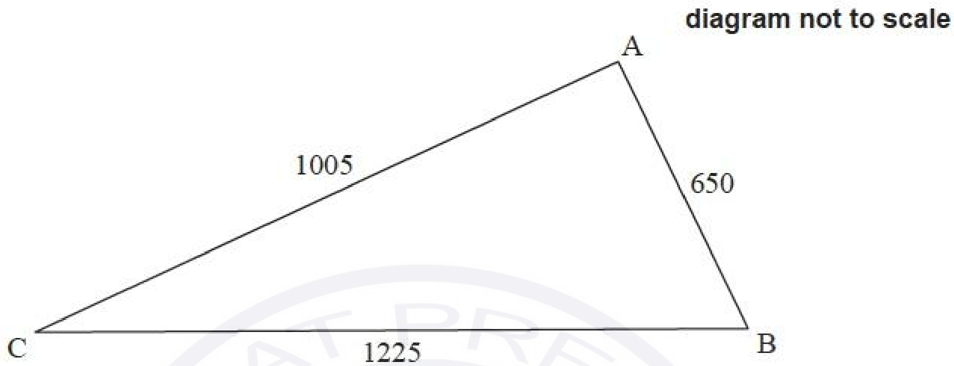
The endpoints of the minute hand and hour hand meet when $\theta = k$.

(i) Find the smallest possible value of k. [2]

Question 3

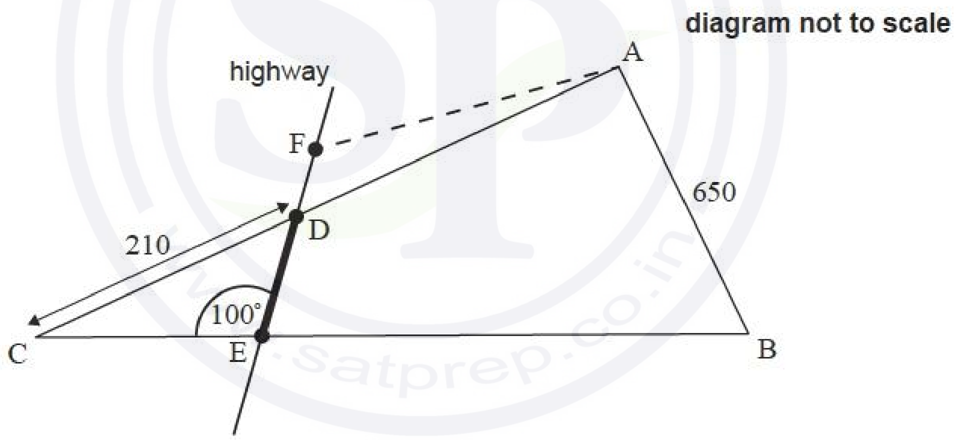
[Maximum mark: 15]

A farmer owns a field in the shape of a triangle ABC such that $\mathrm{AB} = 650\,\mathrm{m}$, $\mathrm{AC} = 1005\,\mathrm{m}$ and $\mathrm{BC} = 1225\,\mathrm{m}$.



(a) Find the size of $\mathrm{A\hat{C}B}$. [3]

The local town is planning to build a highway that will intersect the borders of the field at points D and E, where $\mathrm{DC} = 210\,\mathrm{m}$ and $\mathrm{C\hat{E}D} = 100^\circ$, as shown in the diagram below.



(b) Find DE. [3]

The town wishes to build a carpark here. They ask the farmer to exchange the part of the field represented by triangle DCE. In return the farmer will get a triangle of equal area ADF, where F lies on the same line as D and E, as shown in the diagram above.

(c) Find the area of triangle DCE. [5]

