

Subject – Math AI(Standard Level)
Topic - Number and Algebra
Year - May 2021 – Nov 2024
Paper -1
Questions

Question 1

[Maximum mark: 17]

In this question, give all answers to two decimal places.

Bryan decides to purchase a new car with a price of €14 000, but cannot afford the full amount. The car dealership offers two options to finance a loan.

Finance option A:

A 6 year loan at a nominal annual interest rate of 14% **compounded quarterly**. No deposit required and repayments are made each quarter.

- (a) (i) Find the repayment made each quarter.
- (ii) Find the total amount paid for the car.
- (iii) Find the interest paid on the loan.

[7]

Finance option B:

A 6 year loan at a nominal annual interest rate of $r\%$ **compounded monthly**. Terms of the loan require a 10% deposit and monthly repayments of €250.

- (b) (i) Find the amount to be borrowed for this option.
- (ii) Find the annual interest rate, r .
- (c) State which option Bryan should choose. Justify your answer.

[5]

[2]

Bryan's car depreciates at an annual rate of 25% per year.

- (d) Find the value of Bryan's car six years after it is purchased.

[3]

Question 2

[Maximum mark: 19]

Give your answers in parts (a), (d)(i), (e) and (f) to the nearest dollar.

Daisy invested 37 000 Australian dollars (AUD) in a fixed deposit account with an annual interest rate of 6.4% compounded **quarterly**.

- (a) Calculate the value of Daisy's investment after 2 years. [3]

After m months, the amount of money in the fixed deposit account has appreciated to more than 50 000 AUD.

- (b) Find the minimum value of m , where $m \in \mathbb{N}$. [4]

Daisy is saving to purchase a new apartment. The price of the apartment is 200 000 AUD.

Daisy makes an initial payment of 25% and takes out a loan to pay the rest.

- (c) Write down the amount of the loan. [1]

The loan is for 10 years, compounded monthly, with equal monthly payments of 1700 AUD made by Daisy at the end of each month.

- (d) For this loan, find
- (i) the amount of interest paid by Daisy.
 - (ii) the annual interest rate of the loan. [5]

After 5 years of paying off this loan, Daisy decides to pay the **remainder** in one final payment.

- (e) Find the amount of Daisy's final payment. [3]
- (f) Find how much money Daisy saved by making one final payment after 5 years. [3]

Question 3

[Maximum mark: 16]

A new concert hall was built with 14 seats in the first row. Each subsequent row of the hall has two more seats than the previous row. The hall has a total of 20 rows.

(a) Find:

- (i) the number of seats in the last row.
- (ii) the total number of seats in the concert hall. [5]

The concert hall opened in 2019. The average number of visitors per concert during that year was 584. In 2020, the average number of visitors per concert increased by 1.2%.

(b) Find the average number of visitors per concert in 2020. [2]

The concert organizers use this data to model future numbers of visitors. It is assumed that the average number of visitors per concert will continue to increase each year by 1.2%.

(c) Determine the first year in which this model predicts the average number of visitors per concert will exceed the total seating capacity of the concert hall. [5]

It is assumed that the concert hall will host 50 concerts each year.

(d) Use the average number of visitors per concert per year to predict the total number of people expected to attend the concert hall from when it opens until the end of 2025. [4]

Question 4

[Maximum mark: 16]

The admissions team at a new university are trying to predict the number of student applications they will receive each year.

Let n be the number of years that the university has been open. The admissions team collect the following data for the first two years.

Year, n	Number of applications received in year n
1	12 300
2	12 669

- (a) Calculate the percentage increase in applications from the first year to the second year. [2]

It is assumed that the number of students that apply to the university each year will follow a geometric sequence, u_n .

- (b) (i) Write down the common ratio of the sequence.
(ii) Find an expression for u_n .
(iii) Find the number of student applications the university expects to receive when $n = 11$. Express your answer to the nearest integer. [4]

In the first year there were 10 380 places at the university available for applicants. The admissions team announce that the number of places available will increase by 600 every year.

