# SATPREP

## Assignment : Complex Number

**1.** (a) Express the complex number 8i in polar form.

(b) The cube root of 8i which lies in the first quadrant is denoted by $z$. Express $z$

(i) in polar form;

(ii) in cartesian form.

**2.** Let $P(z) = z^3 + az^2 + bz + c$, where $a$, $b$, and $c \in \mathbb{R}$. Two of the roots of $P(z) = 0$ are $-2$ and $(-3 + 2\text{i})$. Find the value of $a$, of $b$ and of $c$.

**3.** $(z + 2\text{i})$ is a factor of $2z^3 - 3z^2 + 8z - 12$. Find the other two factors.

**4.** (a) Express $z^5 - 1$ as a product of two factors, one of which is linear.

(b) Find the zeros of $z^5 - 1$, giving your answers in the form $r(\cos\theta + \text{i}\sin\theta)$ where $r > 0$ and $-\pi < 6 \le \pi$.

(c) Express $z^4 + z^3 + z^2 + z + 1$ as a product of two real quadratic factors.

**5.** (a) Prove, using mathematical induction, that for a positive integer $n$,

$$(\cos\theta + \text{i}\sin\theta)^n = \cos n\theta + \text{i}\sin n\theta \quad \text{where } \text{i}^2 = -1.$$

(b) The complex number $z$ is defined by $z = \cos\theta + \text{i}\sin\theta$.

(i) Show that $\dfrac{1}{z} = \cos(-\theta) + \text{i}\sin(-\theta)$.

(ii) Deduce that $z^n + z^{-n} = 2\cos n\theta$.

(c) (i) Find the binomial expansion of $(z + z^{-1})^5$.

(ii) Hence show that $\cos^5\theta = \dfrac{1}{16}(a\cos 5\theta + b\cos 3\theta + c\cos\theta)$, where $a$, $b$, $c$ are positive integers to be found.

**6.** Consider the complex number $z = \cos\theta + \text{i}\sin\theta$.

(a) Using De Moivre's theorem show that

$$z^n + \frac{1}{z^n} = 2\cos n\theta.$$

(b) By expanding $\left(z + \dfrac{1}{z}\right)^4$ show that

$$\cos^4\theta = \frac{1}{8}(\cos 4\theta + 4\cos 2\theta + 3).$$