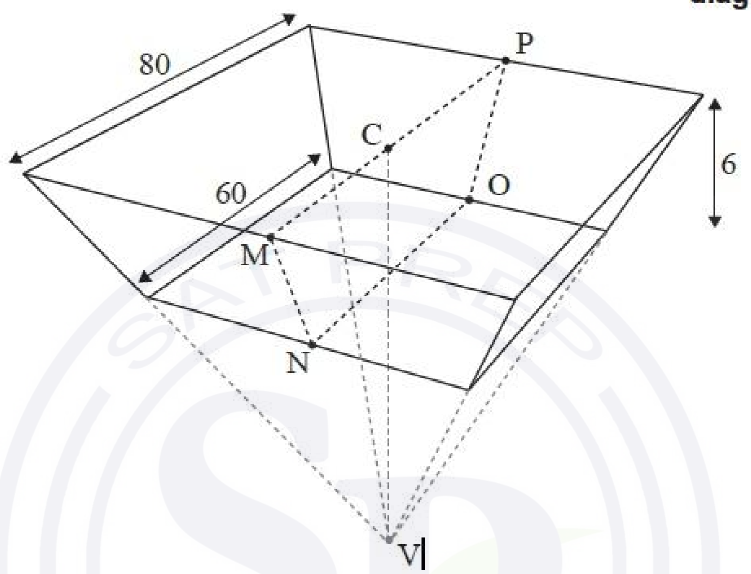
(d) Estimate DF. You may assume the highway has a width of zero. [4]

Question 4

[Maximum mark: 14]

A large water reservoir is built in the form of part of an upside-down right pyramid with a horizontal square base of length 80 metres. The point C is the centre of the square base and point V is the vertex of the pyramid.

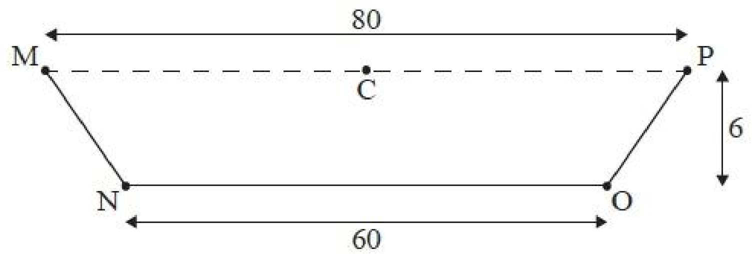
diagram not to scale



The bottom of the reservoir is a square of length 60 metres that is parallel to the base of the pyramid, such that the depth of the reservoir is 6 metres as shown in the diagram.

The second diagram shows a vertical cross section, $MNOPC$, of the reservoir.

diagram not to scale



(a) Find the angle of depression from M to N. [2]

(b) (i) Find CV.

(ii) Hence or otherwise, show that the volume of the reservoir is $29\,600\,\text{m}^3$. [5]

Every day $80\,\text{m}^3$ of water from the reservoir is used for irrigation.

Joshua states that, if no other water enters or leaves the reservoir, then when it is full there is enough irrigation water for at least one year.

(c) By finding an appropriate value, determine whether Joshua is correct. [2]

To avoid water leaking into the ground, the five interior sides of the reservoir have been painted with a watertight material.

(d) Find the area that was painted. [5]

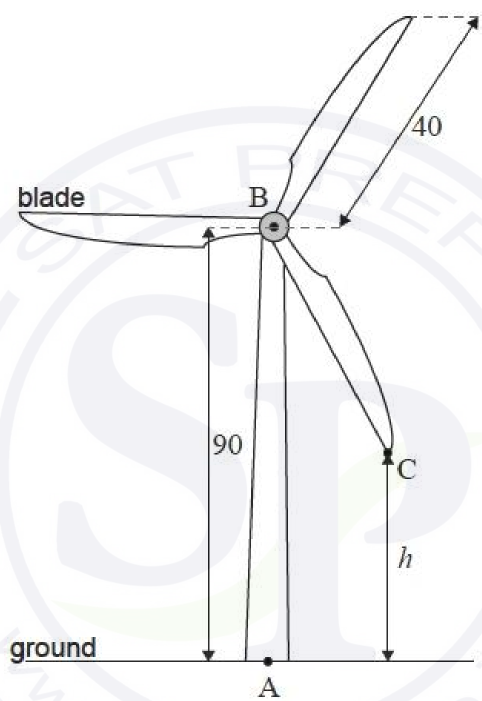
Question 5

[Maximum mark: 20]

A wind turbine is designed so that the rotation of the blades generates electricity. The turbine is built on horizontal ground and is made up of a vertical tower and three blades.

The point A is on the base of the tower directly below point B at the top of the tower. The height of the tower, AB, is $90\,\mathrm{m}$. The blades of the turbine are centred at B and are each of length $40\,\mathrm{m}$. This is shown in the following diagram.

diagram not to scale



The end of one of the blades of the turbine is represented by point C on the diagram. Let h be the height of C above the ground, measured in metres, where h varies as the blade rotates.

(a) Find the

(i) maximum value of h.

