SATPREP

Assignment : Continuous Random Variable

1. A discrete random variable X has its probability distribution given by

$$P(X = x) = k(x + 1), \text{ where } x \text{ is } 0, 1, 2, 3, 4.$$

(a) Show that $k = \frac{1}{15}$.

(b) Find E(X).

2. The random variable X has probability density function f where

$$f(x) = \begin{cases} kx(x+1)(2-x), & 0 \le x \le 2 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Sketch the graph of the function. You are not required to find the coordinates of the maximum.

(b) Find the value of k.

c

3. A continuous random variable X has probability density function

$$f(x) = \begin{cases} 0, & x < 0 \\ ae^{-ax}, & x \ge 0. \end{cases}$$

It is known that $P(X < 1) = 1 - \frac{1}{\sqrt{2}}$.

(a) Show that $a = \frac{1}{2}\ln 2$.

(b) Find the median of X.

(c) Calculate the probability that $X < 3$ given that $X > 1$.

4. A continuous random variable X has the probability density function f given by

$$f(x) = \begin{cases} c(x - x^2), & 0 \le x \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Determine c.

(b) Find E(X).

5. The random variable T has the probability density function

$$f(t) = \frac{\pi}{4}\cos\left(\frac{\pi t}{2}\right), -1 \le t \le 1.$$

Find

(a) $P(T = 0)$;

(b) the interquartile range.

7. The probability density function of the random variable X is given by

$$f(x) = \begin{cases} \frac{k}{\sqrt{4 - x^2}}, & \text{for } 0 \le x \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the value of the constant k.

(b) Show that $E(X) = \frac{6(2 - \sqrt{3})}{\pi}$.

(c) Determine whether the median of X is less than $\frac{1}{2}$ or greater than $\frac{1}{2}$.