Let v_n represent the number of places available at the university in year n .

- (c) Write down an expression for v_n . [2]

For the first 10 years that the university is open, all places are filled. Students who receive a place each pay an \$80 acceptance fee.

- (d) Calculate the total amount of acceptance fees paid to the university in the first 10 years. [3]

When $n = k$, the number of places available will, for the first time, exceed the number of students applying.

- (e) Find k . [3]
(f) State whether, for all $n > k$, the university will have places available for all applicants. Justify your answer. [2]

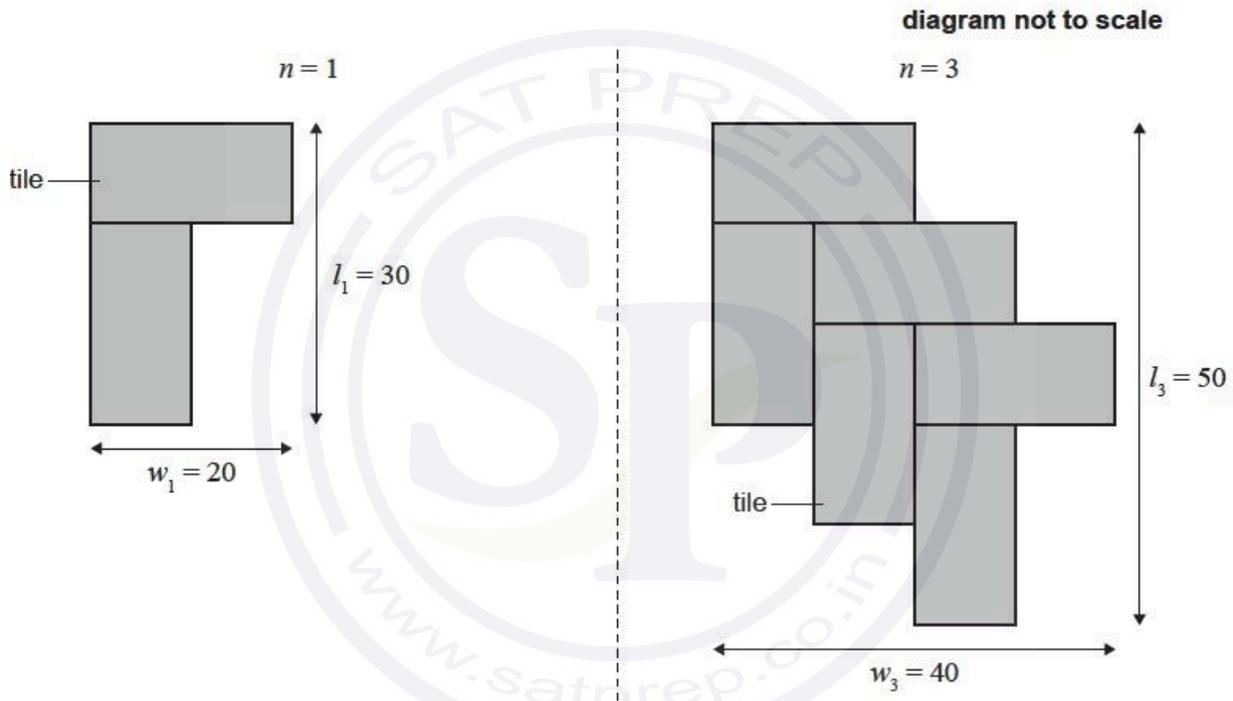
Question 5

[Maximum mark: 19]

Eddie decides to construct a path across his rectangular grass lawn using pairs of tiles.

Each tile is 10 cm wide and 20 cm long. The following diagrams show the path after Eddie has laid one pair and three pairs of tiles. This pattern continues until Eddie reaches the other side of his lawn. When n pairs of tiles are laid, the path has a width of w_n centimetres and a length l_n centimetres.

The following diagrams show this pattern for one pair of tiles and for three pairs of tiles, where the white space around each diagram represents Eddie's lawn.