(ii) minimum value of h. [2]

The blades of the turbine complete 12 rotations per minute under normal conditions, moving at a constant rate.

(b) (i) Find the time, in seconds, it takes for the blade $[\mathrm{BC}]$ to make one complete rotation under these conditions.

(ii) Calculate the angle, in degrees, that the blade $[\mathrm{BC}]$ turns through in one second. [3]

The height, h, of point C can be modelled by the following function. Time, t, is measured from the instant when the blade [BC] first passes [AB] and is measured in seconds.

$$h(t) = 90 - 40\cos(72t^\circ),\ t \geq 0$$

(c) (i) Write down the amplitude of the function.

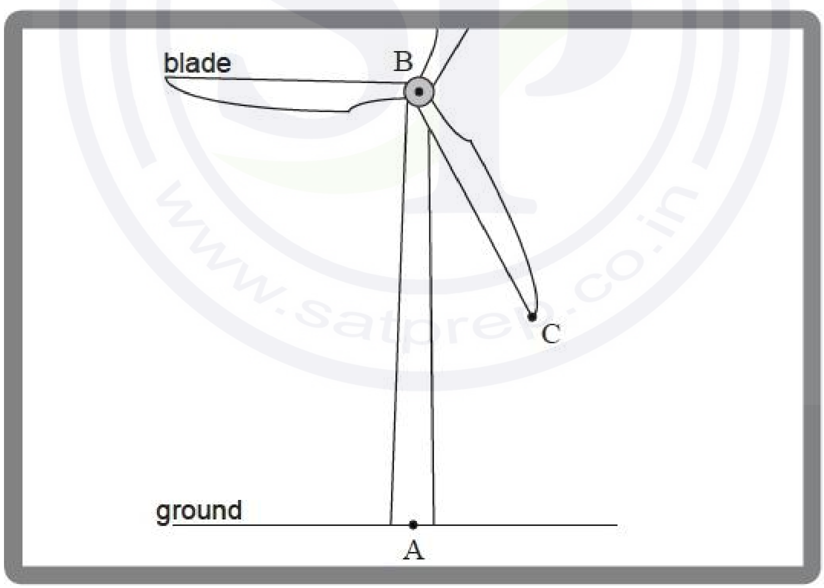
(ii) Find the period of the function. [2]

(d) Sketch the function $h(t)$ for $0 \leq t \leq 5$, clearly labelling the coordinates of the maximum and minimum points. [3]

(e) (i) Find the height of C above the ground when $t = 2$.

(ii) Find the time, in seconds, that point C is above a height of $100\,\mathrm{m}$, during each complete rotation. [5]

Looking through his window, Tim has a partial view of the rotating wind turbine. The position of his window means that he cannot see any part of the wind turbine that is **more than 100 m** above the ground. This is illustrated in the following diagram.



(f) (i) At any given instant, find the probability that point C is visible from Tim's window.

The wind speed increases. The blades rotate at twice the speed, but still at a constant rate.

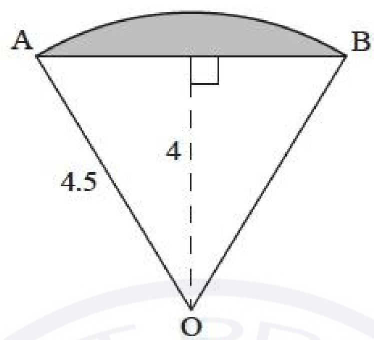
(ii) At any given instant, find the probability that Tim can see point C from his window. Justify your answer. [5]

Question 6

[Maximum mark: 15]

A sector of a circle, centre O and radius $4.5\,\text{m}$, is shown in the following diagram.

diagram not to scale

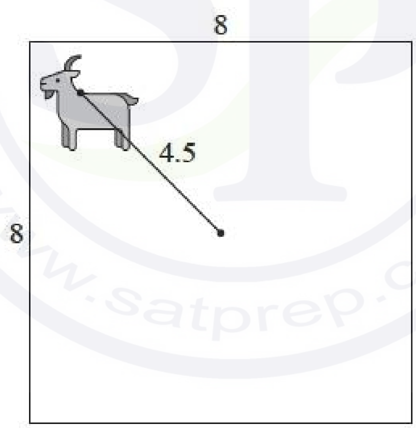


(a) (i) Find the angle $\text{A}\hat{\text{O}}\text{B}$.

