SATPREP

Assignment : Complex Number

1. Let $z = x + yi$. Find the values of x and y if $(1 - i)z = 1 - 3i$.

2. Let $z_1 = \dfrac{\sqrt{6} - i\sqrt{2}}{2}$, and $z_2 = 1 - i$.

(a) Write z_1 and z_2 in the form $r(\cos\theta + i\sin\theta)$, where $r > 0$ and $-\dfrac{\pi}{2} \le \theta \le \dfrac{\pi}{2}$.

(b) Show that $\dfrac{z_1}{z_2} = \cos\dfrac{\pi}{12} + i\sin\dfrac{\pi}{12}$.

(c) Find the value of $\dfrac{z_1}{z_2}$ in the form $a + bi$, where a and b are to be determined exactly in radical (surd) form. Hence or otherwise find the exact values of $\cos\dfrac{\pi}{12}$ and $\sin\dfrac{\pi}{12}$.

3. Find the values of a and b, where a and b are real, given that $(a + bi)(2 - i) = 5 - i$.

4. Given that $z \in \mathbb{C}$, solve the equation $z^3 - 8i = 0$, giving your answers in the form $z = r(\cos\theta + i\sin\theta)$.

5. The complex number z satisfies the equation

$$\sqrt{z} = \frac{2}{1 - i} + 1 - 4i$$

Express z in the form $x + iy$ where $x, y \in \mathbb{Z}$.

6. Let the complex number z be given by

$$z = 1 + \frac{i}{i - \sqrt{3}}.$$

Express z in the form $a + bi$, giving the **exact** values of the real constants a, b.

7. Given that $|z| = 2\sqrt{5}$, find the complex number z that satisfies the equation

$$\frac{25}{z} - \frac{15}{z^*} = 1 - 8i.$$