The following table shows the values of w_n and l_n for the first three values of n .

Number of pairs of tiles, n	Width of lawn crossed by path, w_n (cm)	Length of lawn crossed by path, l_n (cm)
1	20	30
2	a	b
3	40	50

(a) Find the value of

(i) a .

(ii) b .

[2]

(b) Write down an expression in terms of n for

(i) w_n .

(ii) l_n .

[3]

Eddie's lawn has a length 740 cm.

(c) (i) Show that Eddie needs 144 tiles.

(ii) Find the value of w_n for this path.

[3]

(d) Find the total area of the tiles in Eddie's path. Give your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and k is an integer.

[3]

The tiles cost \$24.50 per square metre and are sold in packs of five tiles.

(e) Find the cost of a single pack of five tiles.

[3]

To allow for breakages Eddie wants to have at least 8% more tiles than he needs.

(f) Find the minimum number of packs of tiles Eddie will need to order.

[3]

There is a fixed delivery cost of \$35.

(g) Find the total cost for Eddie's order.

[2]

Question 6

[Maximum mark: 13]

Scott purchases food for his dog in large bags and feeds the dog the same amount of dog food each day. The amount of dog food left in the bag at the end of each day can be modelled by an arithmetic sequence.

On a particular day, Scott opened a new bag of dog food and fed his dog. By the end of the third day there were 115.5 cups of dog food remaining in the bag and at the end of the eighth day there were 108 cups of dog food remaining in the bag.

- (a) Find the number of cups of dog food
- (i) fed to the dog per day;
 - (ii) remaining in the bag at the end of the first day. [4]

- (b) Calculate the number of days that Scott can feed his dog with one bag of food. [2]

In 2021, Scott spent \$625 on dog food. Scott expects that the amount he spends on dog food will increase at an annual rate of 6.4%.

- (c) Determine the amount that Scott expects to spend on dog food in 2025. Round your answer to the nearest dollar. [3]

- (d) (i) Calculate the value of $\sum_{n=1}^{10} (625 \times 1.064^{(n-1)})$.
- (ii) Describe what the value in part (d)(i) represents in this context. [3]

- (e) Comment on the appropriateness of modelling this scenario with a geometric sequence. [1]

Question 7

[Maximum mark: 15]

Daina makes pendulums to sell at a market. She plans to make 10 pendulums on the first day and, on each subsequent day, make 6 more than she did the day before.

- (a) Calculate the number of pendulums she would make on the 12th day. [3]

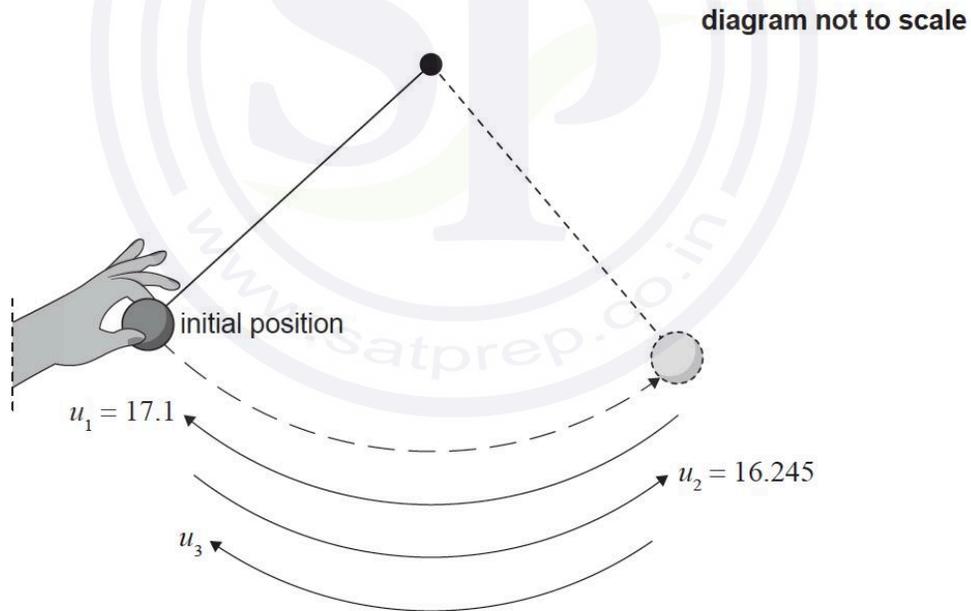
She plans to make pendulums for a **total** of 15 days in preparation for going to the market.