(ii) Find the area of the shaded segment. [8]

A square field with side $8\,\text{m}$ has a goat tied to a post in the centre by a rope such that the goat can reach all parts of the field up to $4.5\,\text{m}$ from the post.

diagram not to scale



(b) (i) Find the area of a circle with radius $4.5\,\text{m}$.

(ii) Find the area of the field that can be reached by the goat. [5]

Let V be the volume of grass eaten by the goat, in cubic metres, and t be the length of time, in hours, that the goat has been in the field.

The goat eats grass at the rate of $\frac{\mathrm{d}V}{\mathrm{d}t} = 0.3\,t\mathrm{e}^{-t}$.

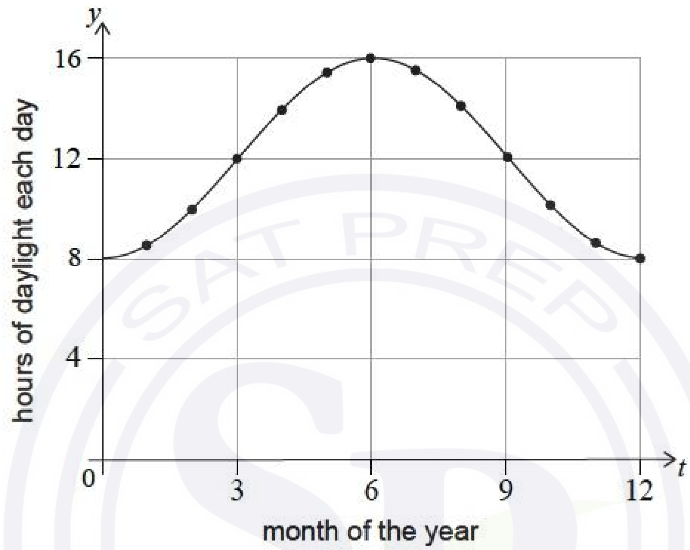
(c) Find the value of t at which the goat is eating grass at the greatest rate. [2]

Question 7

[Maximum mark: 15]

Boris recorded the number of daylight hours on the first day of each month in a northern hemisphere town.

This data was plotted onto a scatter diagram. The points were then joined by a smooth curve, with minimum point $(0, 8)$ and maximum point $(6, 16)$ as shown in the following diagram.



Let the curve in the diagram be $y = f(t)$, where t is the time, measured in months, since Boris first recorded these values.

Boris thinks that $f(t)$ might be modelled by a quadratic function.

(a) Write down one reason why a quadratic function would not be a good model for the number of hours of daylight per day, across a number of years. [1]

Paula thinks that a better model is $f(t) = a\cos(bt) + d,\ t \geq 0$, for specific values of a, b and d.

(b) For Paula's model, use the diagram to write down

(i) the amplitude.

(ii) the period.

(iii) the equation of the principal axis. [4]

(c) Hence or otherwise find the equation of this model in the form: [3]

$$f(t) = a\cos(bt) + d$$

(d) For the first year of the model, find the length of time when there are more than 10 hours and 30 minutes of daylight per day. [4]

The true maximum number of daylight hours was 16 hours and 14 minutes.

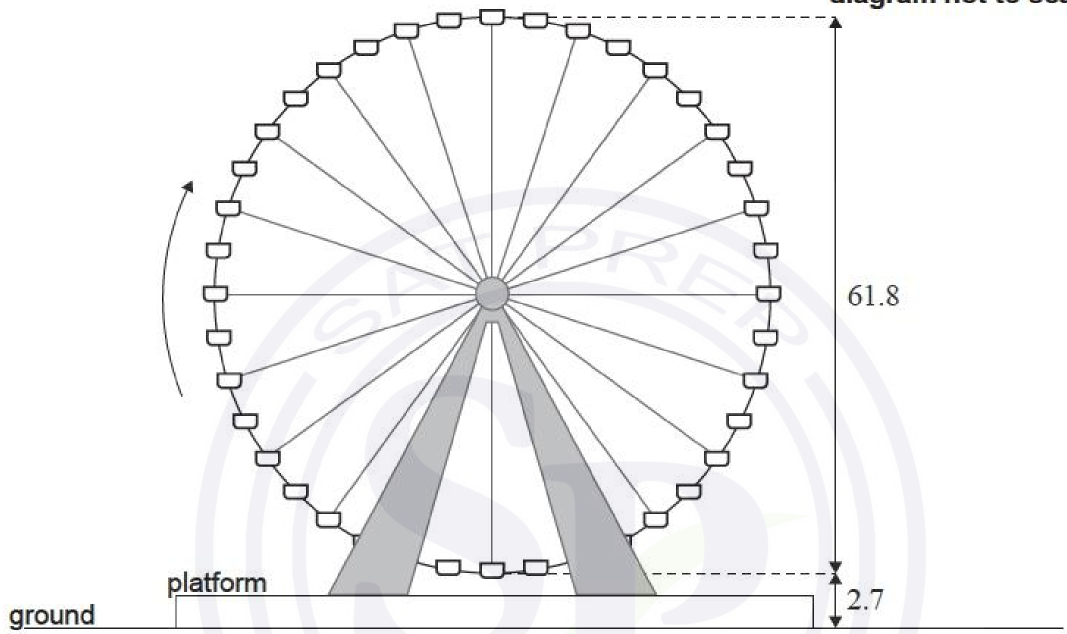
(e) Calculate the percentage error in the maximum number of daylight hours Boris recorded in the diagram. [3]

Question 8

[Maximum mark: 17]

The Texas Star is a Ferris wheel at the state fair in Dallas. The Ferris wheel has a diameter of $61.8\,\text{m}$. To begin the ride, a passenger gets into a chair at the lowest point on the wheel, which is $2.7\,\text{m}$ above the ground, as shown in the following diagram. A ride consists of multiple revolutions, and the Ferris wheel makes 1.5 revolutions per minute.

diagram not to scale



The height of a chair above the ground, h, measured in metres, during a ride on the Ferris wheel can be modelled by the function $h(t) = -a\cos(bt) + d$, where t is the time, in seconds, since a passenger began their ride.

(a) Calculate the value of

(i) a;

(ii) b;

(iii) d. [6]

A ride on the Ferris wheel lasts for 12 minutes in total.

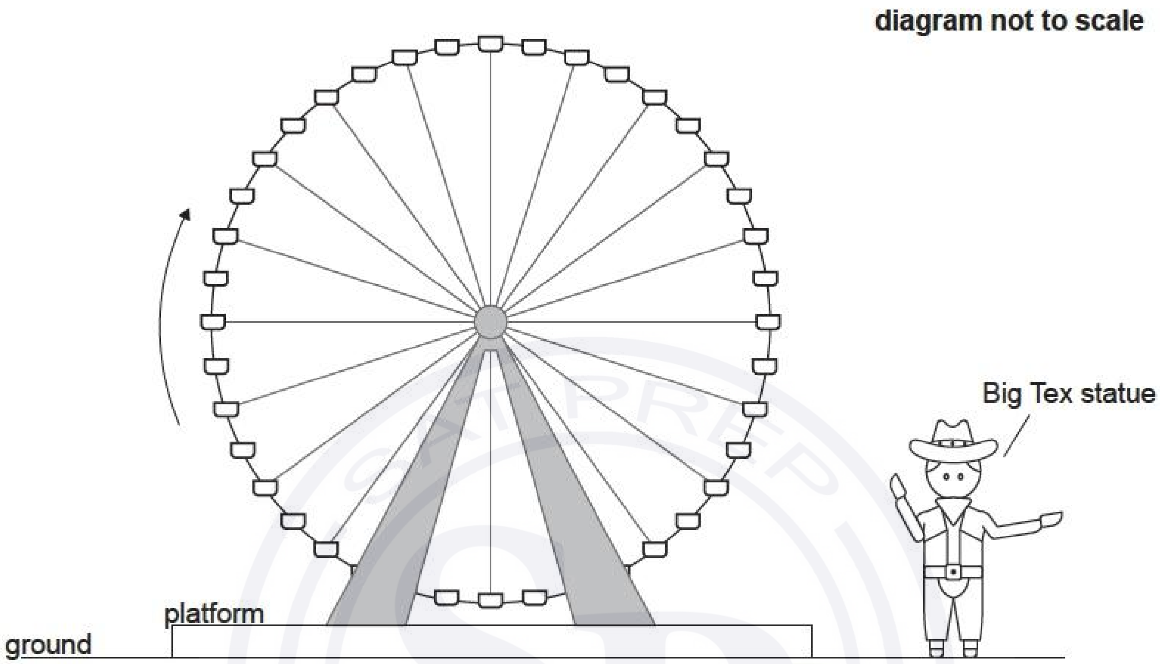
(b) Calculate the number of revolutions of the Ferris wheel per ride. [2]

(c) For exactly one ride on the Ferris wheel, suggest

(i) an appropriate domain for $h(t)$;

(ii) an appropriate range for $h(t)$. [3]

Big Tex is a 16.7 metre-tall cowboy statue that stands on the horizontal ground next to the Ferris wheel.