- (b) Calculate the total number of pendulums she would have available at the market. [2]

Daina would like to have at least 1000 pendulums available to sell at the market and therefore decides to increase her production. She still plans to make 10 pendulums on the first day, but on each subsequent day, she will make x more than she did the day before.

- (c) Given that she will still make pendulums for a total of 15 days, calculate the minimum integer value of x required for her to reach her target. [3]

Daina tests one of her pendulums. She releases the ball at the end of the pendulum to swing freely. The point at which she releases it is shown as the initial position on the left side of the following diagram. Daina begins recording the distances travelled by the ball **after** it has reached the extreme position, represented by the right-hand side of the diagram.



On each successive swing, the distance that the ball travelled was 95% of its previous distance. During the first swing that Daina recorded, the ball travelled a distance of 17.1 cm. During the second swing that she recorded, it travelled a distance of 16.245 cm.

- (d) Calculate the distance that the ball travelled during the 5th recorded swing. [3]
- (e) Calculate the total distance that the ball travelled during the first 16 recorded swings. [2]
- (f) Calculate the distance that the ball travelled before Daina started recording. [2]

Question 8

[Maximum mark: 16]

Ruben wants to buy a car for a price of 285 000 South African rand (ZAR). He goes to a bank to get a loan to buy the car. To be eligible for the loan, Ruben must make an initial down payment equal to 25% of the price of the car.

The bank offers him a 5-year loan for the remaining balance, with a 4.5% nominal interest rate per annum, compounded monthly. Ruben will pay the loan in fixed payments at the end of each month.

- (a) (i) Find the original amount of the loan after the down payment is paid. Give the exact answer.
- (ii) Calculate Ruben's monthly payment for this loan, to two decimal places. [5]
- (b) Using your answer from part (a)(ii), calculate the total amount Ruben will pay over the life of the loan, to the nearest ZAR. Do **not** include the initial down payment. [2]

Ruben would like to repay the loan faster and increases his payments such that he pays 4600 ZAR each month.

- (c) Find the total number of monthly payments he will need to make to pay off the loan. [2]

This strategy will result in Ruben's final payment being less than 4600 ZAR.

- (d) Determine the amount of Ruben's final payment, to two decimal places. [4]
- (e) Hence, determine the total amount Ruben will save, to the nearest ZAR, by making the higher monthly payments. [3]

Question 9

[Maximum mark: 16]

Tiffany wants to buy a house for a price of 285 000 US Dollars (USD). She goes to a bank to get a loan to buy the house. To be eligible for the loan, Tiffany must make an initial down payment equal to 15% of the price of the house.

The bank offers her a 30-year loan for the remaining balance, with a 4% nominal interest rate per annum, compounded monthly. Tiffany will pay the loan in fixed payments at the end of each month.

- (a) (i) Find the original amount of the loan after the down payment is paid.
Give the exact answer.
- (ii) Calculate Tiffany's monthly payment for this loan, to two decimal places. [5]
- (b) Using your answer from part (a)(ii), calculate the total amount Tiffany will pay over the life of the loan, to the nearest dollar. Do **not** include the initial down payment. [2]

Tiffany would like to repay the loan faster and increases her payments such that she pays 1300USD each month.

- (c) Find the total number of monthly payments she will need to make to pay off the loan. [2]

This strategy will result in Tiffany's final payment being less than 1300USD.

- (d) Determine the amount of Tiffany's final payment, to two decimal places. [4]
- (e) Hence, determine the total amount Tiffany will save, to the nearest dollar, by making the higher monthly payments. [3]