(d) By considering the graph of $h(t)$, determine the length of time during one revolution of the Ferris wheel for which the chair is higher than the cowboy statue. [3]

There is a plan to relocate the Texas Star Ferris wheel onto a taller platform which will increase the maximum height of the Ferris wheel to $65.2\,\mathrm{m}$. This will change the value of one parameter, a, b or d, found in part (a).

(e) (i) Identify which parameter will change.

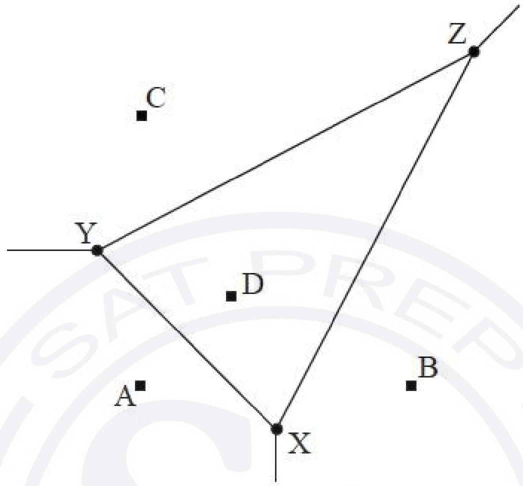
(ii) Find the new value of the parameter identified in part (e)(i). [3]

Question 9

[Maximum mark: 18]

The Voronoi diagram below shows four supermarkets represented by points with coordinates $A(0, 0)$, $B(6, 0)$, $C(0, 6)$ and $D(2, 2)$. The vertices X, Y, Z are also shown. All distances are measured in kilometres.

diagram not to scale



(a) Find the midpoint of $[BD]$. [2]

(b) Find the equation of (XZ). [4]

The equation of (XY) is $y = 2 - x$ and the equation of (YZ) is $y = 0.5x + 3.5$.

(c) Find the coordinates of X. [3]

The coordinates of Y are $(-1, 3)$ and the coordinates of Z are $(7, 7)$.

(d) Determine the exact length of $[YZ]$. [2]

(e) Given that the exact length of $[XY]$ is $\sqrt{32}$, find the size of $X\hat{Y}Z$ in degrees. [4]

(f) Hence find the area of triangle XYZ. [2]

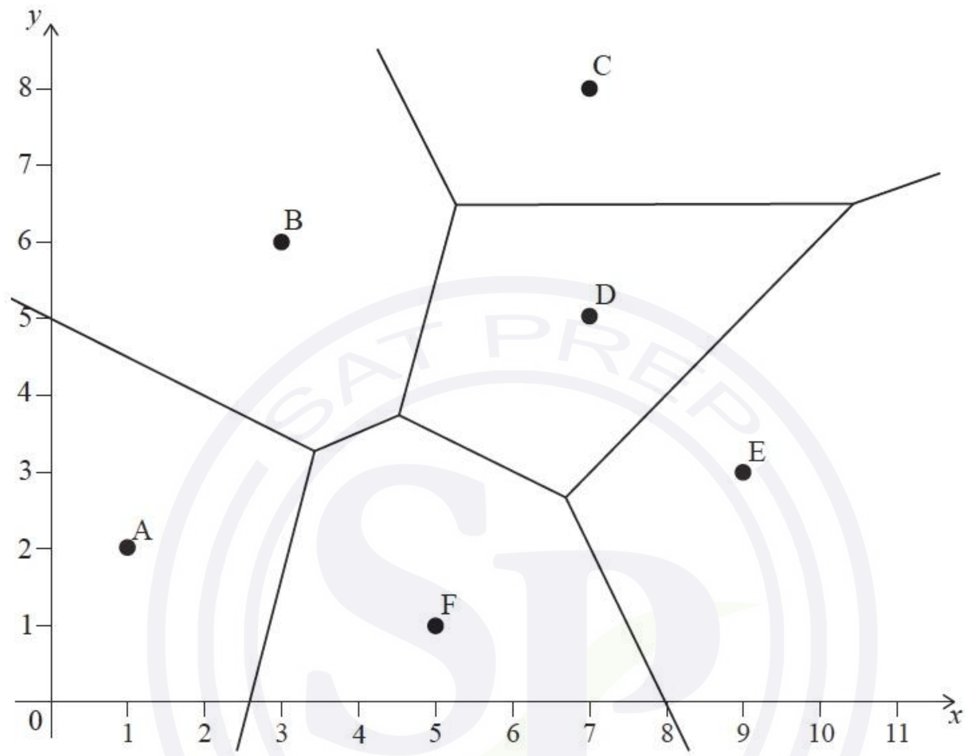
A town planner believes that the larger the area of the Voronoi cell XYZ, the more people will shop at supermarket D.

(g) State one criticism of this interpretation. [1]

Question 10

[Maximum mark: 13]

Six restaurant locations (labelled A, B, C, D, E and F) are shown, together with their Voronoi diagram. All distances are measured in kilometres.



(a) Elena wants to eat at the closest restaurant to her. Write down the restaurant she should go to, if she is at

(i) $(2, 7)$.

(ii) $(0, 1)$, when restaurant A is closed. [2]

Restaurant C is at $(7, 8)$ and restaurant D is at $(7, 5)$.

(b) Find the equation of the perpendicular bisector of CD. [2]

Restaurant B is at $(3, 6)$.

(c) Find the equation of the perpendicular bisector of BC. [5]

(d) Hence find

(i) the coordinates of the point which is of equal distance from B, C and D.

(ii) the distance of this point from D. [4]

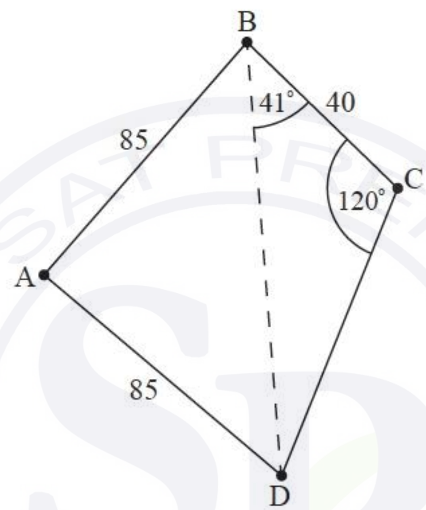
Question 11

[Maximum mark: 17]

The following diagram shows a park bounded by a fence in the shape of a quadrilateral ABCD. A straight path crosses through the park from B to D.

$$\mathrm{AB} = 85\,\mathrm{m},\ \mathrm{AD} = 85\,\mathrm{m},\ \mathrm{BC} = 40\,\mathrm{m},\ \mathrm{C\hat{B}D} = 41^\circ,\ \mathrm{B\hat{C}D} = 120^\circ$$

diagram not to scale



(a) (i) Write down the value of angle BDC.

(ii) Hence use triangle BDC to find the length of path BD. [4]

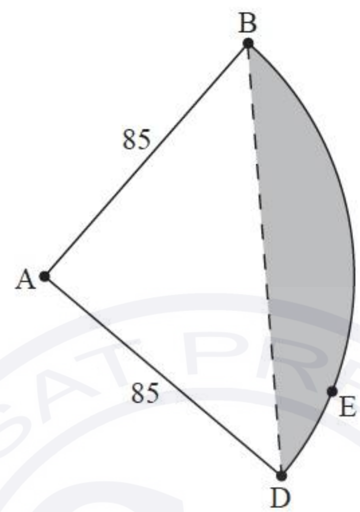
(b) Calculate the size of angle $\mathrm{B\hat{A}D}$, correct to five significant figures. [3]

The size of angle $\mathrm{B\hat{A}D}$ rounds to 77°, correct to the nearest degree. Use $\mathrm{B\hat{A}D} = 77^\circ$ for the rest of this question.

(c) Find the area bounded by the path BD, and fences AB and AD. [3]

A landscaping firm proposes a new design for the park. Fences BC and CD are to be replaced by a fence in the shape of a circular arc BED with center A. This is illustrated in the following diagram.

diagram not to scale



(d) Write down the distance from A to E. [1]

(e) Find the perimeter of the proposed park, ABED. [3]

(f) Find the area of the shaded region in the proposed park. [3]